Quantum frame covariance and subsystem relativity

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Quantum Information and Quantum Matter
@ NYU Abu Dhabi
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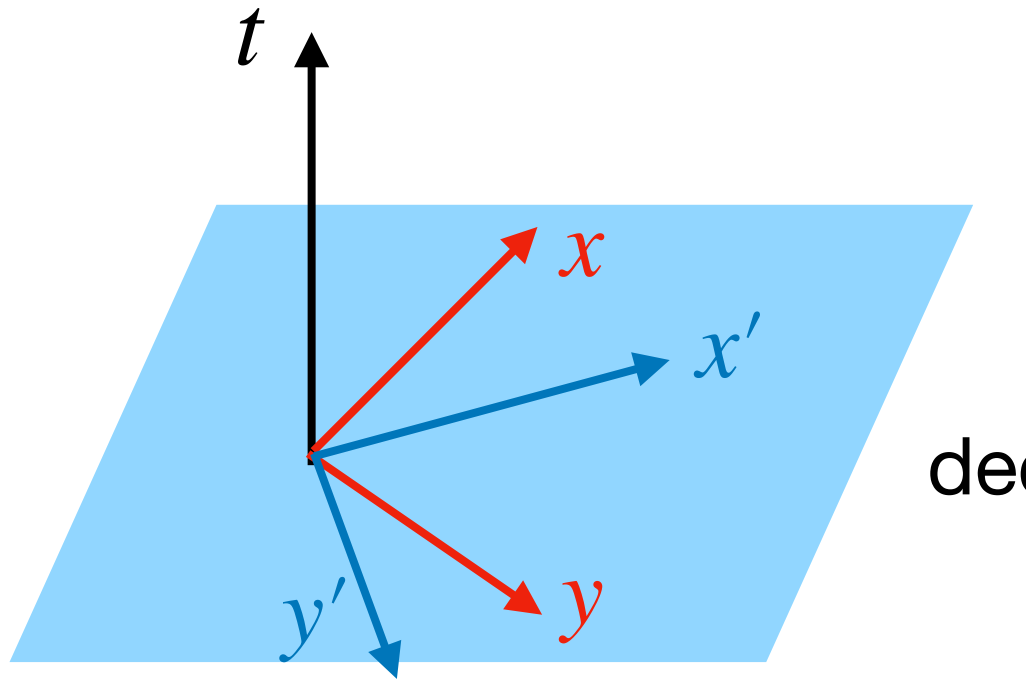
loosely based on:

Ahmad, Galley, PH, Lock, Smith, PRL 128 (2022) 170401
de la Hamette, Galley, PH, Loveridge, Müller 2110.13824
Vanrietvelde, PH, Giacomini, Castro-Ruiz, Quantum 4 (2020) 225
Mele, Kotecha, PH, *to appear soon*

Relativities

Galilean

"All the laws of mechanics the same in every inertial frame"



frame-dependent decomposition of space

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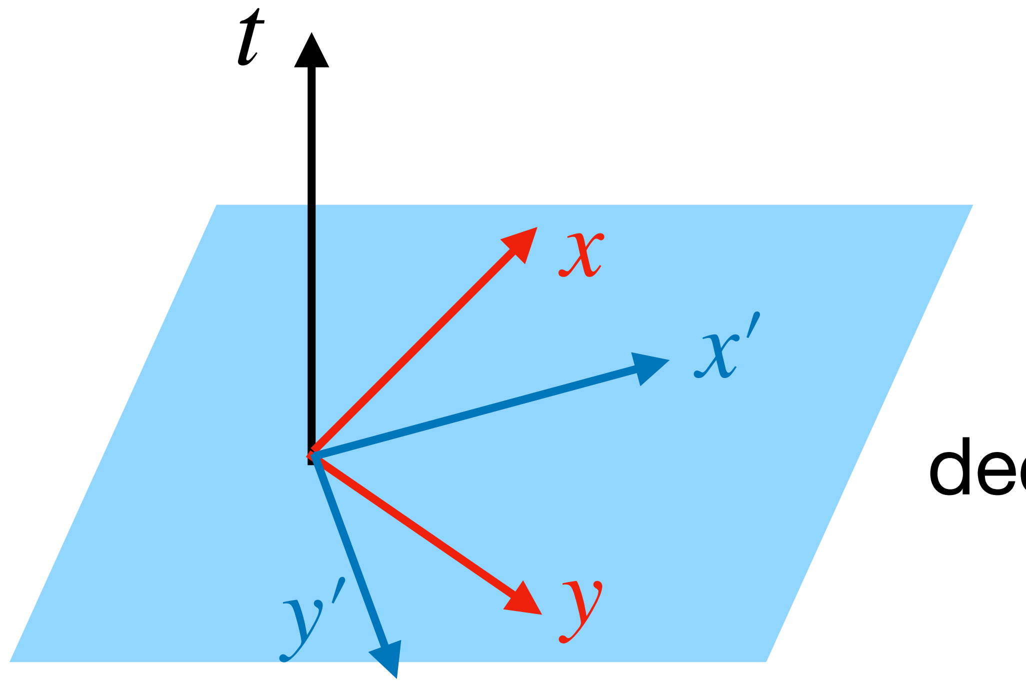
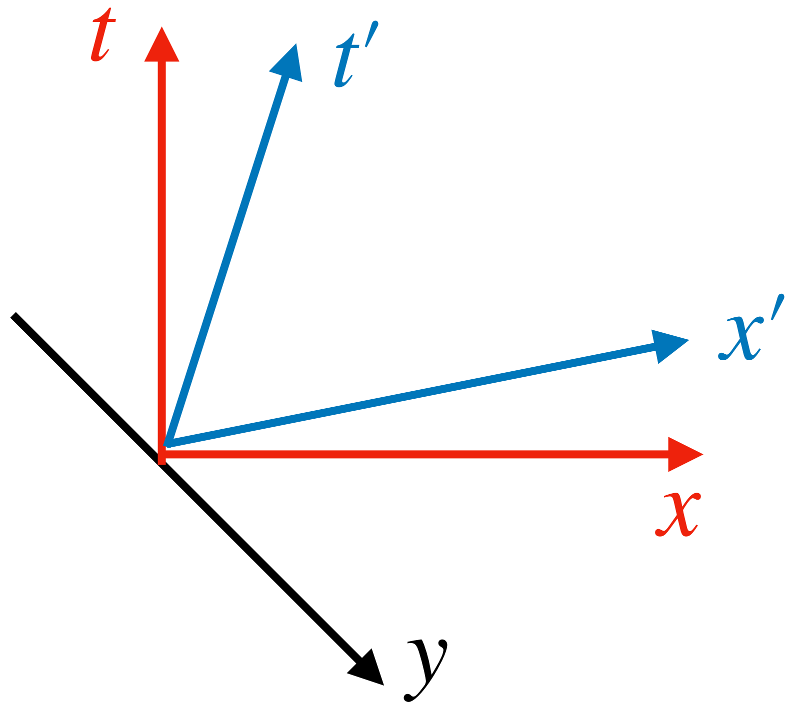


frame-dependent decomposition of space

$$\frac{1}{c} \neq 0$$

Special

"All the laws of physics (exc. Newt. gravity) the same in every inertial frame"



frame-dependent decomposition of spacetime into space and time

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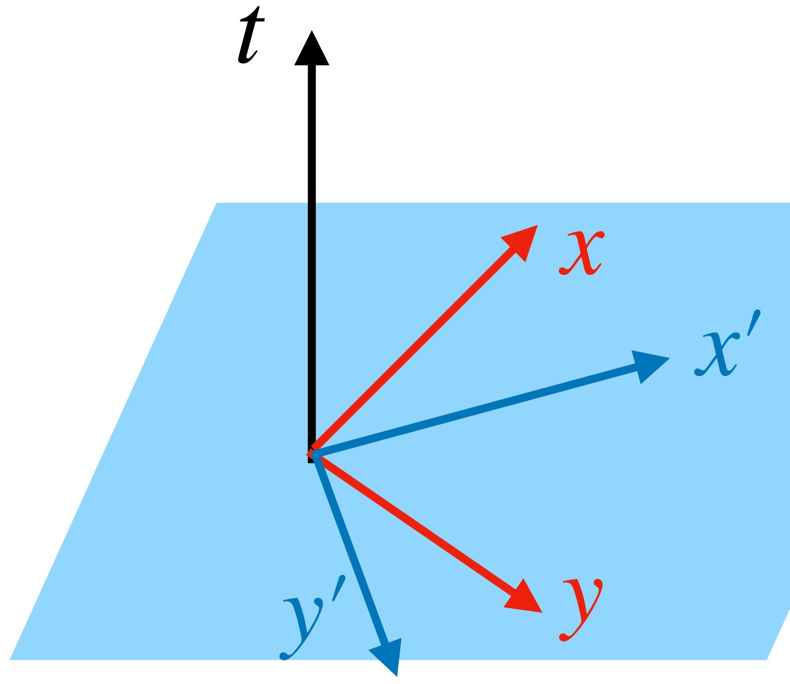

frame-dependent decomposition of space

$$\xrightarrow{\frac{1}{c} \neq 0}$$

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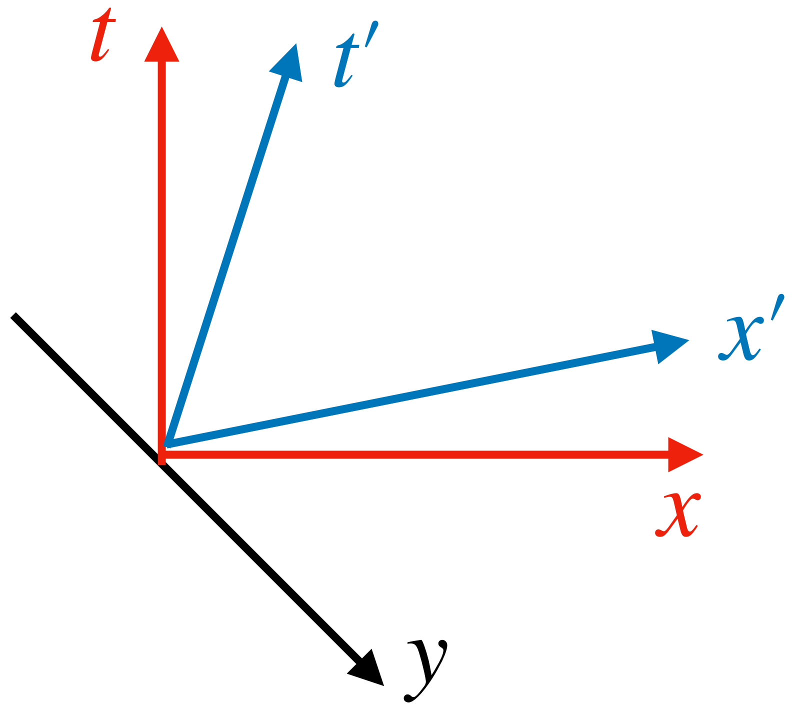
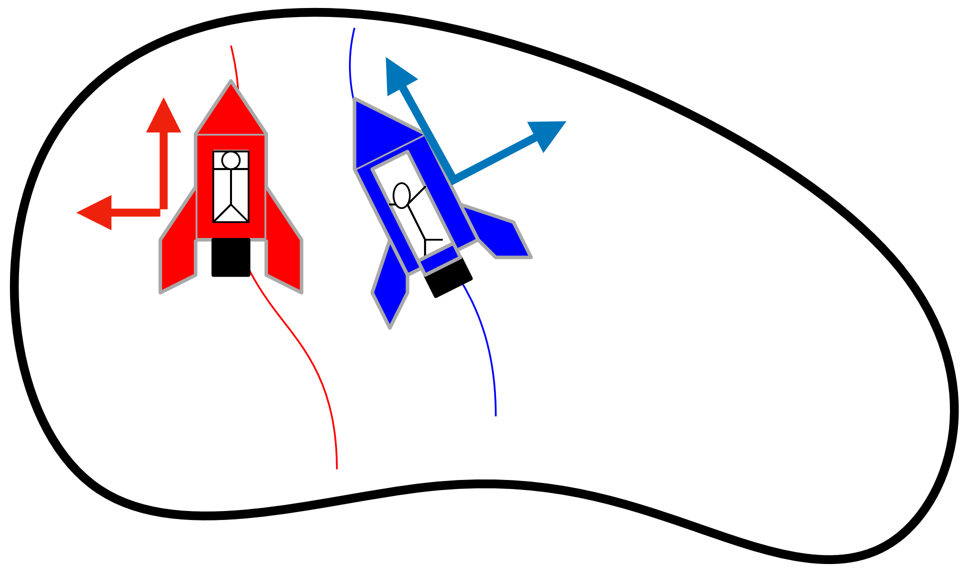


frame-dependent decomposition of spacetime into space and time

$$\Big\downarrow G \neq 0$$

General

"All the laws of physics the same in every frame"



frame-dependent local decomposition of spacetime into space and time

Relativities

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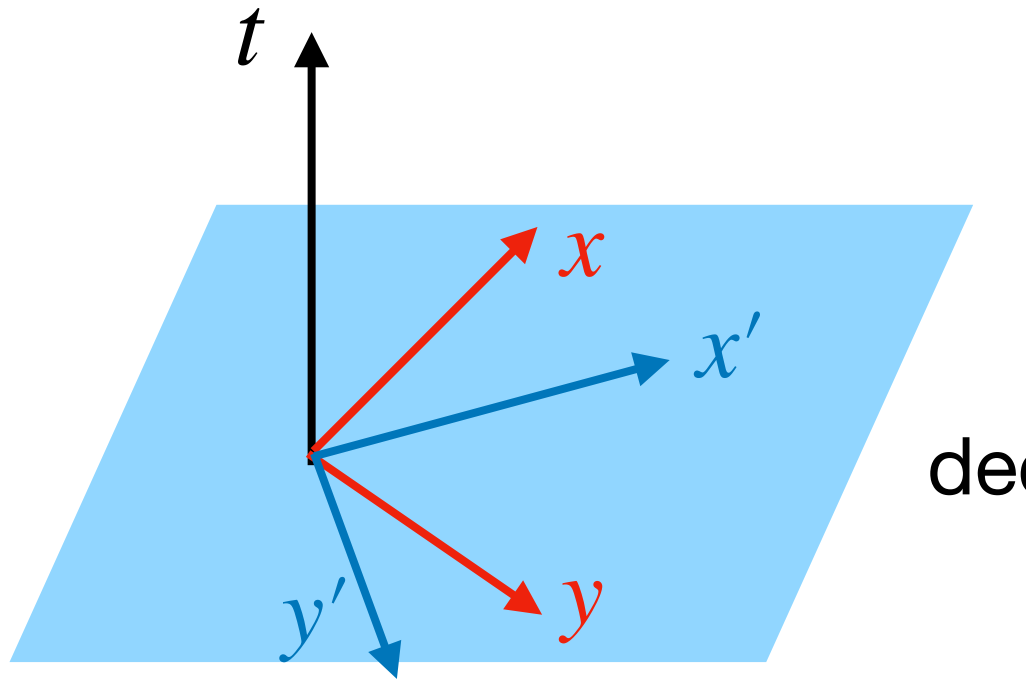

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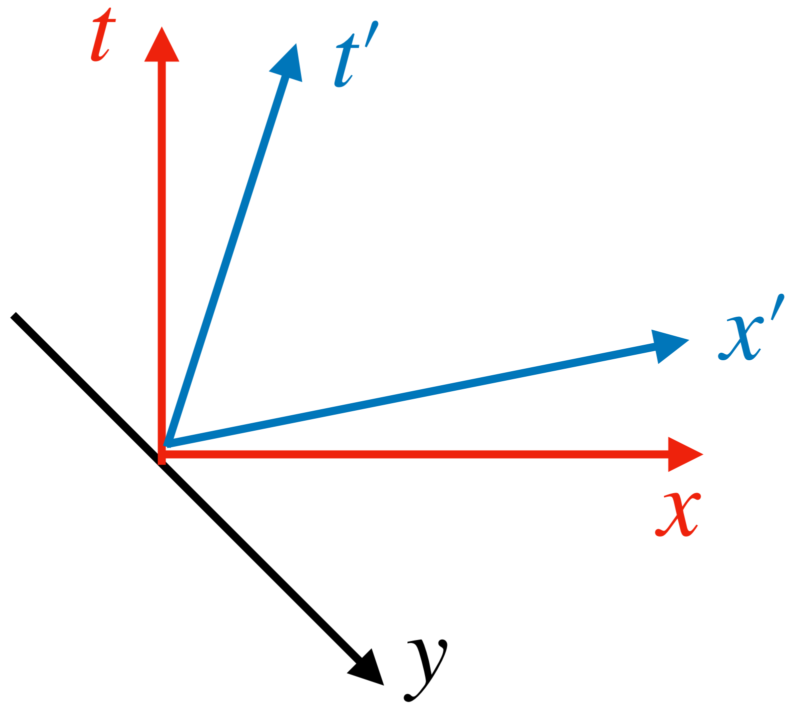


frame-dependent decomposition of spacetime into space and time

$$G \neq 0$$

$$\hbar \neq 0$$

Quantum?

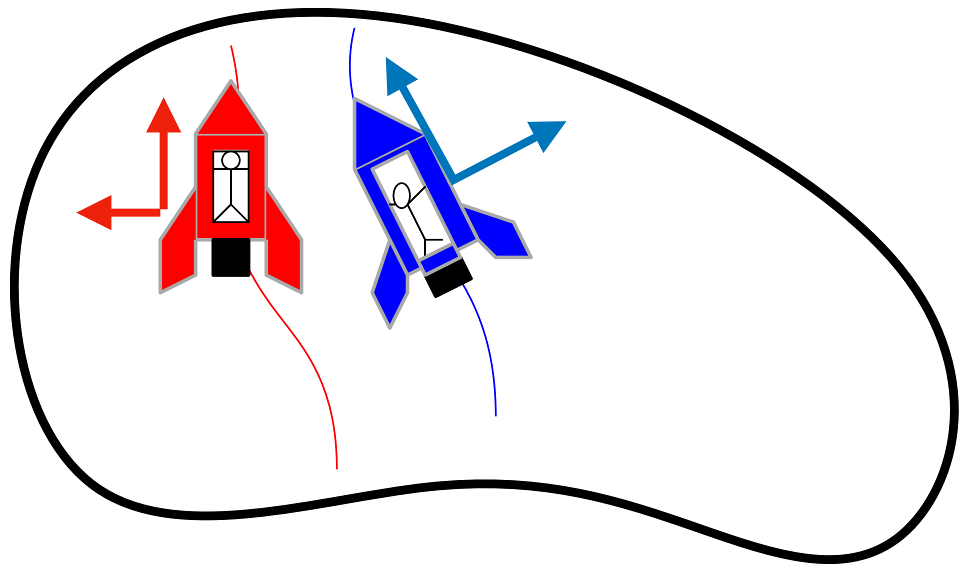
“All the … laws of … the same in every … QRF”

?

QRF-dependent decomposition of ….

General

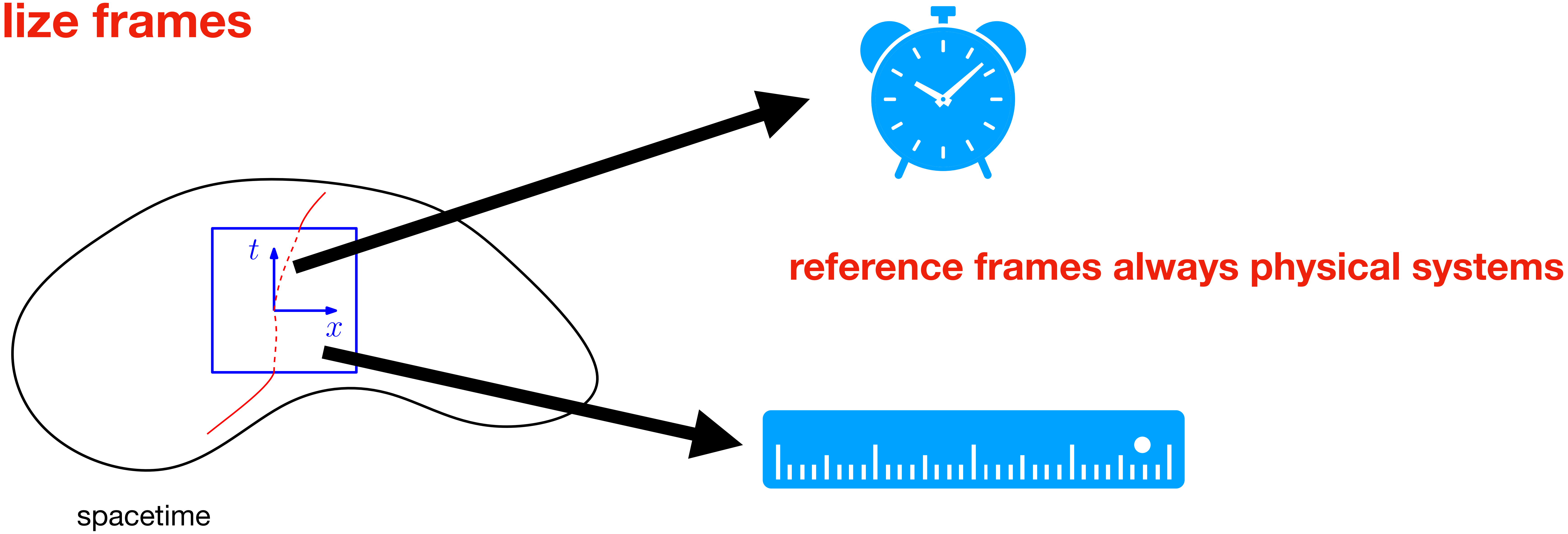
“All the laws of physics the same in every frame”



frame-dependent local decomposition of spacetime into space and time

Quantum reference frames

internalize frames



- no background structure
- universality of QT (ext. of Heisenberg cut) $\Rightarrow$ RF subject to QM itself

"RFs in relative superposition"

Why care?

- **Foundational interest**
  classical frame relations ⇔ classical spacetime structure

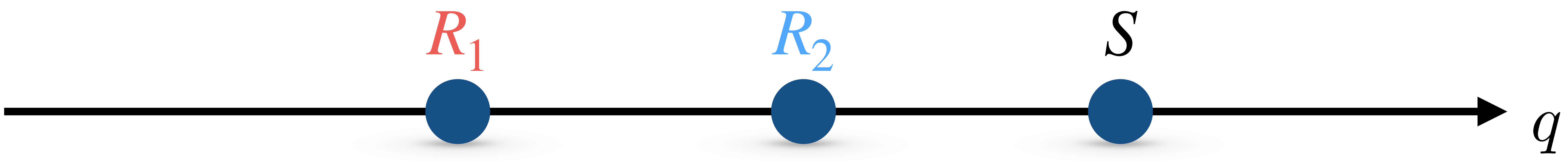
  ⇒ **quantum frame relations ⇔ quantum spacetime structure?**

- **systems with gauge symmetry (gauge-inv. descriptions implicitly invoke internal frames)**

- **gravity: no background frame**

- **quantum info: agents may not share a common external lab frame**

Intuition: spatial QRFs in 1D



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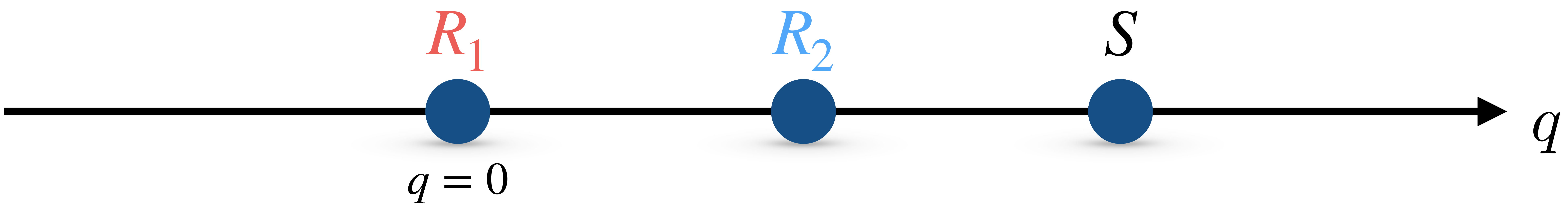

aim: "jump" into perspective of particle R_1, defining origin, and describe R_2S relative to it

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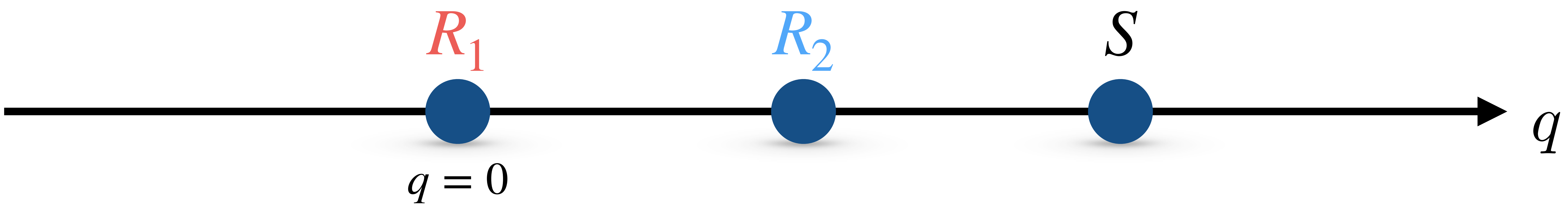

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$$|q_1\rangle_{R_2} \otimes |x\rangle_S$$

R_1 perspective

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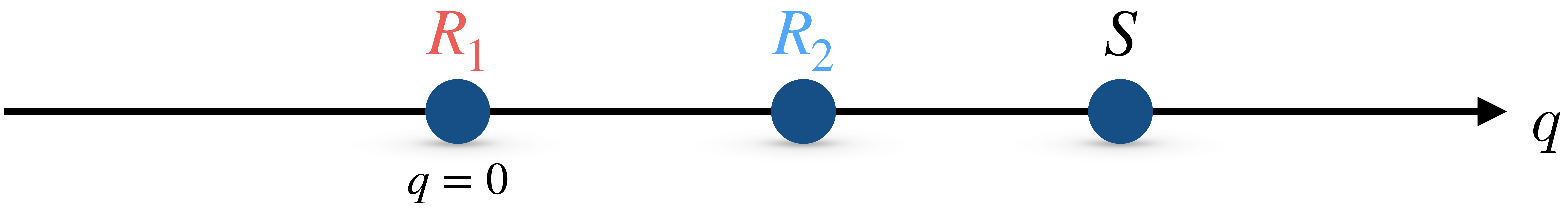

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how will R_2 "see" the same configuration?

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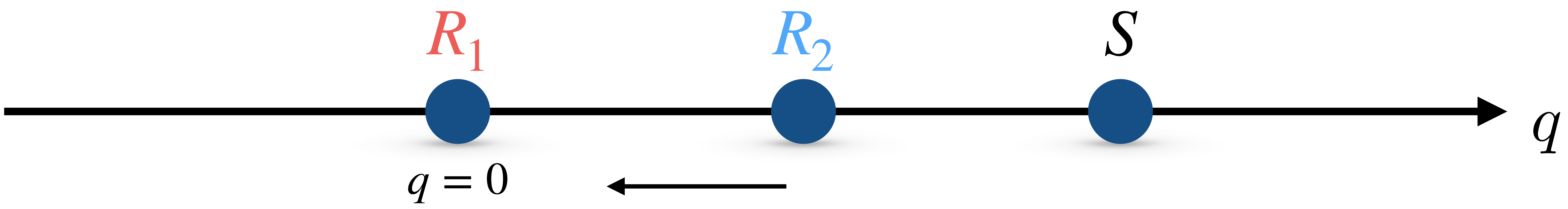

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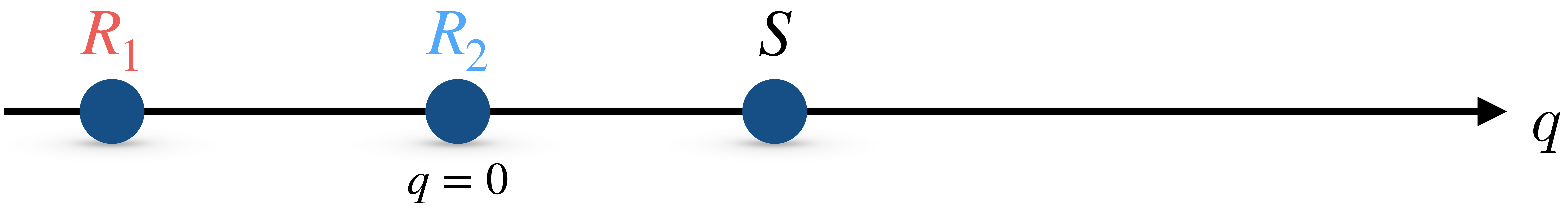

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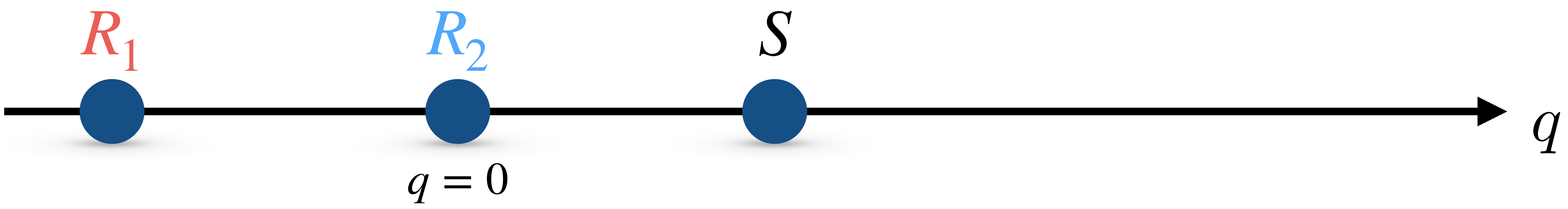

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$$|q_1\rangle_{R_2} \otimes |x\rangle_S \longrightarrow |-q_1\rangle_{R_1} \otimes |x - q_1\rangle_S$$

R_1 perspective $\qquad$ R_2 perspective

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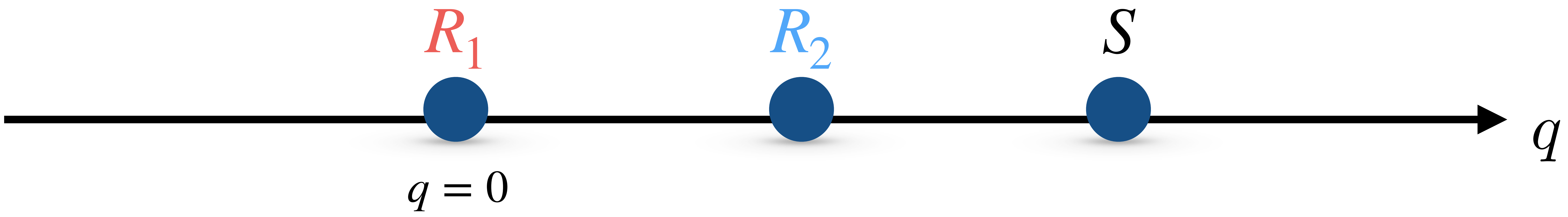

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R_1 perspective

R_2 perspective

how will R_2 "see" the same configuration? $\Rightarrow$ **what about superpositions?**

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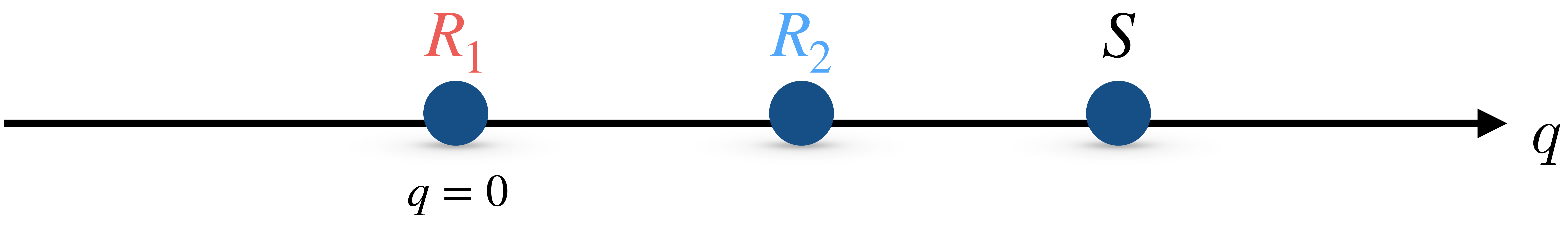

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assuming linearity

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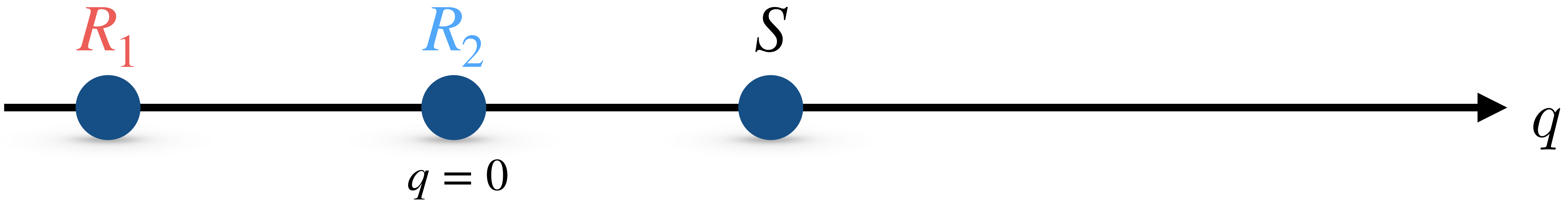

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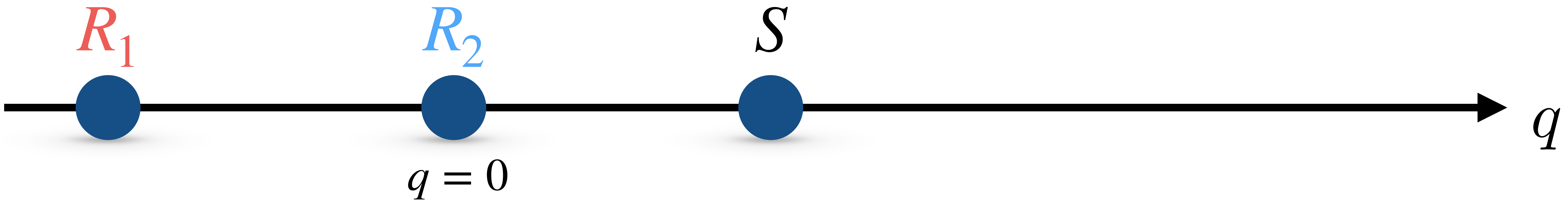

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R_1 perspective

R_2 perspective

QRF transformation a conditional unitary:

$$V_{R_1\to R_2} = \mathbb{F}_{12}\int dq\,|-q\rangle\langle q|_{R_2} \otimes U_S(-q)$$

swaps particles R_2 and R_1

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[Giacomini et al Nat. Comm. '19]

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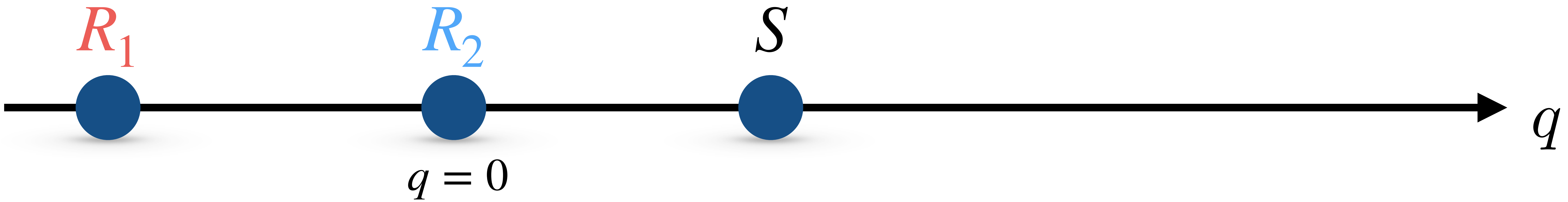

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Example illustrates: superposition and entanglement of subsystem S QRF relative

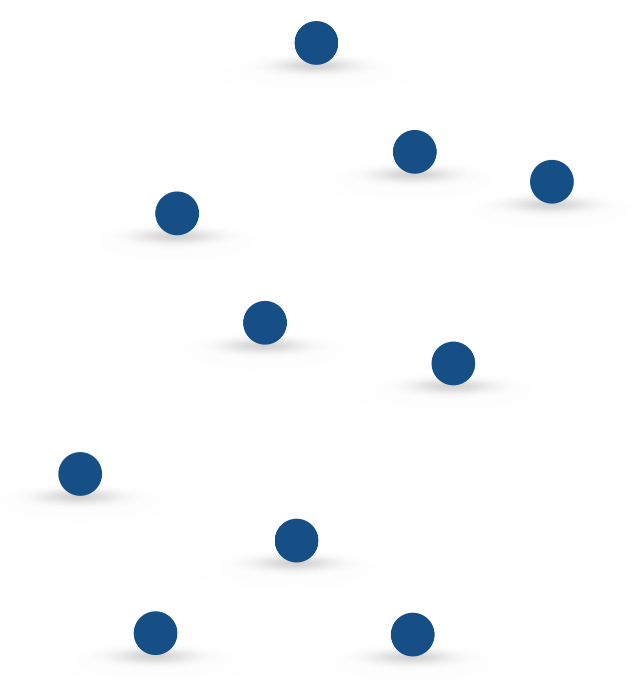
The story more generally

RFs and symmetries

Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

RFs and symmetries



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pair (G, S) could be, e.g.:

- spatial symmetry + group of particles
- diffeos + all dynamical fields in spacetime
- …

RFs and symmetries

external frame

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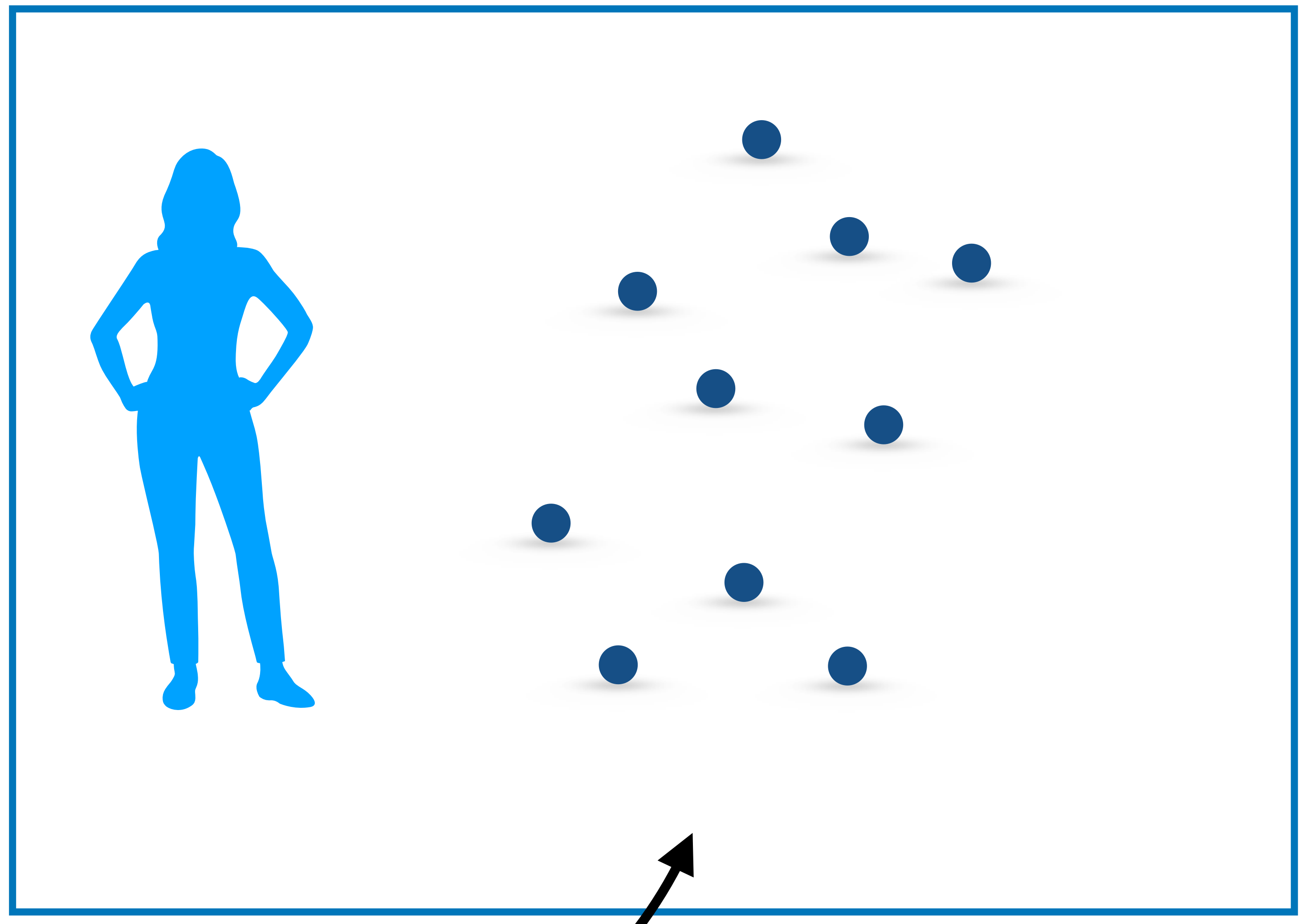

quantum information/foundations:
lab frame

gravity:
fictitious (or edge modes)

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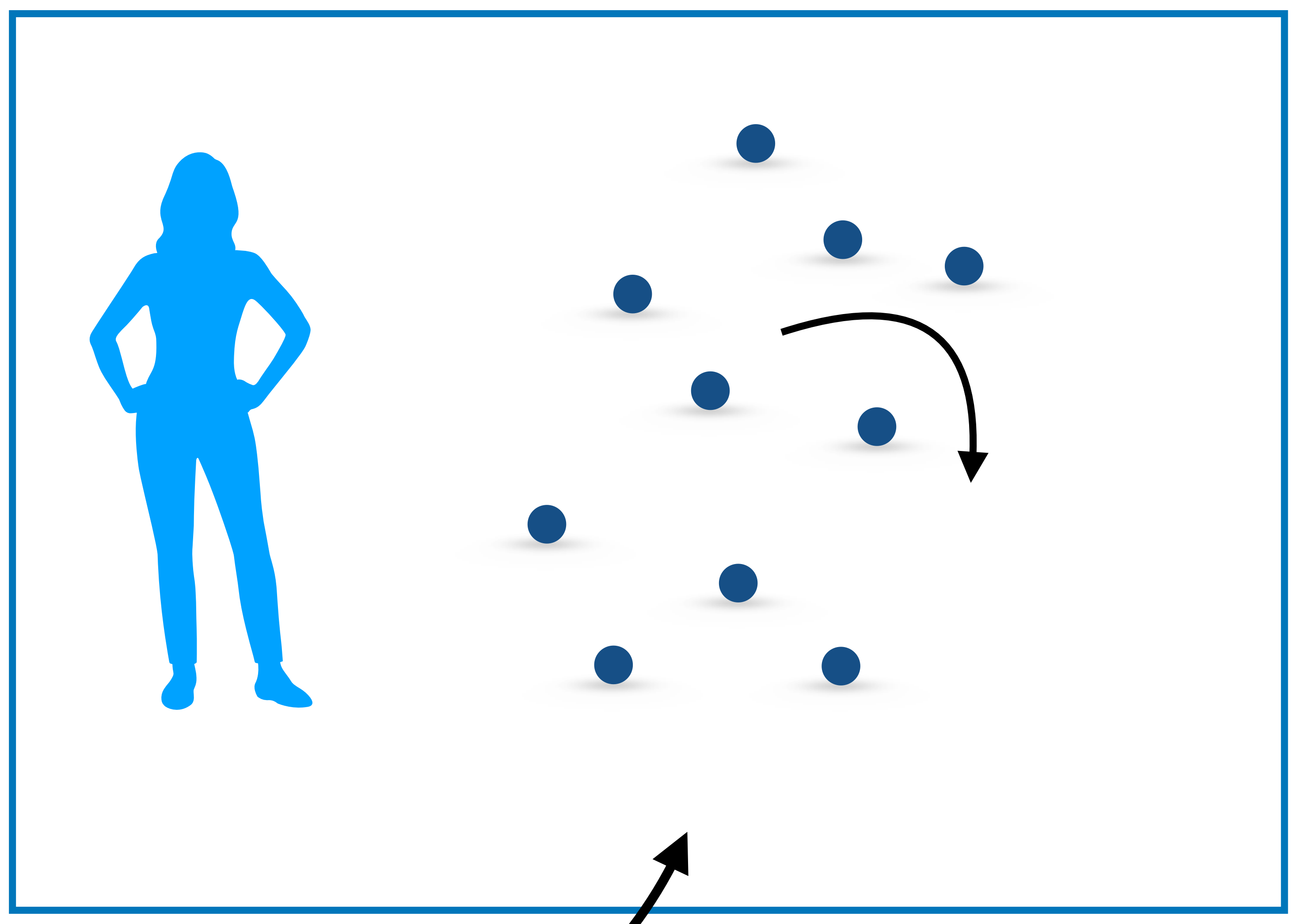

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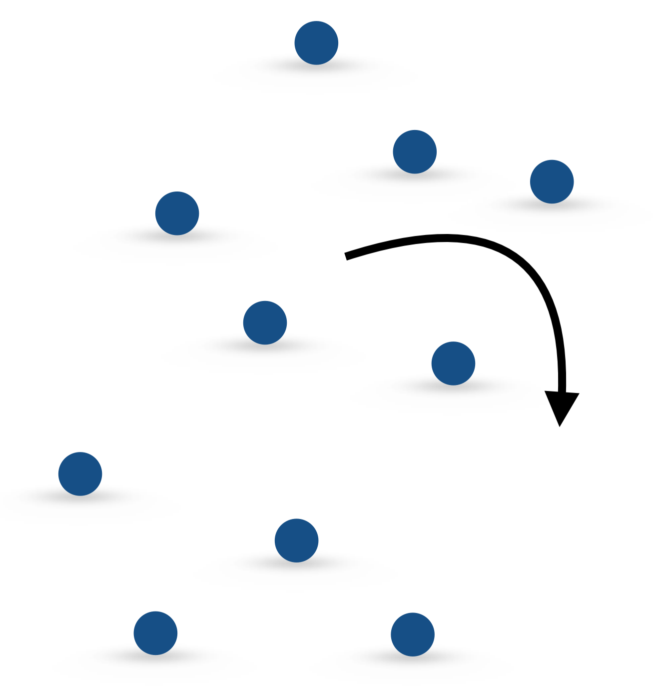

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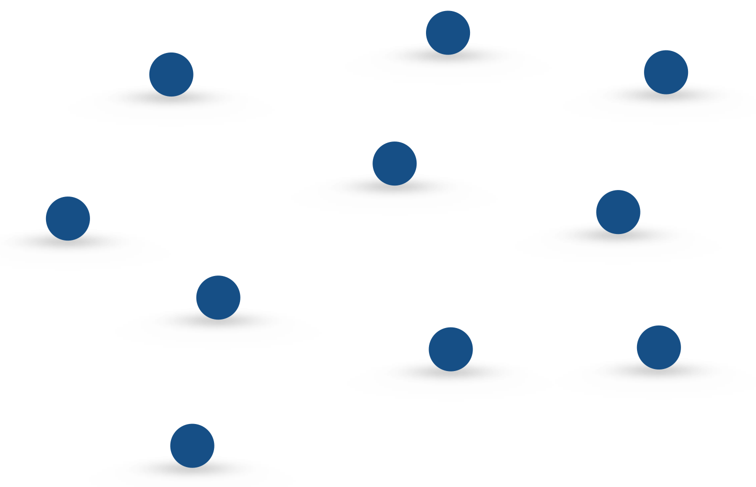

RFs and symmetries

G-transformations = ext. frame transf. = "gauge transformations"

$\Rightarrow$ "gauge inv." = external frame indep.

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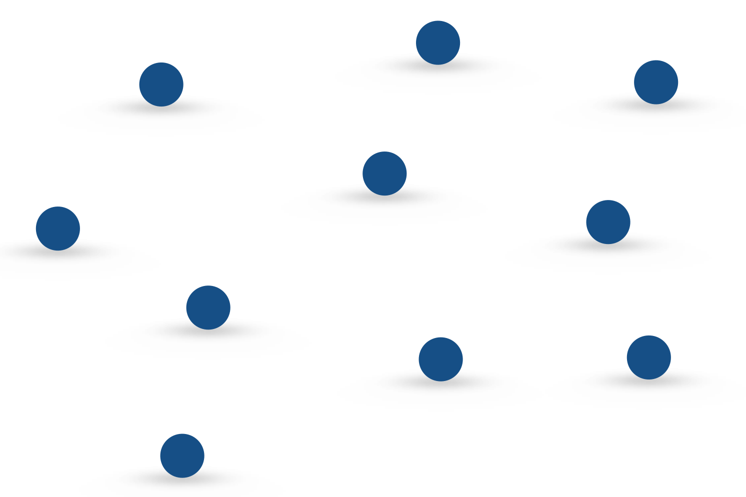

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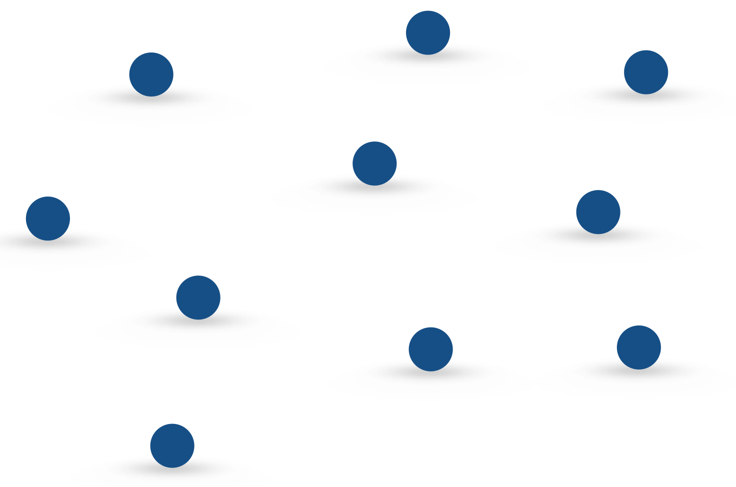

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quantum information/foundations:
change of ext. lab frame

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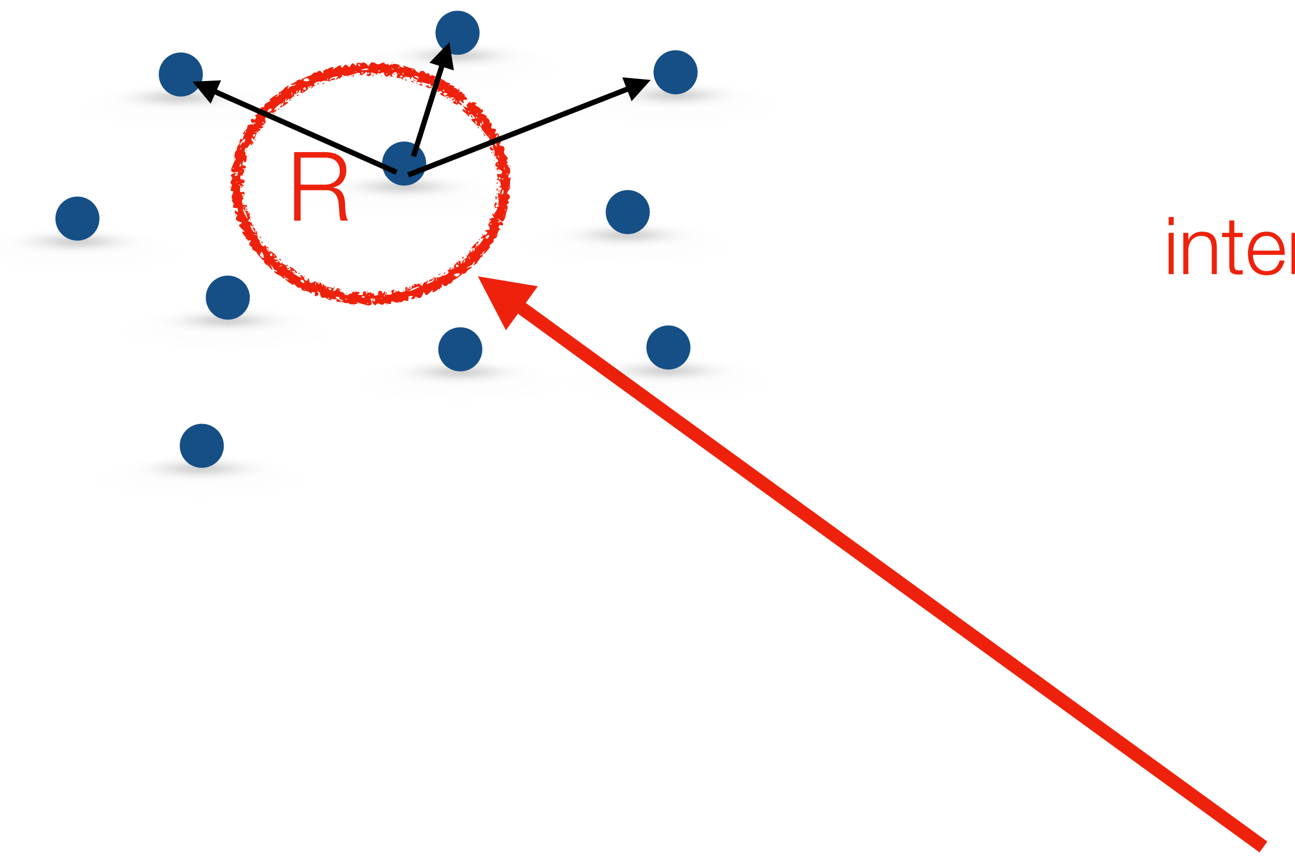

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Describe S relative to internal reference subsystem R

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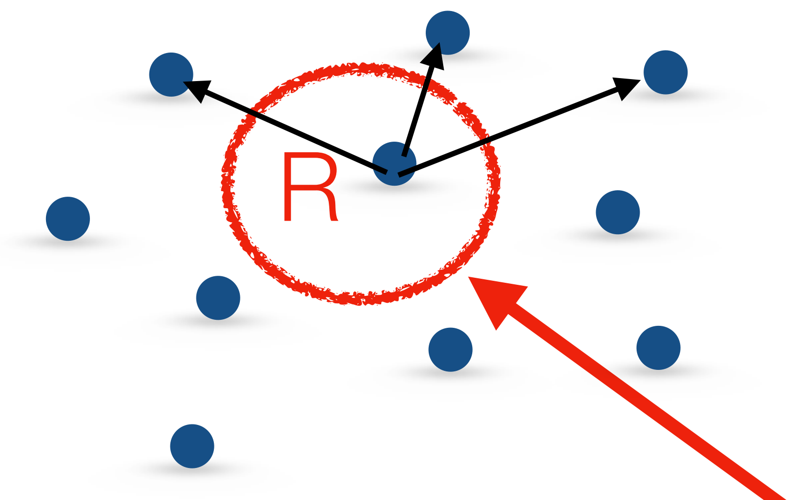

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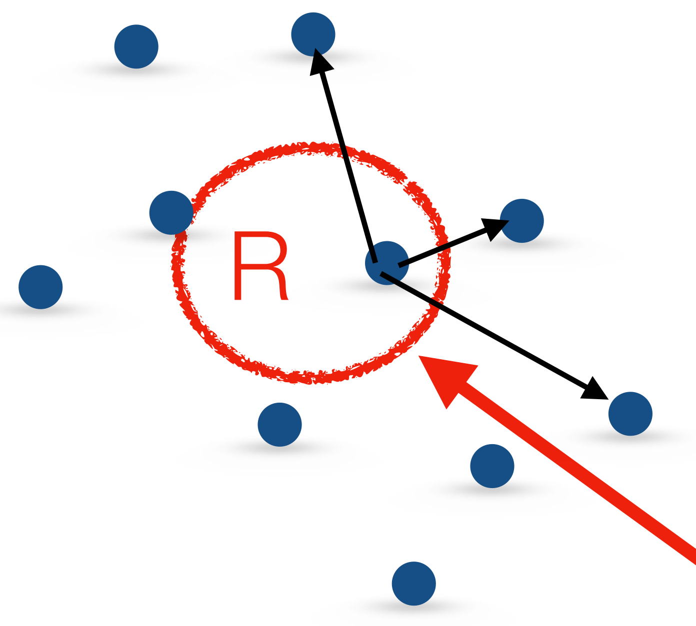

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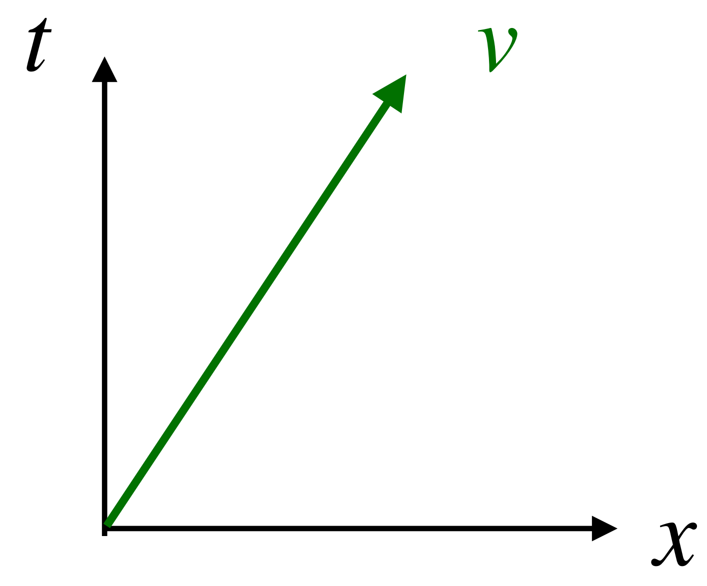
ρ and $g \cdot \rho$ members of same relational equivalence class of states, <u>different descriptions</u> of same relational state

Example: Special relativity with internal frames



$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in \mathrm{SO}_+(3{,}1)$, **internally indistinguishable**

fictitious/external coord. frame

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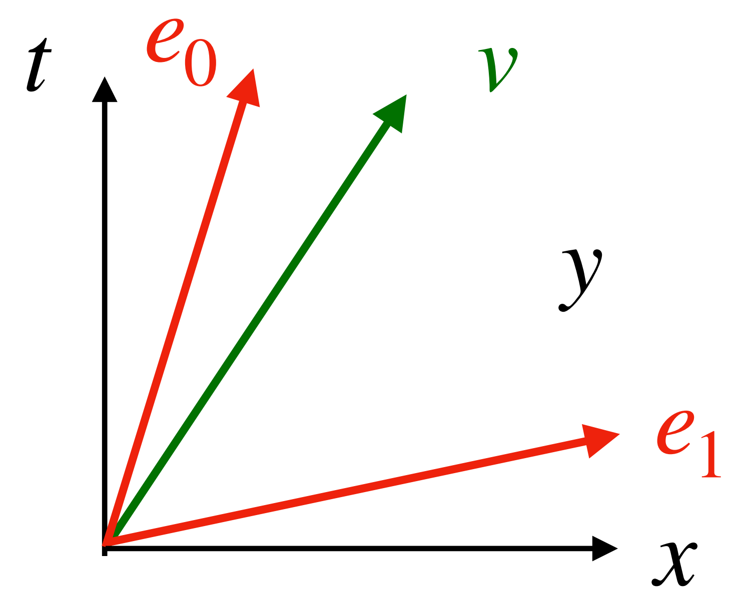

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introduce internal frame (tetrad)

$e_a^\mu \qquad \mu = t, x, y, z$ spacetime index, $\quad a = 0,1,2,3$ frame index

frame orientations

fictitious/external coord. frame

$\Rightarrow$ **group acts on itself since** $\quad \eta_{ab} = e_a^\mu \, e_b^\nu \, \eta_{\mu\nu} \quad \Rightarrow \quad e_a^\mu \in \mathrm{SO}_+(3,1)$ **group valued frame orientations**

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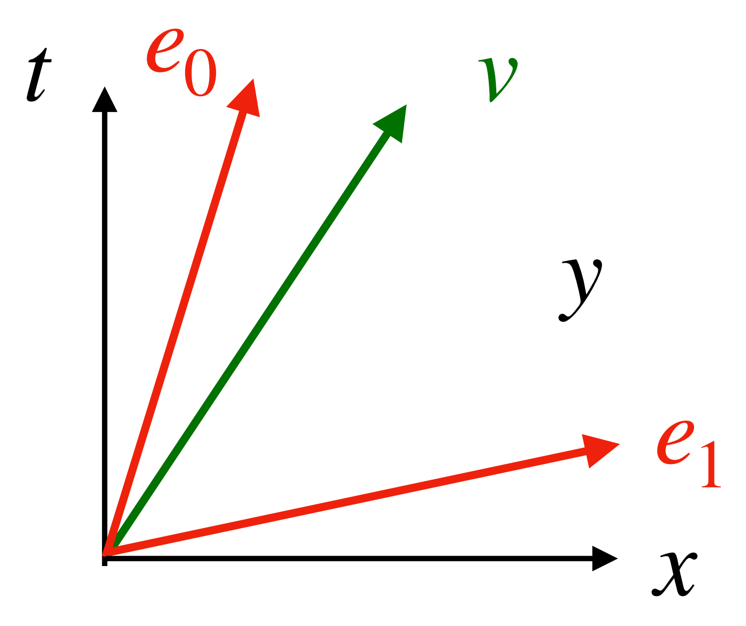

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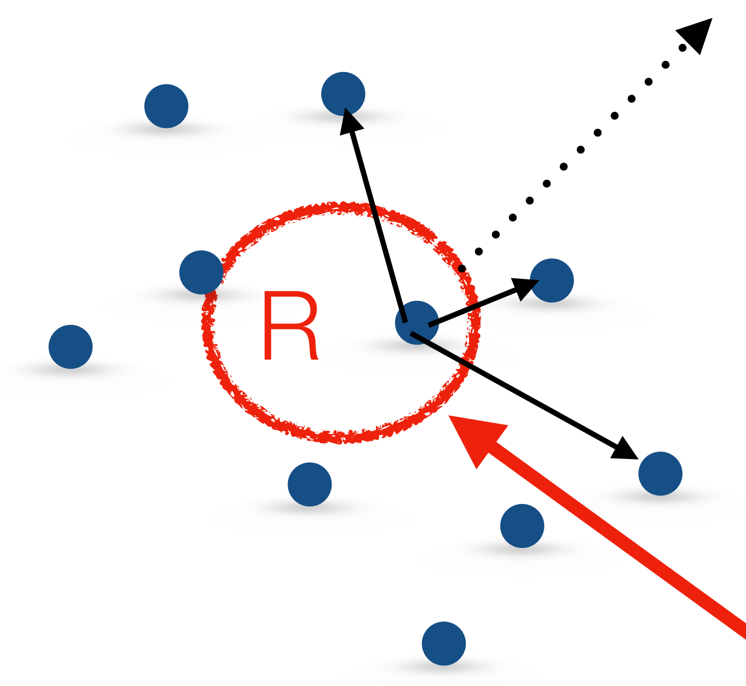

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Interested in internally distinguishable (relational) states/ observables

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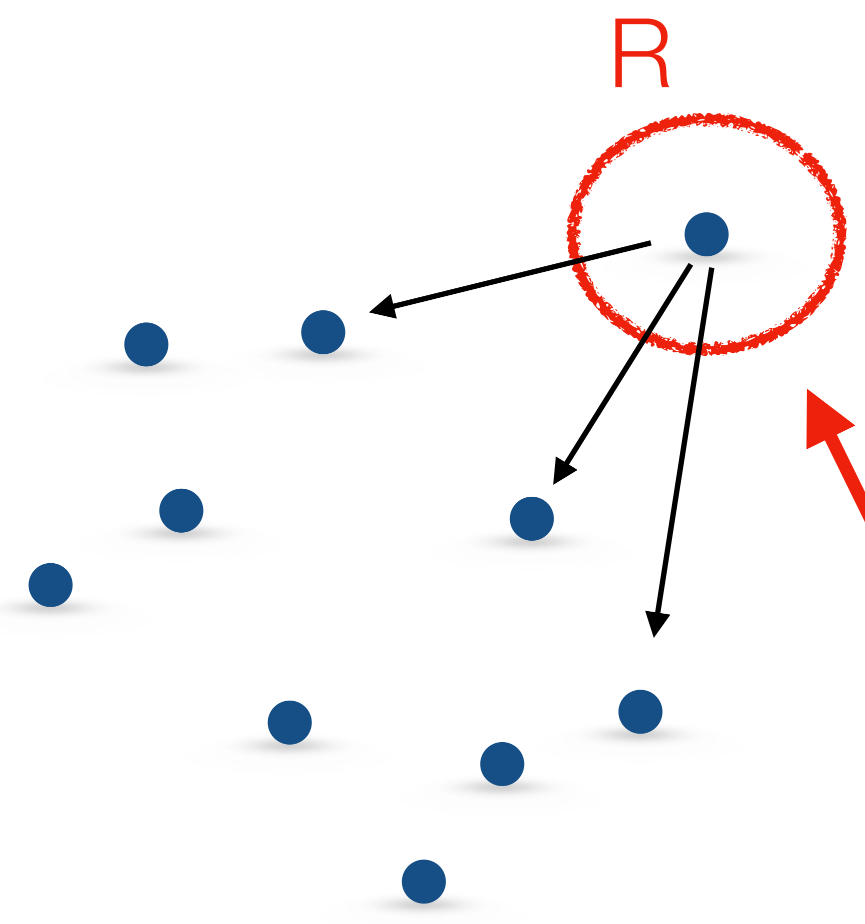

internally distinguishable
(relations changed)

$\Rightarrow$ **internal frame reorientation (“symmetry”)**

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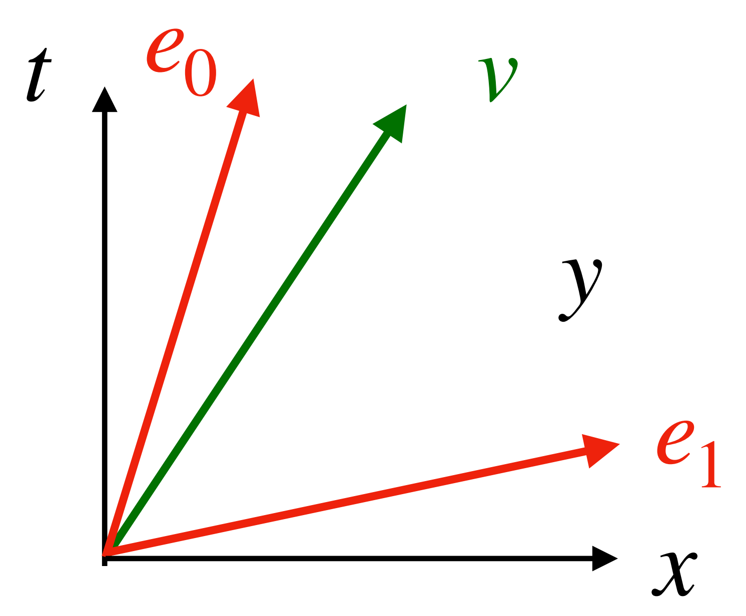

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frame orientations

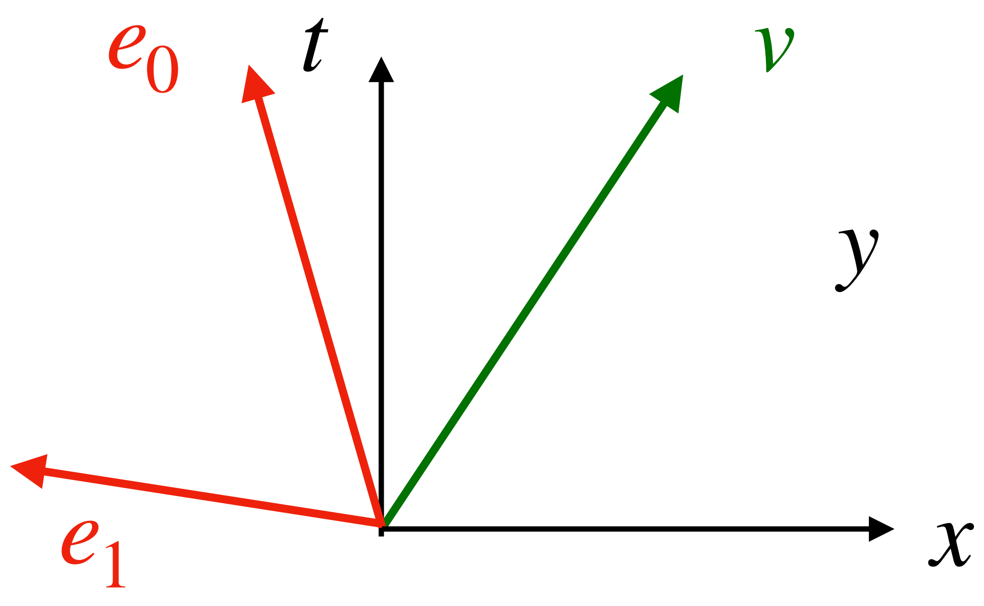
2 indices, 2 commuting group actions:

- **"gauge transformations":** $\Lambda^\mu{}_\nu \, e^\nu_a \qquad \Lambda^\mu{}_\nu \in SO_+(3,1)$
- **"symmetries" (frame reorientations):** $\Lambda_a{}^b \, e^\mu_b \qquad \Lambda_a{}^b \in SO_+(3,1)$

only acts on frame

$\Rightarrow$ **group acts on itself since** $\quad \eta_{ab} = e^\mu_a \, e^\nu_b \, \eta_{\mu\nu} \quad \Rightarrow \quad e^\mu_a \in SO_+(3,1)$ **group valued frame orientations**

Example: Special relativity with internal frames



fictitious/external coord. frame

$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in \mathrm{SO}_+(3,1),$ **internally indistinguishable**

introduce internal frame (tetrad)

$e_a^\mu \qquad \mu = t, x, y, z$ spacetime index, $\quad a = 0,1,2,3$ frame index

frame orientations

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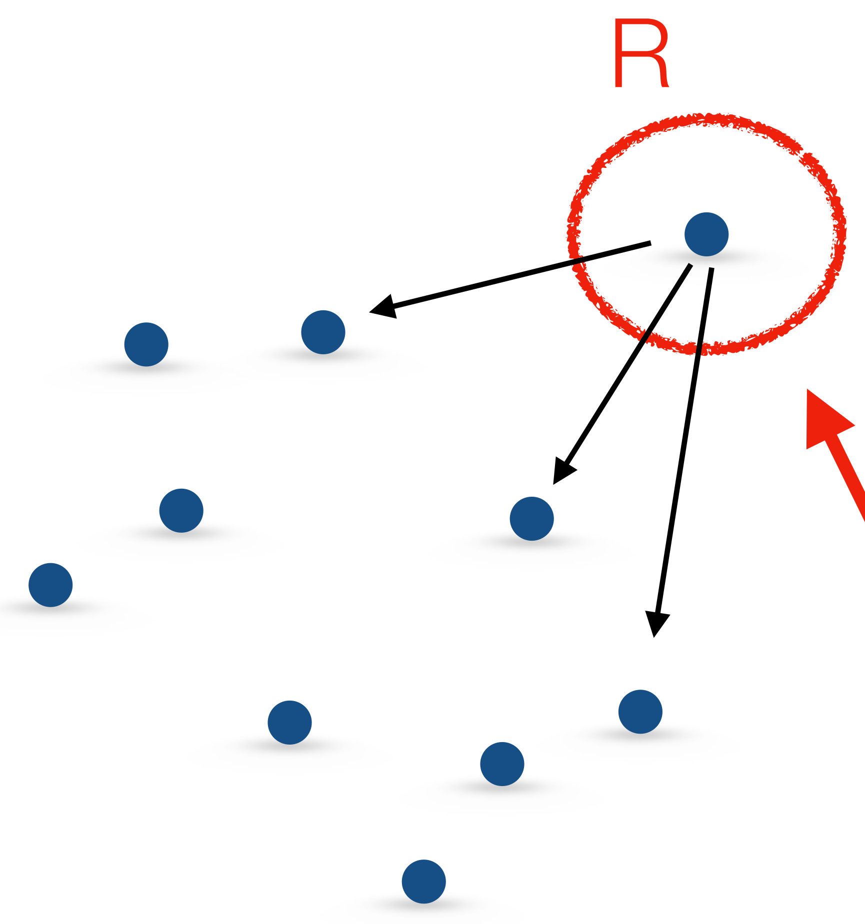
2 ways of “jumping into a RF perspective”

<u>Premise:</u>

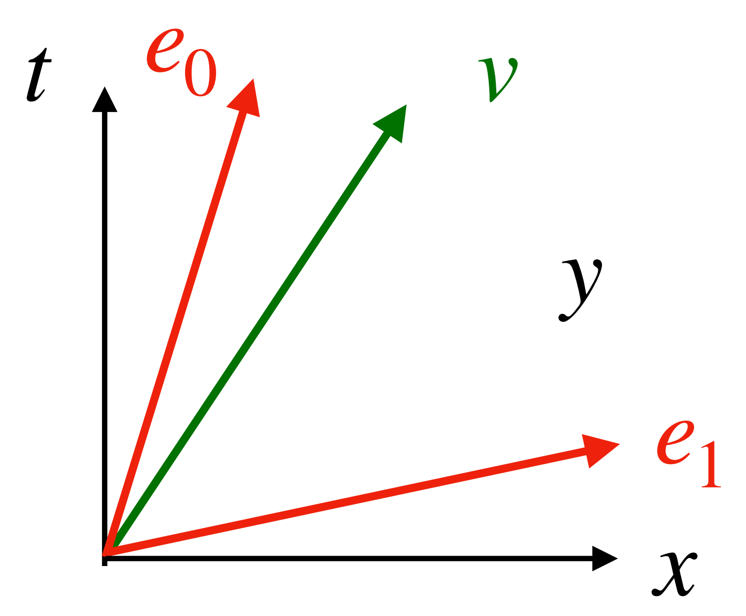
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



Describe S relative to internal reference subsystem R

1. relational observables relative to R (gauge inv.)

Example: Special relativity with internal frames



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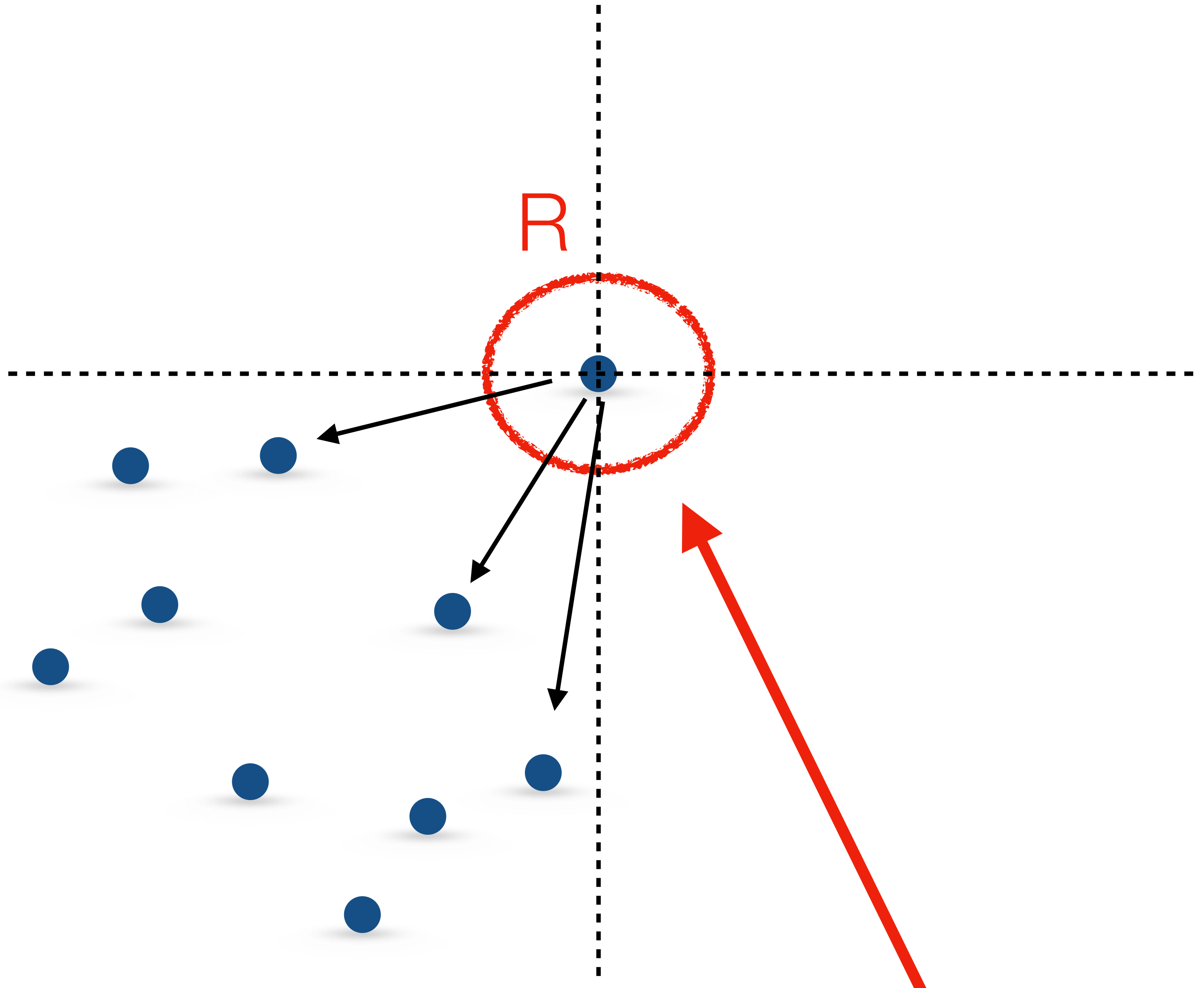
$\Rightarrow$ **"gauge-invariant" description of v:** $v_a = (v, e_a) = \eta_{\mu\nu} v^\mu e_a^\nu$ **"relational/frame dressed observables"**

(describes v relative to frame)

2 ways of "jumping into a RF perspective"



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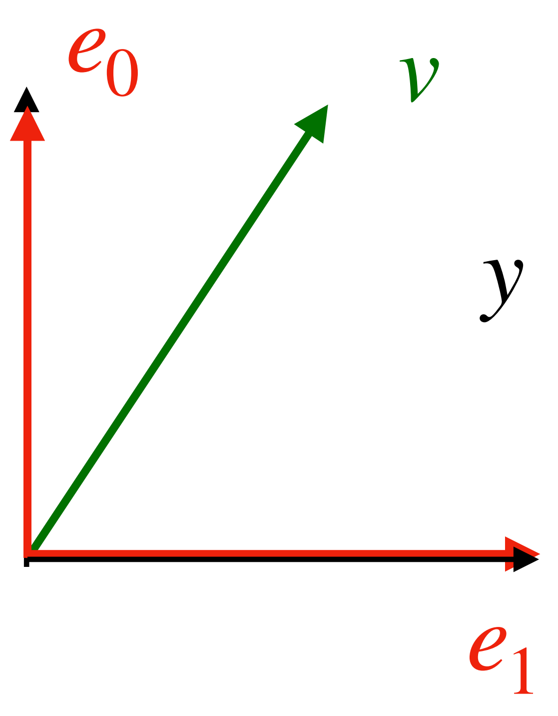
Describe S relative to internal reference subsystem R

1. relational observables relative to R (gauge inv.)

2. put R into "origin" (gauge fix)

Example: Special relativity with internal frames



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$\Rightarrow$ **gauge fix background frame to align with tetrad**

The multiple choice problem

Premise:
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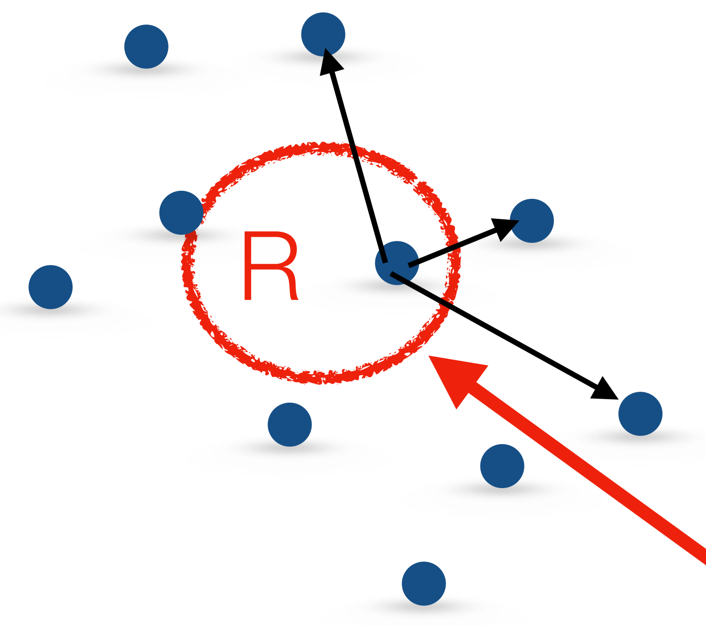

which frame to choose?

Describe S relative to internal reference subsystem R

The multiple choice problem

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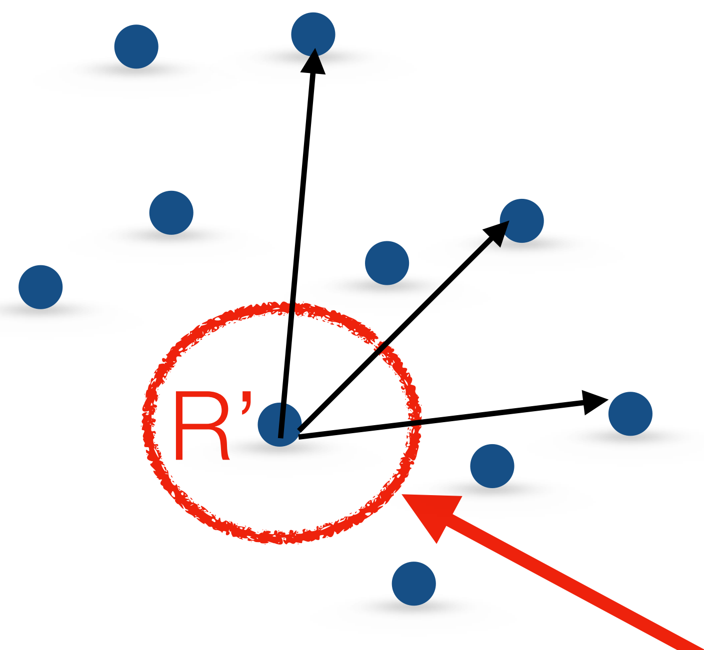
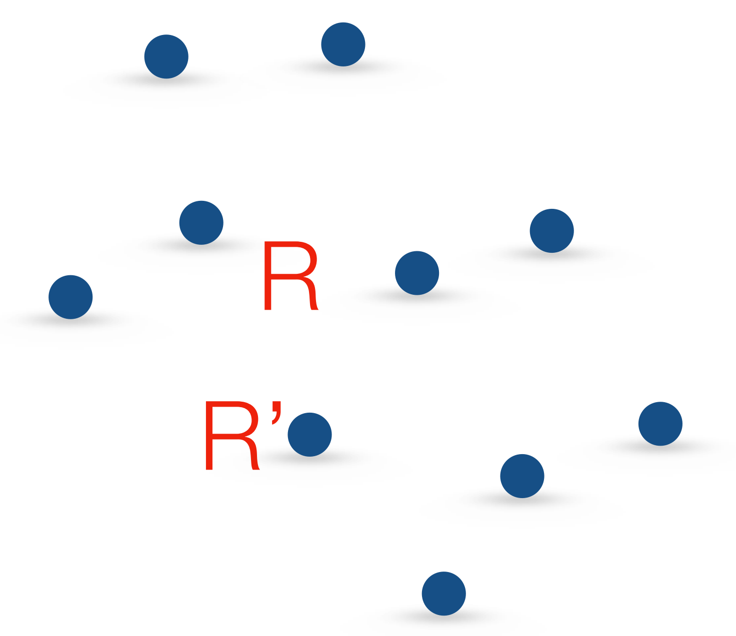


which frame to choose?

Describe S relative to internal reference subsystem R'

2 ways of changing RF



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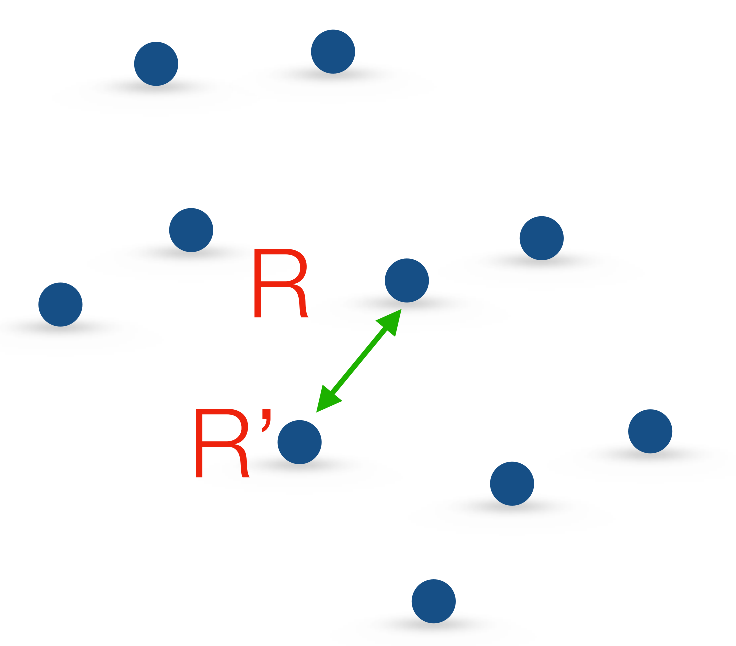

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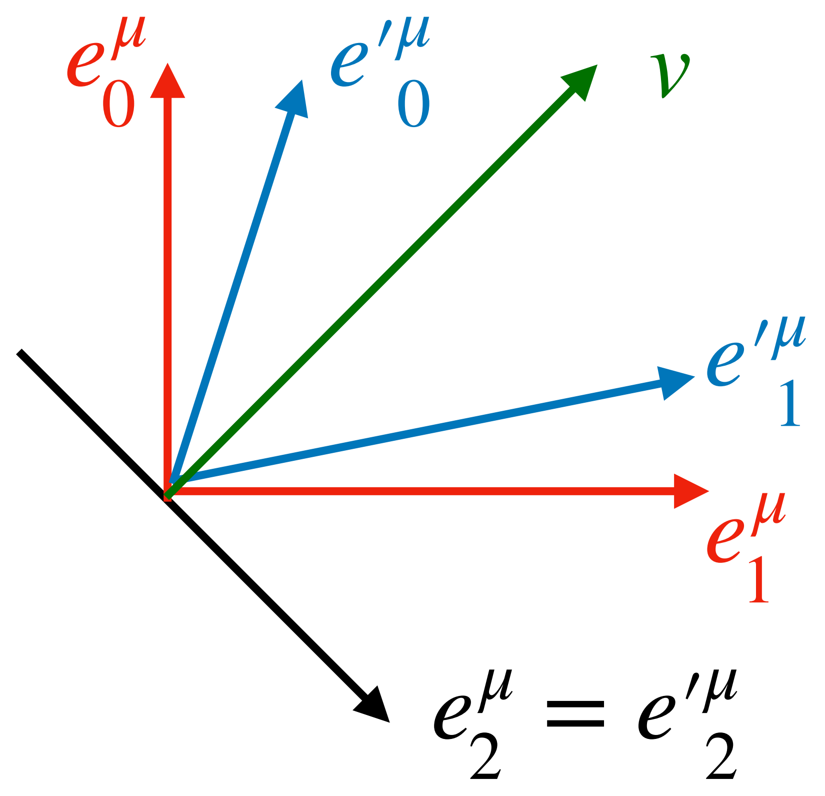
1. relation-conditional reorientation

Warmup: Special relativity with internal frames



introduce second internal frame $e'_{a'}$

$$v_a = v^\mu \eta_{\mu\nu} e^\nu_a = v^\mu e'_{\mu a'} e'^{a'}_\nu e^\nu_a = v_{a'} \Lambda^{a'}{}_a$$

relational observable rel. to e

relational observable rel. to e'

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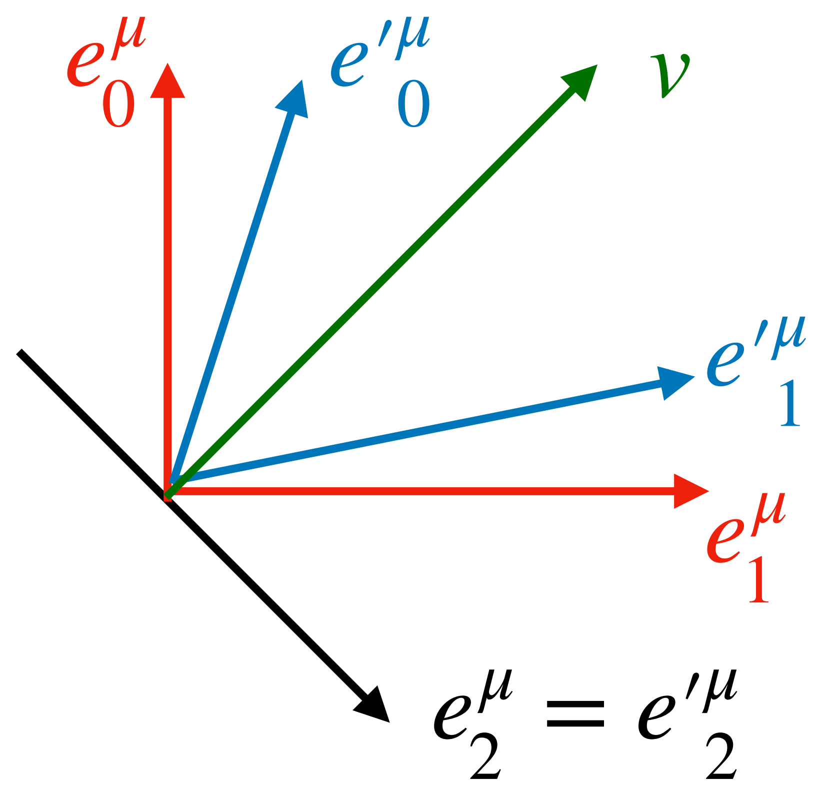

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$$\Lambda^{a'}{}_a = e'^{a'}_\mu e^\mu_a \in \mathrm{SO}_+(3,1)$$

is relational observable describing 1st rel. to 2nd frame

$$\begin{tikzcd}
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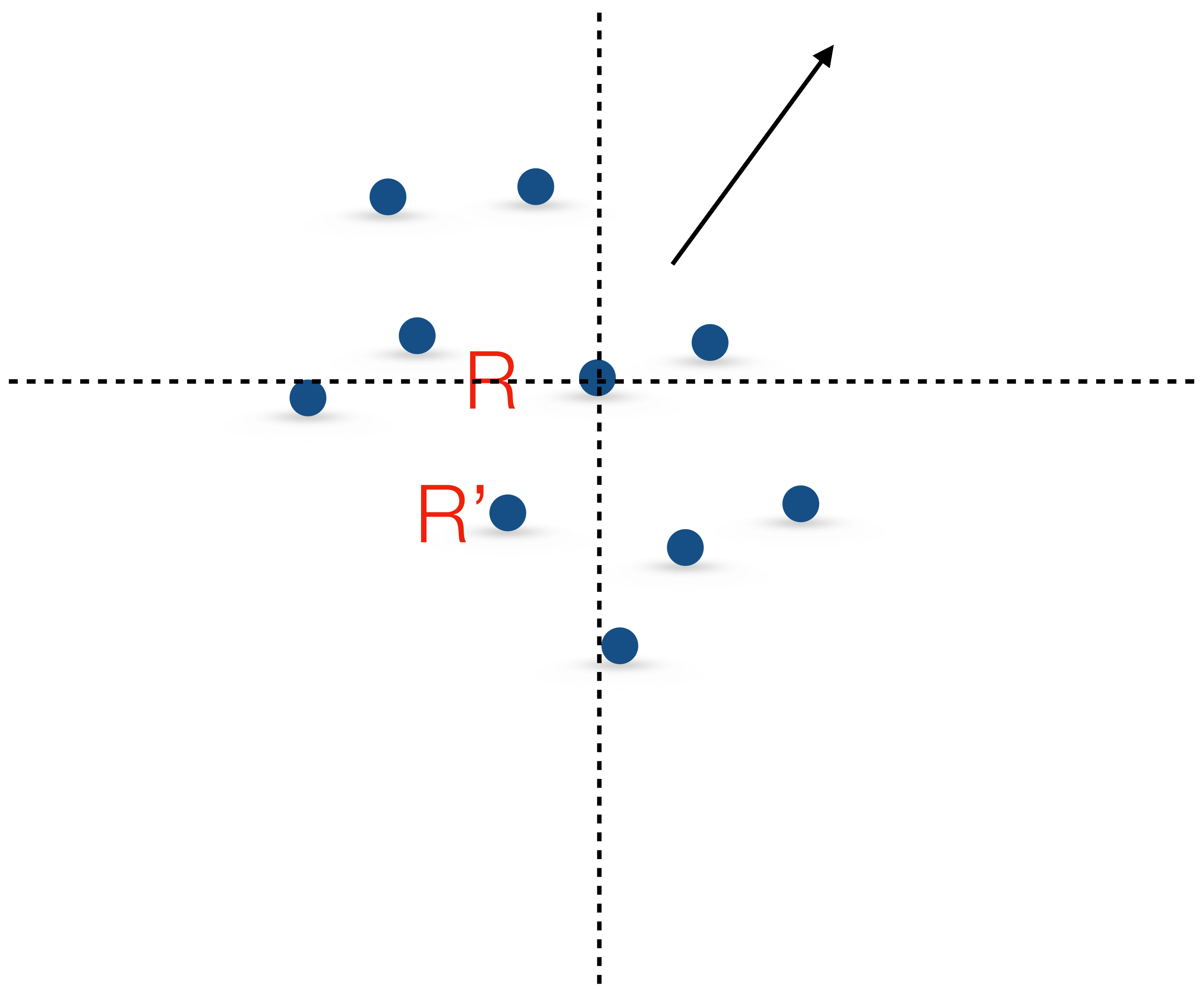

1. **relation-conditional reorientation**
2. **relation-conditional gauge-transf.**

2 ways of changing RF

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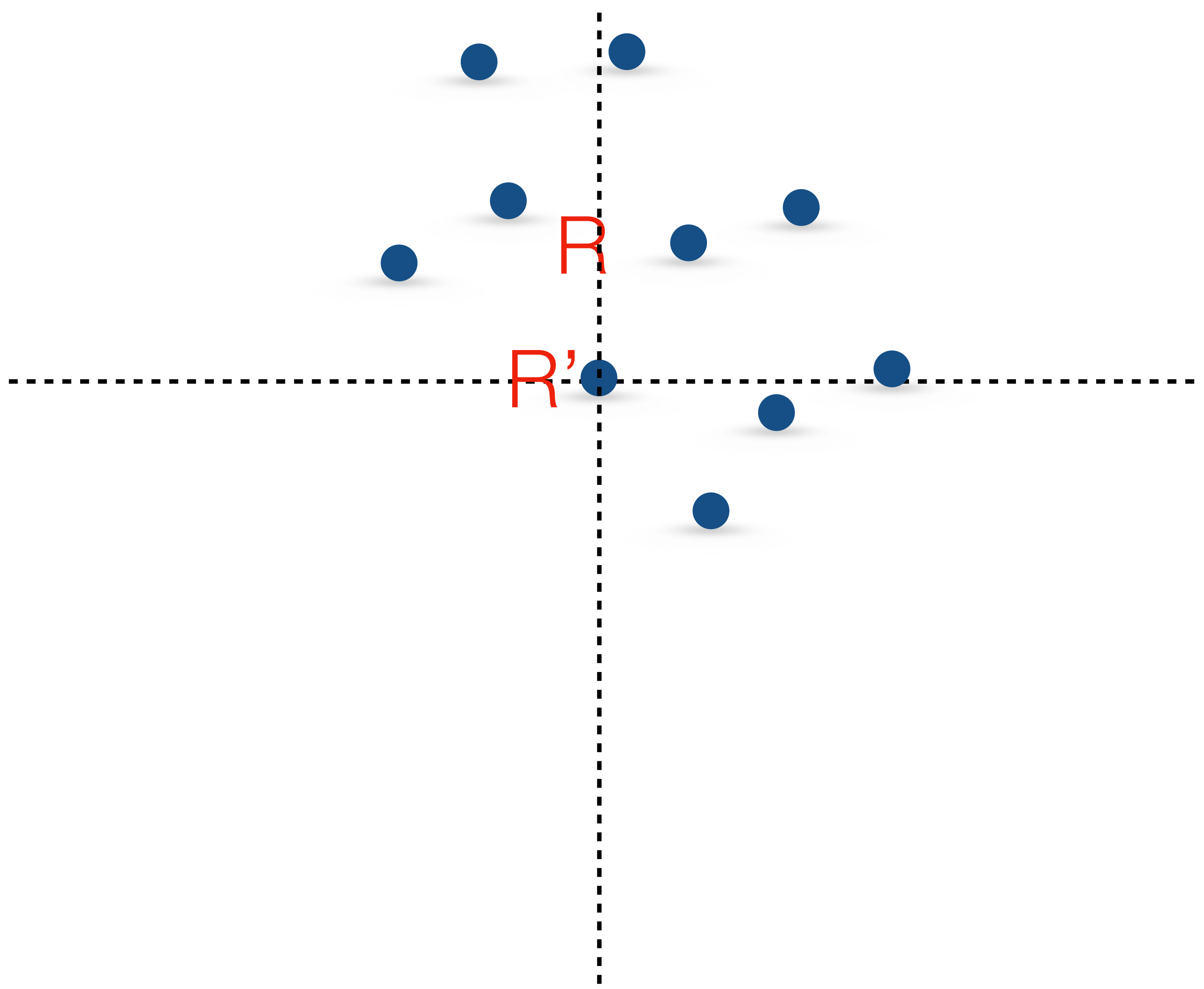

1. relation-conditional reorientation

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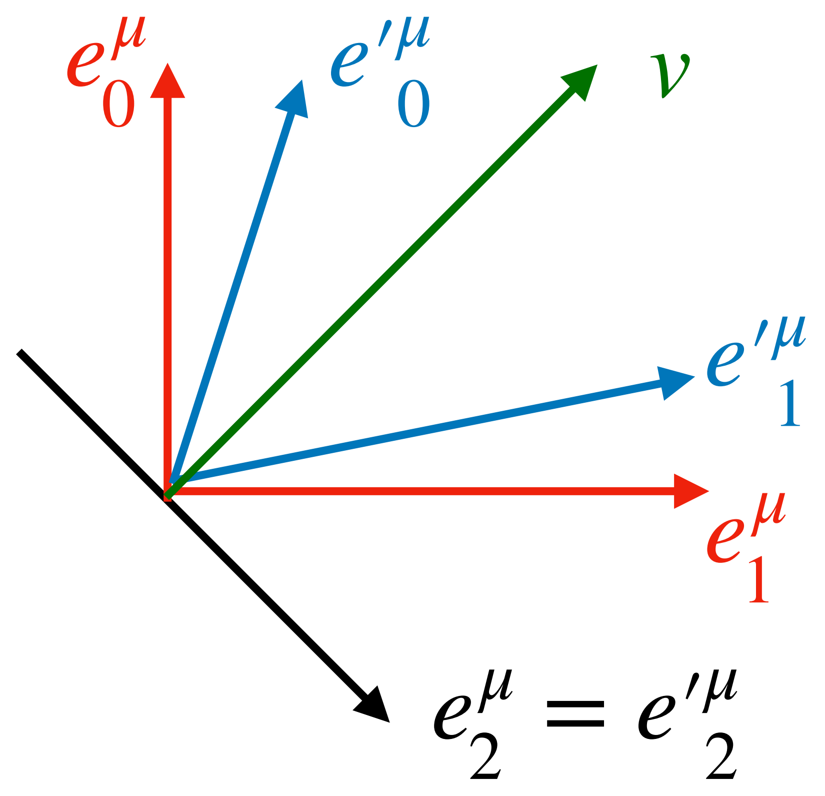
Warmup: Special relativity with internal frames



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\end{tikzcd}$$

gauge induced RF transformation:

$\Lambda^{\nu'}{}_\mu \in \mathrm{SO}_+(3,1)$ **looks the same as** $\Lambda^{a'}{}_a$

coordinate change via gauge fixings

$$\begin{tikzcd}
 & v_a = \eta_{\mu\nu} e^\nu_a v^\mu \arrow[dl, "e^\nu_a = \delta^\nu_a"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\
v_a \hat{=} v_\mu & & v_a \hat{=} \Lambda^{\nu'}{}_\mu v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_\mu \in \mathrm{SO}_+(3,1)"']
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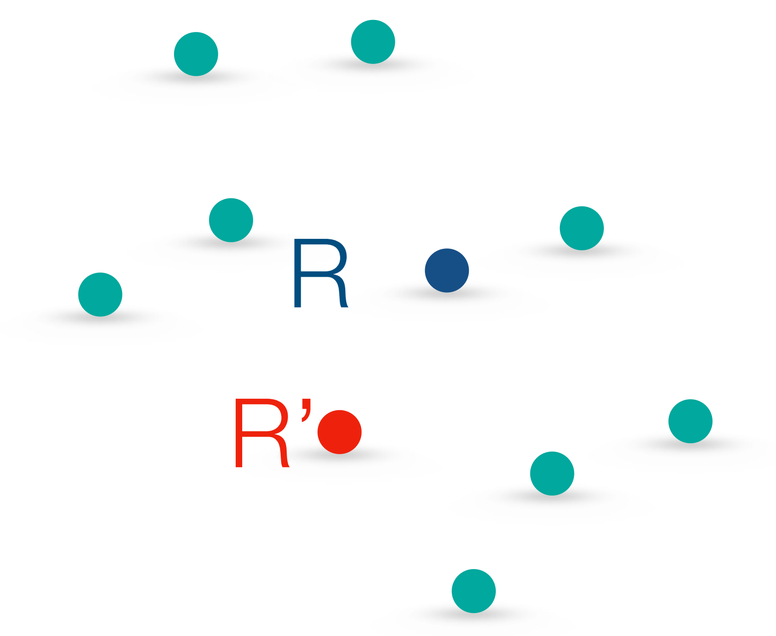
kinematical vs. relational subsystems

green balls: subsystem S'



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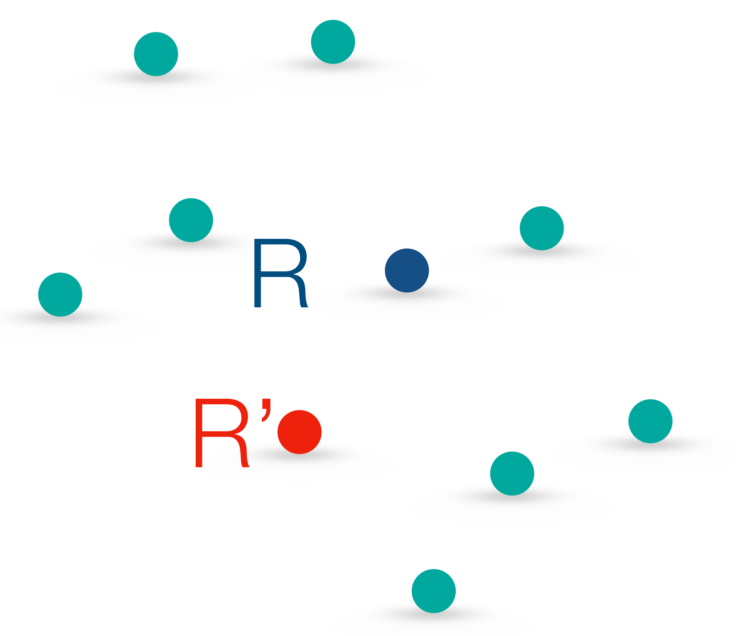

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kinematical vs. relational subsystems

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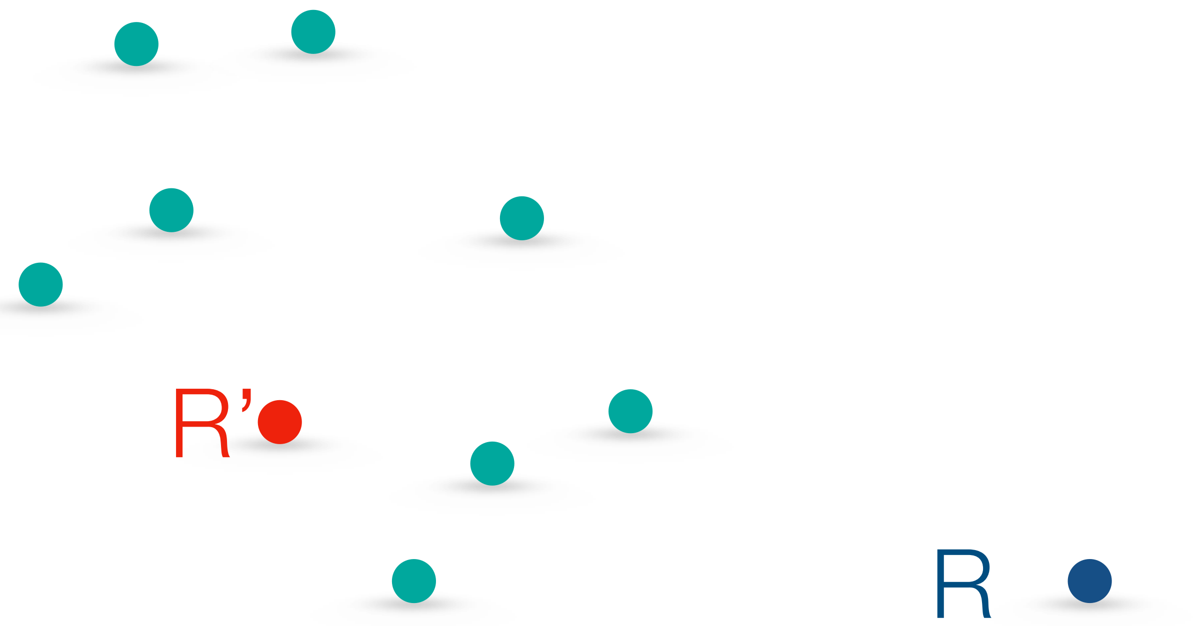

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leaves description of S' rel. to external frame invariant, but changes description relative to frame R

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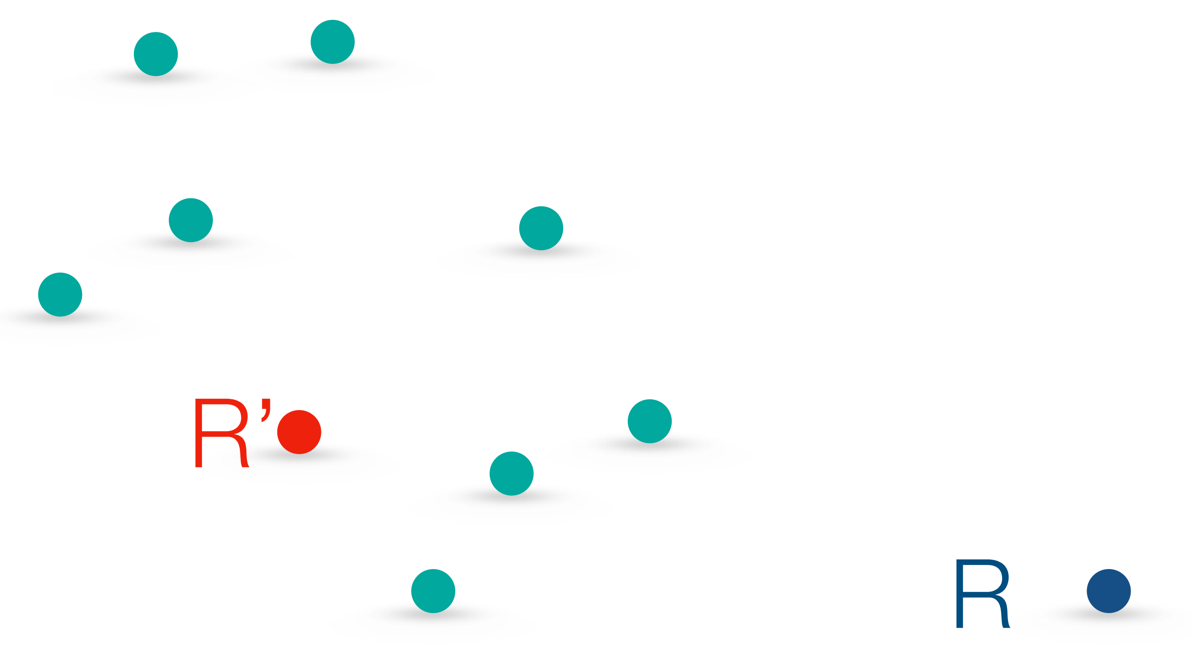

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kinematical vs. relational subsystems

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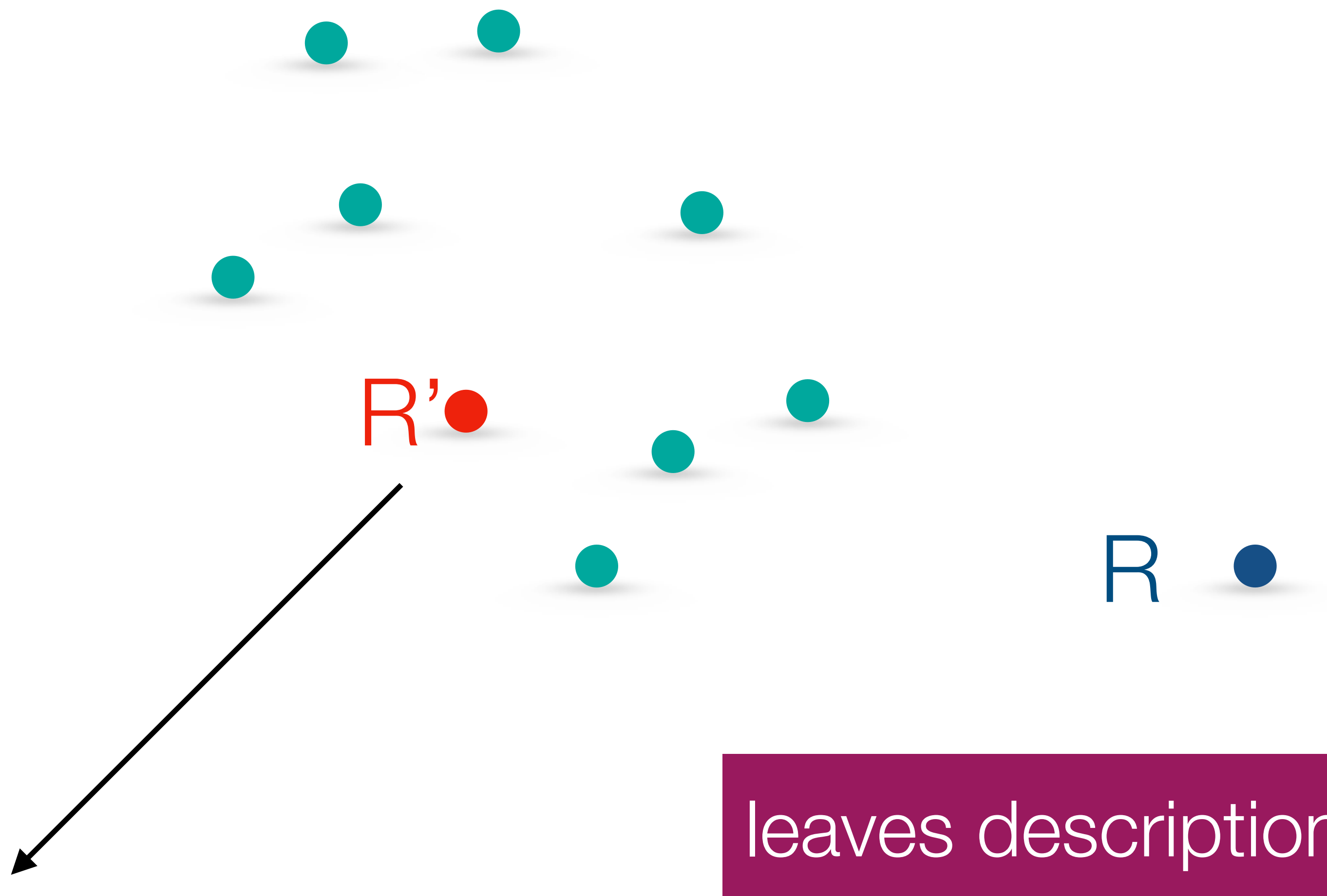

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kinematical vs. relational subsystems

leaves description of S' relative to R invariant, but changes it relative to R'

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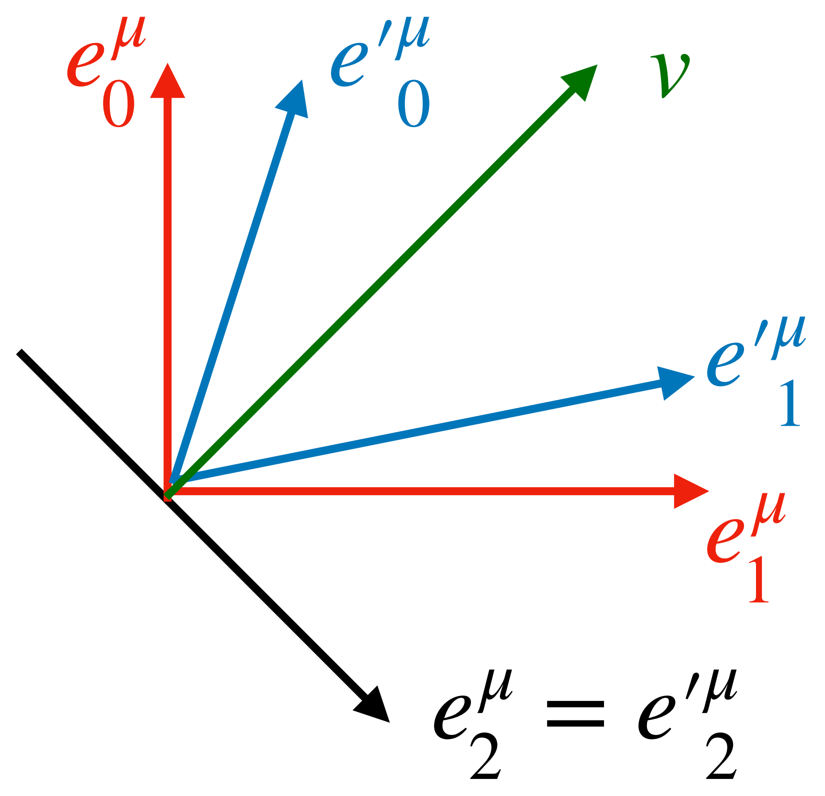
2. **gauge inv. notion of subsystem depends on choice of RF**

$\Rightarrow$ **gauge inv. correlations, thermal properties, … are RF dependent**

Warmup: Special relativity with internal frames



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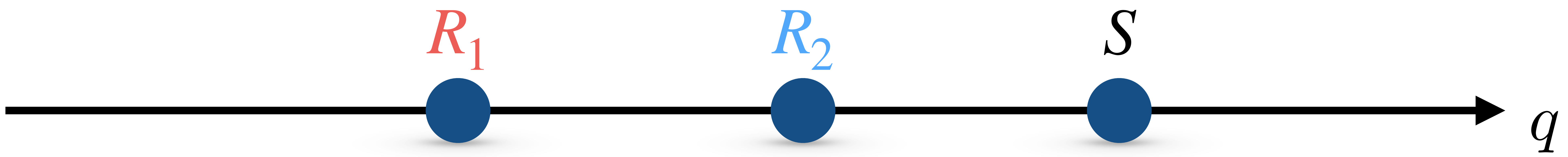
\end{tikzcd}$$

subsystem relativity $\Rightarrow$ relativity of simultaneity

Quantum reference frames

… or frames in relative superposition

Example: spatial QRFs in 1D

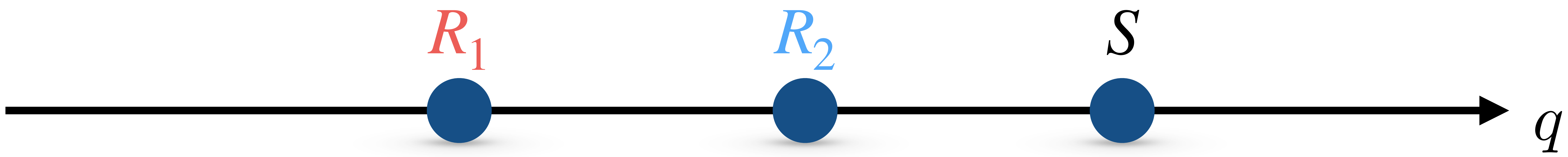


setup relative to external (possibly fictitious) frame:

$$\mathscr{H}_{\text{kin}} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S$$

space of externally distinguishable states

Example: spatial QRFs in 1D



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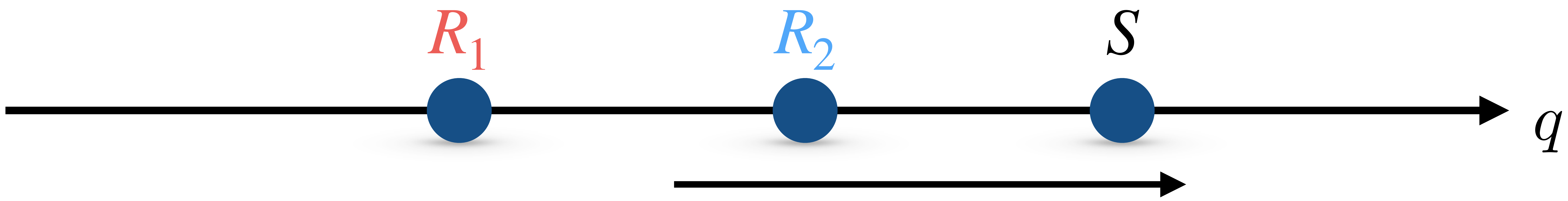
space of externally distinguishable states

global translations as external frame transformations, i.e. gauge transformations

$$U_{R_1 R_2 S}(x) = e^{ix(p_1+p_2+p_S)}$$

analog of $\Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$ in SR

Example: spatial QRFs in 1D



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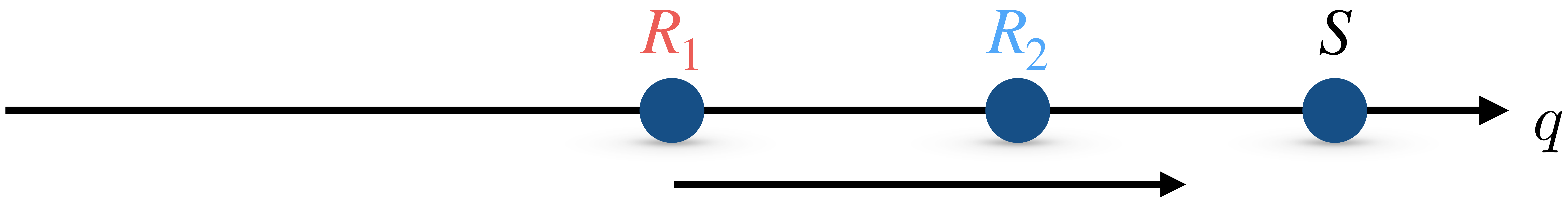

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frame orientation states for R_1 $\quad |q\rangle_{R_1}$

analog of tetrad $e^\mu{}_a \in \mathrm{SO}_+(3,1)$ in SR

frame reorientations (only act on frame)

$$U_{R_1}(x) = e^{ixp_1}$$

analog of $\Lambda^a{}_b e^\mu_a$ in SR

$\Rightarrow$ **2 commuting group actions (here trivial)**

Example: spatial QRFs in 1D

recall relational observables from SR $\quad v_a = v^\mu \, \eta_{\mu\nu} \boxed{e^\nu_a}$

frame-orientation conditional gauge transformation

<u>Relational observables through G-twirl:</u>

$$O_{f_{R_2S},R_1}(x) = \int dq \, U_{R_1R_2S}(q) \Big(|x\rangle\langle x|_{R_1} \otimes f_{R_2S} \Big) \, U^\dagger_{R_1R_2S}(q)$$

"what's the value of f_{R_2S} when R_1 is in position x?"

frame-orientation conditional gauge transformation (controlled unitary)

Example: spatial QRFs in 1D

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gauge-inv. $\quad [O_{f_{R_2S},R_1}(x), U_{R_1R_2S}(y)] = 0$

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"what's the value of $f_{R_2 S}$ when R_1 is in position x?"

frame-orientation conditional gauge transformation (controlled unitary)

gauge-inv. $\quad [O_{f_{R_2 S},R_1}(x), U_{R_1 R_2 S}(y)] = 0$

for example, get $O_{q_2,R_1}(0) = q_2 - q_1$ **and** $O_{q_S,R_1}(0) = q_S - q_1$

Perspective-neutral formulation of QRF covariance

$$\mathscr{H}_{\text{kin}} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S$$

space of externally distinguishable states

gauge-inv. (external frame-indep.) physical Hilbert space $\mathscr{H}_{\text{phys}}$ **with** $|\psi\rangle_{\text{phys}} = U_{R_1 R_2 S}(x)\,|\psi\rangle_{\text{phys}}$

space of internally distinguishable states

Perspective-neutral formulation of QRF covariance

$\mathscr{H}_{\text{kin}} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S$

space of externally distinguishable states

gauge-inv. (external frame-indep.) physical Hilbert space $\mathscr{H}_{\text{phys}}$ **with** $|\psi\rangle_{\text{phys}} = U_{R_1R_2S}(x)\,|\psi\rangle_{\text{phys}}$

states such that $(p_1 + p_2 + p_S)\,|\psi\rangle_{\text{phys}} = 0$

space of internally distinguishable states

$\Rightarrow$ redundancy: many different ways in describing same invariant $|\psi_{\text{phys}}\rangle$

$\Rightarrow$ associate with different internal QRF choices: redundant = reference DoFs

$\mathscr{H}_{\text{phys}}$ is a (internal frame) perspective-neutral space: description of physics prior to having chosen internal reference system relative to which remaining DoFs are described

gauge-induced QRF changes: quantum coordinate changes

$$\begin{tikzcd} & v_{a} = \eta_{\mu\nu} e^{\nu}_{a} v^{\mu} \arrow[dl, "e^{\nu}_{a} = \delta^{\nu}_{a}"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\ v_{a} \hat{=} v_{\mu} & & v_{a} \hat{=} \Lambda^{\nu'}{}_{\mu} v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_{\mu} \in \mathrm{SO}_{+}(3,1)"'] \end{tikzcd}$$

recall: “jumping into frame perspective” through gauge-fixing

gauge-induced QRF changes: quantum coordinate changes

Vanrietvelde, PH, Giacomini, Castro Ruiz, Quantum 2020

$$v_a = \eta_{\mu\nu} e^{\nu}_a v^{\mu}$$

$e^{\nu}_a = \delta^{\nu}_a$

$e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}$

$\Lambda^{\nu'}{}_{\mu} \in \mathrm{SO}_+(3,1)$

$v_a \hat{=} v_\mu \qquad v_a \hat{=} \Lambda^{\nu'}{}_{\mu} v_{\nu'}$

recall: "jumping into frame perspective" through gauge-fixing

perspective-neutral

$$\mathscr{H}_{\text{phys}}$$

$\varphi_{R_1} = \langle q = 0|_{R_1} \otimes I_{R_2 S}$

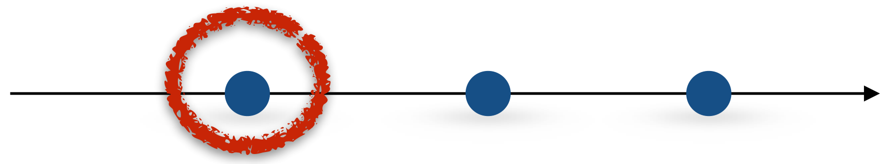
$\varphi_{R_2} = \langle q = 0|_{R_2} \otimes I_{R_1 S}$

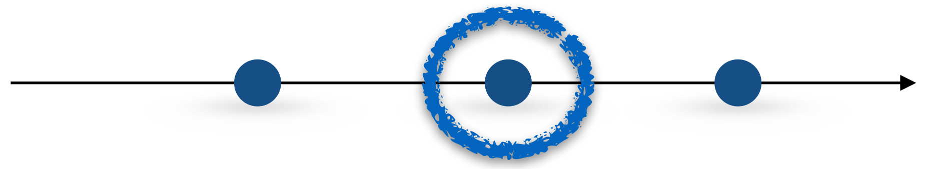
R_1 perspective

R_2 perspective

$$\mathscr{H}_{R_2 S|R_1} = L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S \longrightarrow \mathscr{H}_{R_1 S|R_2} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_S$$

$$V_{R_1 \to R_2} = \varphi_{R_2} \circ \varphi_{R_1}^{-1}$$





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$$\begin{tikzcd}[column sep=small]
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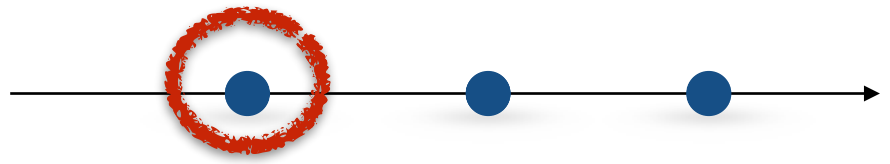
\end{tikzcd}$$

recall: “jumping into frame perspective” through gauge-fixing

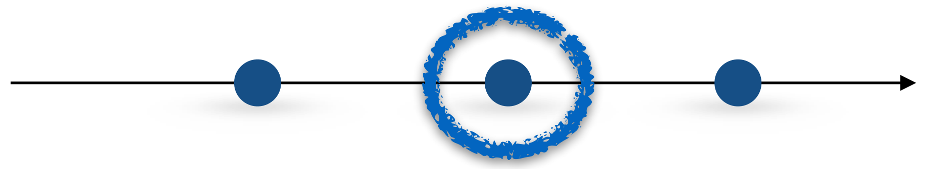
perspective-neutral

$$\begin{tikzcd}[column sep=small]
 & \mathscr{H}_{\text{phys}} \arrow[dl, "\varphi_{R_1} = \langle q=0|_{R_1} \otimes I_{R_2 S}"'] \arrow[dr, "\varphi_{R_2} = \langle q=0|_{R_2} \otimes I_{R_1 S}"] & \\
\mathscr{H}_{R_2 S|R_1} = L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S \arrow[rr] & & \mathscr{H}_{R_1 S|R_2} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_S
\end{tikzcd}$$

R_1 perspective



R_2 perspective



$$V_{R_1 \to R_2} = \varphi_{R_2} \circ \varphi_{R_1}^{-1}$$

$$= \mathbb{F}_{12} \int dq \, |-q\rangle\langle q|_{R_2} \otimes U_S(q)$$

reproduces QRF transf. from earlier

[Giacomini et al Nat. Comm. ’19]

gauge-induced QRF changes: quantum coordinate changes

QRF relativity of entanglement and superposition

perspective-neutral

$$\mathscr{H}_{\text{phys}}$$

$$\varphi_{R_1} = \langle q = 0|_{R_1} \otimes I_{R_2 S}$$

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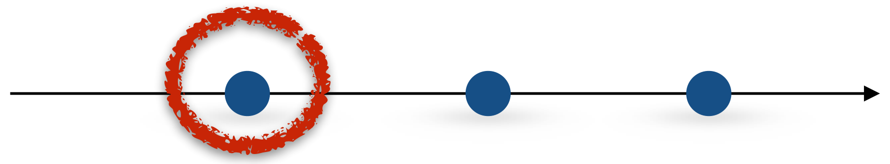
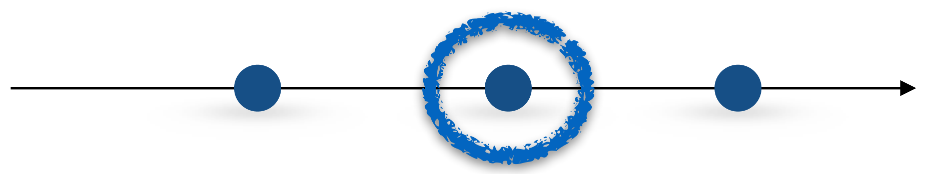
$$V_{R_1 \to R_2} = \mathbb{F}_{12} \int dq \, |-q\rangle\langle q|_{R_2} \otimes U_S(q)$$





$$|q_1\rangle_{R_2} \otimes |x\rangle_S + |q_2\rangle_{R_2} \otimes |x\rangle_S$$

$$|-q_1\rangle_{R_1} \otimes |x-q_1\rangle_S + |-q_2\rangle_{R_1} \otimes |x-q_2\rangle_S$$

QRF relativity of subsystems

QRF perspectives φ_{R_1} and φ_{R_2} are nothing but two tensor product structures on gauge-inv. $\mathscr{H}_{\text{phys}}$

perspective-neutral

$$\mathscr{H}_{\text{phys}}$$

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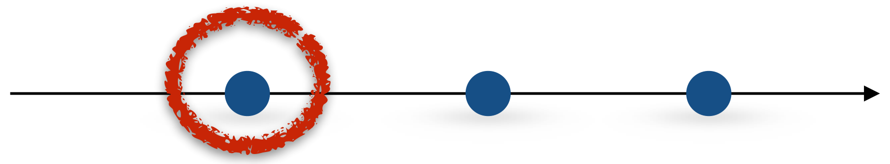

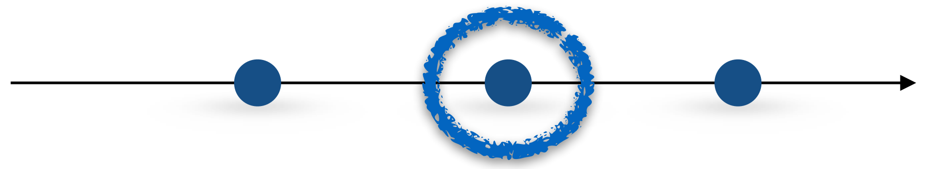

$$| q_1 \rangle_{R_2} \otimes | x \rangle_S + | q_2 \rangle_{R_2} \otimes | x \rangle_S$$

$$| -q_1 \rangle_{R_1} \otimes | x - q_1 \rangle_S + | -q_2 \rangle_{R_1} \otimes | x - q_2 \rangle_S$$

QRF relativity of subsystems

QRF perspectives φ_{R_1} and φ_{R_2} are nothing but two tensor product structures on gauge-inv. $\mathscr{H}_{\text{phys}}$

$\Rightarrow$ **inequivalent because QRF transf. $V_{R_1 \to R_2}$ a nonlocal unitary**

perspective-neutral

$\mathscr{H}_{\text{phys}}$

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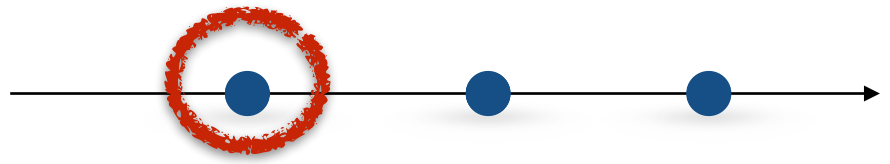

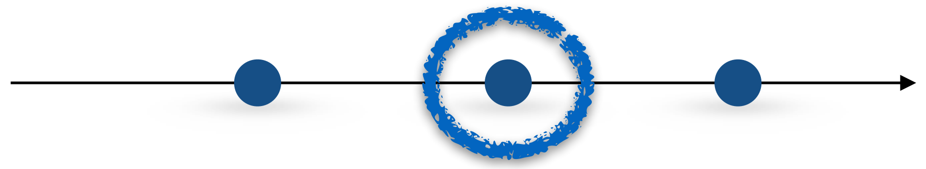

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QRF relativity of subsystems

different factorization of total gauge-inv. algebra into commuting subalgebras

$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{S|R_1} \otimes \mathscr{A}_{R_2|R_1}$

$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{S|R_2} \otimes \mathscr{A}_{R_1|R_2}$

perspective-neutral

$\mathscr{H}_{\text{phys}}$

generated by canonical pairs $(q_S - q_1, p_S)$ and $(q_2 - q_1, p_1)$ which become local in

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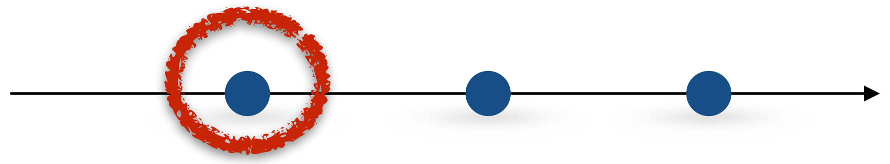

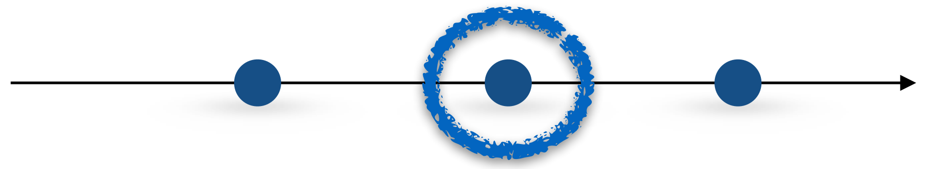

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$V_{R_1 \to R_2} = \mathbb{F}_{12} \int dq \, | -q \rangle\langle q |_{R_2} \otimes U_S(q)$

have $\mathscr{A}_{S|R_2} \neq \mathscr{A}_{S|R_1}$

realization of virtual subsystems, Zanardi '00s

QRFs for general unimodular Lie groups

de la Hamette, Galley, PH, Loveridge, Müller
2110.13824;

works similarly, essentially

global translations $U_{R_1R_2S}(x)$ $\longrightarrow$ **gauge transf.** $U_{R_1}(g) \otimes U_{R_2}(g) \otimes U_S(g)$

frame orientation states $|q\rangle_R$ $\longrightarrow$ **coherent states** $|\phi(g)\rangle_R$

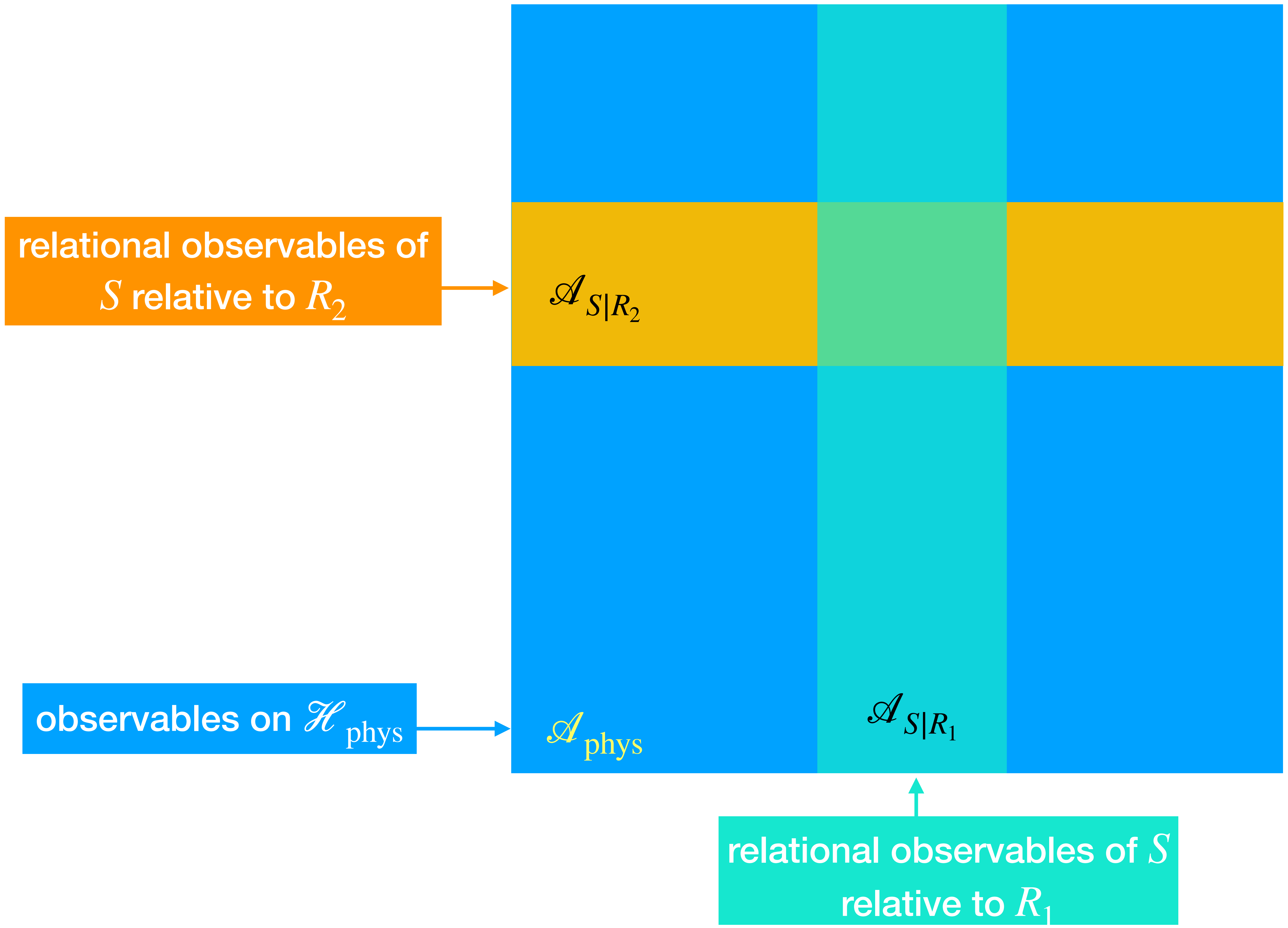
spatial integration $\int dq$ $\longrightarrow$ **group integration** $\int_G dg$

.....

Quantum relativity of subsystems

Ahmad, Galley, PH, Lock, Smith PRL ’22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824;*
Kotecha, Mele, PH *to appear*

3 kinematical subsystems $\mathcal{H}_{\mathrm{kin}} = \mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_S$



Recall: kinematical vs. relational subsystems

leaves description of S' relative to R invariant, but changes it relative to R'

green balls: subsystem S'

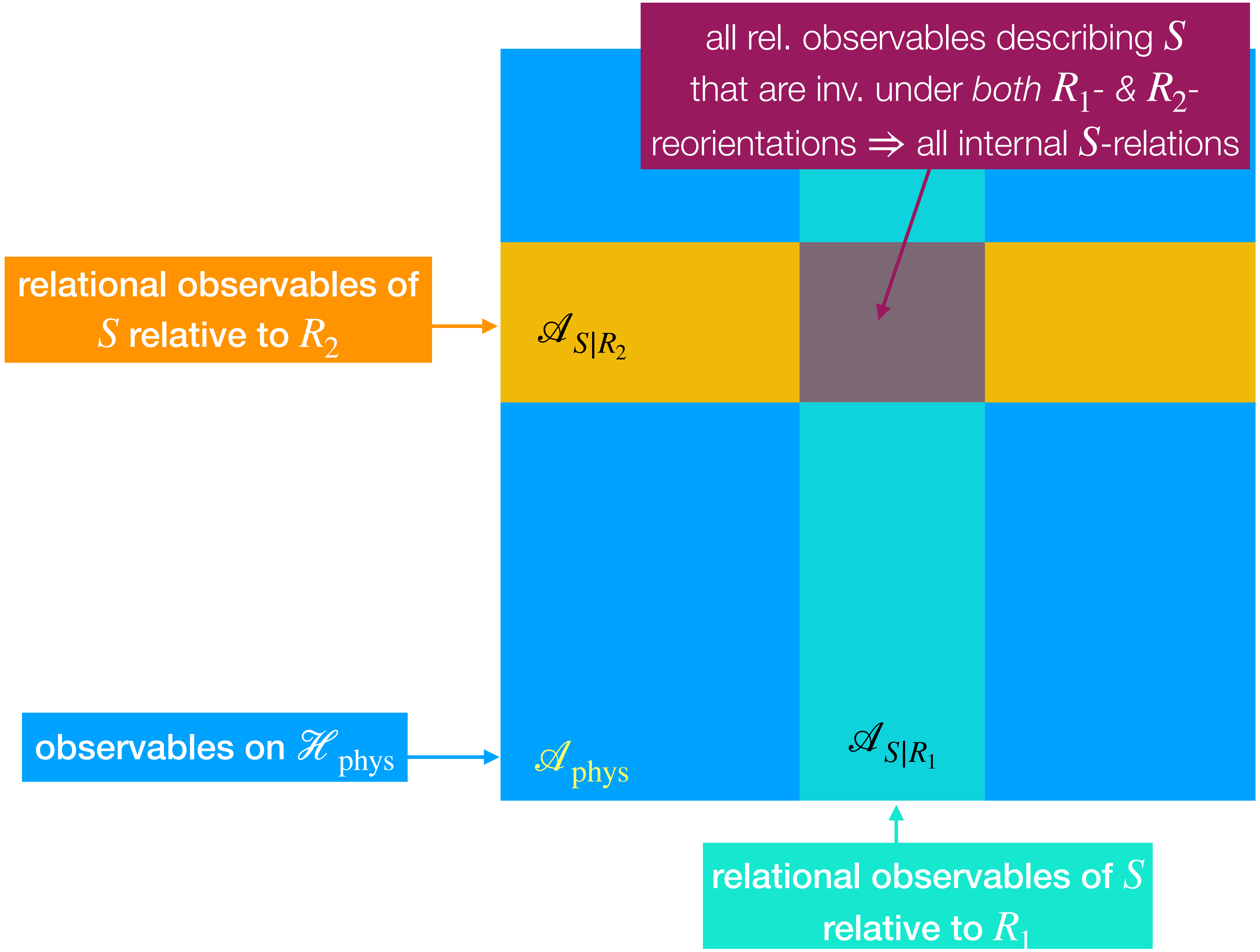
leaves description of S' rel. to external frame invariant, but changes description relative to frame R

$\Rightarrow$ **overlap of rel. observable algebras** $\mathscr{A}_{S|R} \cap \mathscr{A}_{S|R'} = \{\text{internal rel. obs. of S}\}$ **(but don't coincide)**

Quantum relativity of subsystems

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824;*
Kotecha, Mele, PH *to appear*

3 kinematical subsystems $\mathscr{H}_{\rm kin} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$



Quantum relativity of subsystems

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824;*
Kotecha, Mele, PH *to appear*

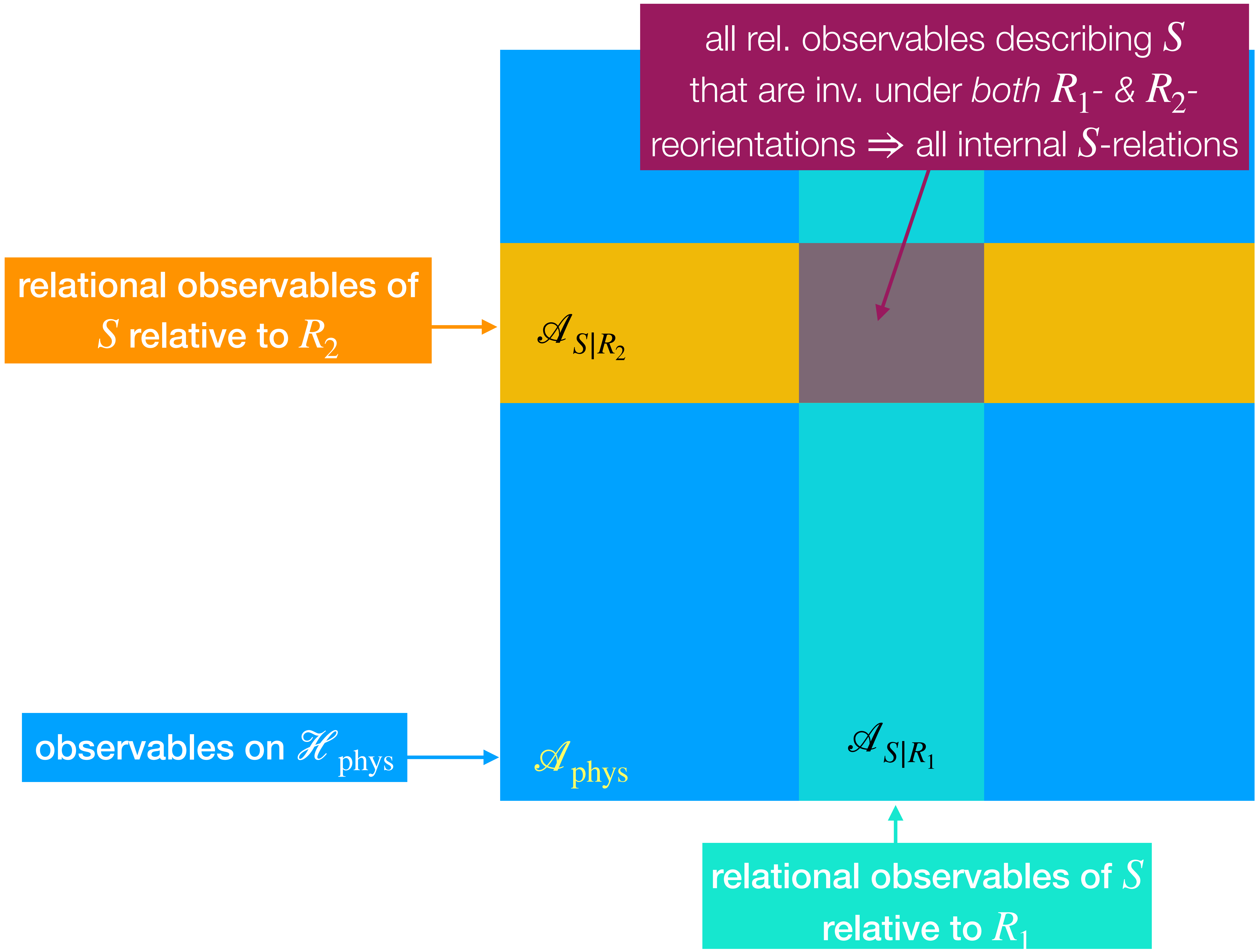
3 kinematical subsystems $\mathcal{H}_{\text{kin}} = \mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_S$

$\Rightarrow$ different relational observable subalgebras inside total invariant algebra

$\Rightarrow$ induce different gauge-inv. tensor factorizations

(not in general factorization across $R_j | R_i$ and $S | R_i$)

$\Rightarrow$ different appearance of same physics



Upshot: frame-dependent gauge-inv. properties

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller
2110.13824

"frames R_1 and R_2 mean different inv. DoFs when they refer to subsystem S"

if factorizability in two frame perspectives, i.e.

$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{S|R_1} \otimes \mathscr{A}_{R_2|R_1} \simeq \mathscr{A}_{S|R_2} \otimes \mathscr{A}_{R_1|R_2}$ but $\mathscr{A}_{S|R_2} \neq \mathscr{A}_{S|R_1}$

then correlations/entanglement of S with its complement will in general differ in two perspectives

(see also Giacomini, Castro-Ruiz, Brukner '19; Castro-Ruiz, Oreshkov '21)

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then correlations/entanglement of S with its complement will in general differ in two perspectives

(see also Giacomini, Castro-Ruiz, Brukner '19; Castro-Ruiz, Oreshkov '21)

$\Rightarrow$ gauge-inv. entanglement entropy in general $S(\rho_{S|R_2}) \neq S(\rho_{S|R_1})$ for same global physical state

$\Rightarrow$ dynamics of S can be closed/isolated relative to R_1 and open relative to R_2

(can map unitary dynamics/zero heat & work exchange into
open dynamics/ non-zero heat & work exchange in other perspective)

Kotecha, Mele, PH *to appear*

$\Rightarrow$ QRF relativity of superpositions, correlations, equilibrium, thermodynamics, …

Conclusions

- **Natural extension of relativity principle into quantum realm**
  based on internal QRFs ⇒ in terms of group structures really the same as in SR
- **Systematic method for changing QRF perspectives**
  accommodates RFs in relative superposition
- **Gauge-inv. subsystems depend on choice of QRF (“quantum relativity of subsystems”)**
  ⇒ correlations, thermal properties, dynamics, …. depend on frame

⇒ works completely analogously with (so far classical) dynamical frames in gauge theory and gravity

Carrozza, PH 2109.06184; Carrozza, Eccles, PH 2205.00913; Goeller, PH, Kirklin 2206.01193

Appendix

Symmetry-induced QRF changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*

changes of relational observables, recall:

$$v_a = \boxed{v_{a'} \Lambda^{a'}{}_a}$$

relational observable rel. to $\boldsymbol{e}$

relational observable rel. to $\boldsymbol{e}'$

- **RF transformation between two frames is** $\Lambda^{a'}{}_a = e'^{a'}_\mu e^\mu_a \in \mathrm{SO}_+(3,1)$

  relational observable describing 1st rel. to 2nd frame

relation-conditional frame reorientation

Symmetry-induced QRF changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*

can do analog in QT: G-twirl for symmetries $V_{R_1}(g) \otimes \mathbf{1}_{R_2 S}$

relation-conditional frame reorientation