M-theory and matter
via
Twisted equivariant differential (TED) K-theory

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M-Theory and Mathematics:
Classical and Quantum Aspects

12-16 Jan. 2023
NYUAD

جامعة نيويورك أبوظبي
NYU | ABU DHABI
Center for Quantum & Topological Systems

Overview of a program (series of papers) joint with: Urs Schreiber.

Related work also with: Domenico Fiorenza, Dan Grady, Alexander Voronov.

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[S]= S.
[SS]= S.-Schreiber
[FSS]= Fiorenza-S.-Schreiber
[GS]= Grady-S.
[SV]=S.-Voronov
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TED K-theory: High energy vs. condensed matter

General wisdom:
Twisted Equivariant Differential (TED) Topological K-theory
classifies

stable D-branes in string theory		free topological phases in condensed matter theory
	and *some* enhancement to	
non-perturbative effects		interacting phases
	is needed to account for	
M-branes		topological order
	harboring	
$N \sim 1$ YM theory for hadrodynamics		anyon statistics for topological quantum gates

Outline/Theme

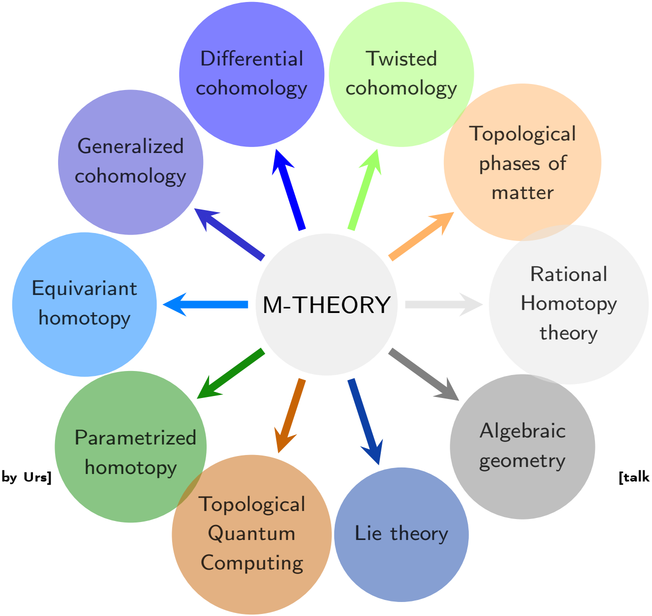
The main theme:

Issue	Solution
TED-K had never really been constructed	$\rightarrow$ Systematic construction of TED K-theory
M-theory had remained notoriously elusive	$\rightarrow$ Precise proposal for interacting enhancement via Hypothesis H
Nonperturbative aspects of field theory (M-theory) are actually practically relevant	$\rightarrow$ Concrete implementation of topological order via TED-K
	Embedding in M-theory via M-branes

Physics context:

- a mix of high energy and condensed matter.
- Exact constructions/duality and not just analogies.

Mathematical Richness of M-theory



[talk by Urs]

[talk by Sasha]

Generalities on what physics wants

- Nontrivial physical entities, such as fields, charges, etc., generically take values in cohomology.
- Anomalies and quantum considerations require generalized versions (cobordism, elliptic cohomology, K-theory, etc.) depending on context.

$$\begin{tikzcd} & \boxed{\text{Generalized Cohomology}} \arrow[dl, squiggly] \arrow[d, squiggly] \arrow[dr, squiggly] & \\ \boxed{\text{Twisted}} & \boxed{\text{Equivariant}} & \boxed{\text{Differential}} \end{tikzcd}$$

I. **Twisting**: Account for symmetries via automorphisms. Physically: it is the coupling between fields that are being twisted with those doing the twist.

interesting $\updownarrow$ interplay

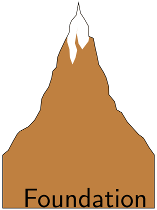
II. **Equivariant**: Account for (spacetime) singularities/symmetries via group actions, e.g. orbifolds, orientifolds ... $\leftarrow$ $\boxed{\text{Quite subtle}}$

III. **Differentially refined**: Include geometric data, such as connections, Chern character form, smooth structure, smooth representatives of maps ...

Motivation from modelling of **fields** (in QFT, string theory and M-theory).

Schematically:

$$
\begin{tikzcd}
E^\bullet(M) & \textit{Quantum} & \textit{M-theory} \\
H^\bullet(M;\mathbb{Z}) \arrow[u, squiggly, "\text{anomaly cancellation}"] & & \\
H^\bullet_{\mathrm{dR}}(M) \arrow[u, squiggly, "\text{quantization}"] & & \\
\Omega^\bullet(M) \arrow[u, squiggly, "\text{gauge}"] & \textit{Classical} \arrow[uuu, squiggly] & \textit{Sugra} \arrow[uuu, squiggly]
\end{tikzcd}
$$



$$
\begin{tikzcd}
\boxed{\text{Generalized}} \arrow[d, shift right] \\
\boxed{\text{Rational}} \arrow[u, dotted, shift right]
\end{tikzcd}
$$

2. Twists

- We would like to introduce automorphisms.
- These arise from geometric and physical considerations.
- Homotopy p.o.v.: moduli/family setting; bundles of spectra $\begin{tikzcd} \mathcal{S} \arrow[r] & \mathcal{E} \arrow[d] \\ & X \end{tikzcd}$

$$\begin{tikzcd} \Omega^\bullet(M) \arrow[loop above, "\mathit{twist}_\Omega"] \arrow[r, squiggly, "\text{exact}", "\text{complex}"'] & H^\bullet_{\mathrm{dR}}(M) \arrow[loop above, "\mathit{twist}_{\mathrm{dR}}"] \arrow[r, squiggly, "\text{quantization}"] & H^\bullet(M;\mathbb{Z}) \arrow[loop above, "\mathit{twist}_H"] \arrow[r, squiggly, "\text{anomaly}", "\text{cancellation}"'] & E^\bullet(M) \arrow[loop above, "\mathit{twist}_E"] \end{tikzcd}$$

Relations among various twists?

Example (twist_Ω)

Twisted differential forms are forms valued in the orientation line bundle.
Top such form is a density (pseudo-volume form).

Example: Twisted de Rham cohomology

- The de Rham complex $(\Omega^\bullet, d)$: $\quad \ldots \xrightarrow{d} \Omega^i(X) \xrightarrow{d} \Omega^{i+1}(X) \xrightarrow{d} \ldots$
- Twist by a 1-form built out of scalar ftn: $d \rightsquigarrow d_\phi := d + d\phi\wedge$ with $d_\phi^2 = 0$.

Example (Witten's deformation of Morse theory)

For smooth $f : M \to \mathbb{R}$, the Witten differential is $d_s = e^{-sf} d e^{sf} = d + sdf\wedge$, where $s \in \mathbb{R}$. Then $d_s^2 = 0$, $d_s : \Omega^p \to \Omega^{p+1}$. The term e^{-sf} is a quasi-isomorphism

$$
\begin{tikzcd}
\ldots \arrow[r] & \Omega^p \arrow[r, "d"] \arrow[d, "e^{-sf}"'] \arrow[dr, phantom, "\circlearrowright"] & \Omega^{p+1} \arrow[r] \arrow[d, "e^{-sf}"] & \ldots \\
\ldots \arrow[r] & \Omega^p \arrow[r, "d_s"] & \Omega^{p+1} \arrow[r] & \ldots
\end{tikzcd}
$$

and d_s yields isomorphic cohomology groups.

- Twist by a closed 3-form: $d_{H_3} = d - H_3\wedge$, with $d_{H_3}^2 = 0$.

Definition

Twisted de Rham cohomology: $H^i(X, H_3) := \ker(d_{H_3})/\mathrm{im}(d_{H_3})$

Example (*The Ramond-Ramond (RR) fields in string theory)*

Rationally, $F = \sum_{i \le 5} u^{-i} F_{2i+\epsilon}$, $\epsilon = 0$ or 1 for type IIA or type IIB string theory. These are twisted by a closed 3-form, the NS-field H_3.

Reverse engineering for twisted generalized cohomology

Rational twisted cohomology arises as image of some Chern character.

The Chern-Dold character: Primary image of any generalized cohomology theory is rationalization. [FSS book in press: The Chern character in abelian and nonabelian cohomology]

$$\begin{tikzcd} E^n(X) \arrow[d, "\mathrm{ch}_E"] \\ \bigoplus_k H^{n+k}(X; \pi_k(E) \otimes_{\mathbb{Z}} \mathbb{Q}) \end{tikzcd}$$

$$\begin{tikzcd} \boxed{\text{Integral}} \\ \boxed{\text{Rational}} \arrow[u, dotted, "\text{Lifting back:}"', "\text{Postnikov towers, AHSS,}\cdots"] \end{tikzcd}$$

Example (Twisted K-theory)

Degree three twist H_3:

$$\mathrm{ch}_{H_3} : \underbrace{K^\bullet(X, H_3)}_{\text{twisted K}-\text{theory}} \longrightarrow \underbrace{H^{\mathrm{ev}}(X, H_3)}_{\textit{twisted de Rham cohomology}}$$

[Rosenberg, Bouwknegt-Carey-Mathai-Murray-Stevenson, Atiyah-Segal, $\cdots$]

Differential refinement: cohomology

- Introduce geometric data via differential forms (connections, Chern forms, $\cdots$), i.e., retain *differential form representatives* of cohomology classes.

$$
\begin{tikzcd}
 & \Omega^\bullet(M) \arrow[dl, squiggly, "adjoin"'] \arrow[d, squiggly, "adjoin"] \arrow[dr, squiggly, "adjoin"] & \\
H^\bullet(M;\mathbb{Q}) \arrow[d, squiggly, "\text{refinement}"] & H^\bullet(M;\mathbb{Z}) \arrow[d, squiggly, "\text{refinement}"] & E^\bullet(M) \arrow[d, squiggly, "\text{refinement}"] \\
\widehat{H^\bullet}(M;\mathbb{Q}) & \widehat{H^\bullet}(M;\mathbb{Z}) & \widehat{E^\bullet}(M)
\end{tikzcd}
$$

- Amalgam of an underlying (topological) cohomology theory and the data of differential forms:

$$
\begin{tikzcd}
\boxed{\text{Differential gen. cohomology}} \arrow[r] \arrow[d] & \text{Forms} \arrow[d] \\
\text{Gen. cohomology} \arrow[r] & \text{de Rham cohomology}
\end{tikzcd}
$$

- That is, we have a homotopy fiber product (of sheaves of spectra classifying)

$$\text{“Differential cohomology} = \text{Cohomology} \times_{\text{de Rham}} \text{Forms”}$$

Differential generalized cohomology

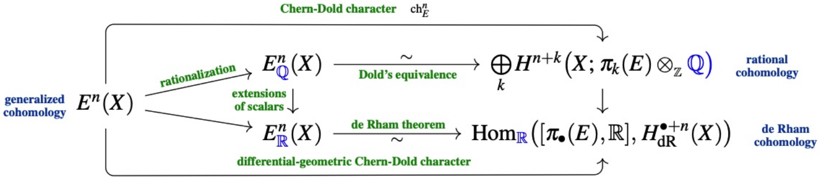
- Start with a generalized cohomology theory E
- $\Omega(X, E_*) := \Omega(X) \otimes_{\mathbb{Z}} E_*$ Smooth diff. forms with coefficients in $E_* := E(*)$
- $\Omega_{\mathrm{cl}}(X, E_*) \subseteq \Omega(X, E_*)$ closed forms
- $H_{\mathrm{dR}}(X, E_*)$ cohomology of the complex $\big(\Omega(X, E_*), d\big)$

Definition

A differential/smooth extension of h is a contravariant functor

$$\widehat{E} : \text{Compact Smooth Manifolds} \longrightarrow \text{Graded Abelian Grps}$$

$$\begin{tikzcd}
 & \Omega_{\mathrm{cl}}(X, E_*) \arrow[d] \\
\widehat{E}(X) \arrow[ur, "R"] \arrow[dr, two heads, "I"'] & H_{\mathrm{dR}}(X, E_*) \\
 & E(X) \arrow[u]
\end{tikzcd}$$

[FSS21]: 

This has a nonabelian/nonlinear generalization – cf. Cohomotopy for M-theory

The full structure: TED

Twisted $\cap$ Equivariant $\cap$ Differential $\cap$ Generalized

$$
\begin{tikzcd}
 & \Omega^\bullet_{G}(M) \arrow[dl, squiggly, "adjoin"'] \arrow[d, squiggly] \arrow[dr, squiggly, "adjoin"] & & \text{Geometric} \\
H^\bullet_{G}(M;\mathbb{Q}) \arrow[loop above, "twist_H"] \arrow[d, squiggly, "\text{refinement}"] & H^\bullet_{G}(M;\mathbb{Z}) \arrow[loop above, "twist_H"] \arrow[d, squiggly, "\text{refinement}"] & E^\bullet_{G}(M) \arrow[loop above, "twist_E"] \arrow[d, squiggly, "\text{refinement}"] & \text{Topological} \\
\widehat{H^\bullet}_{G}(M;\mathbb{Q}) \arrow[loop below, "twist_{\widehat{H}}"] & \widehat{H^\bullet}_{G}(M;\mathbb{Z}) \arrow[loop below, "twist_{\widehat{H}}"] & \widehat{E^\bullet}_{G}(M) \arrow[loop below, "twist_{\widehat{E}}"] & \text{Combined}
\end{tikzcd}
$$

Proper Orbifold Cohomology [arX:2008.01101]
The twisted non-abelian character map [arX:2009.11909]
Equivariant Principal ∞-bundles [arX:2112.13654]
Anyonic Defect Branes in TED-K-Theory [arX:2203.11838]
The twisted equivariant character map [SS23-TEC in preparation]
Twisted equivariant differential generalized cohomology [SS23-TED work in progress]

+ earlier work [GS] ...

Motivation/Application in string theory

Examples ([GS])

1. Type I (II) RR fields live in twisted differential KO-theory $\widehat{KO}_{\hat{\tau}}$ (K-theory $\widehat{K}_{\hat{\tau}}$).
2. Differential refinements of various twisted cohomology theories.

- $G \circlearrowleft$ *spacetime* with RR fields/D-branes in TED K-theory.

$$\boxed{\text{charges, fields} \in \text{TED K-theory}}$$

[Grady-Schreiber-S.] in progress.

- Later we will consider TED K-theory of *configuration spaces* to connect to cohomotopy that describes fields/flux quantization in M-theory.

Rosetta stone

Topological phases	Topological K theory	String/M theory
Single-electron state in d-dim crystal	Line bundle over Brillouin d-torus	Single probe D-brane of codimension d
Single positron state	Virtual line bundle over Brillouin torus	Single anti $\overline{\text{D}}$-brane of codimension d
Bloch-Floquet transform	Hilbert space bundle over Brillouin d-torus	Unstable (tachyonic) D9/$\overline{\text{D9}}$-brane state
Dressed Dirac vacuum operator	Family of Fredholm operators	Tachyon field
Valence bundle of electron/positron states	Virtual bundle of their kernels and cokernels	stable D-brane state f after tachyon condensation
Topological phase	K-theory class	Stable D-brane charge

Symmetry protection	Twisted equivariance	Global symmetries
CPT symmetry	KR/KU/KO-theory	Type I/IIA/IIB
Crystallographic symmetry	Orbifold K-theory	Spacetime orbifolding
Gauged internal symmetry	Inner local system-twist	Inside of orbi-singularity

Rosetta stone

Topological order	Twisted differentiality	Gauge symmetries
Berry connection	Differential K-theory	Chan-Paton gauge field
Mass terms	Differential K-LES	Axio-Dilaton RR-field
Nodal point charge	Flat K-theory	Defect brane charge

Anyonic defects	TED-K of Configurations	Defect branes
N band nodes	N-punctured Brillouin torus	N defect branes
Interacting n-electron states around N band nodes	Vector bundle over n-point configuration space in N-punctured Brillouin torus	Interacting n probe branes around N defect branes
$\mathfrak{su}_2$-anyon species	Holonomy of inner local system	$\mathrm{SL}(2,\mathbb{Z})$-charges of defect branes

novel

Subtleties with the equivariant setting

Key subtlety:

Constructing the twisted equivariant Chern character as map of equivariant moduli stacks, to give the differential theory.

- this is previously under-developed, because equivariant classifying spaces are generally *far from simply-connected/nilpotent* (complications with RHT).
- Equivariant Eilenberg-MacLane spaces pick a *local system* $\Rightarrow$ deg. 1 twist. (i.e. Equivariant classifying space for equivariant gerbes)

We will see how some of this works in the applications.

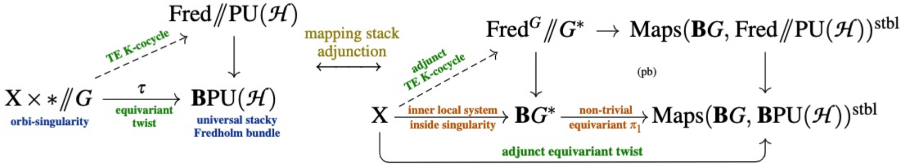
Turn on its head

The Galois-theoretic/local systems effect hidden in this technicality is responsible for the appearance of conformal blocks and braid group statistics in TED-K

$\Rightarrow$ Mathematically accounts for anyons in topological phases of matter.

Novel mathematical reflection of the physics folklore that something special happens inside the singularities.
[Matches all the expectations of strings/D-branes on ADE singularties]

Get a rich structure from the singularities/fixed points of the group action:



$G^* := \mathrm{Hom}\big(G, \mathrm{U}(1)\big)$ denotes the Pontrjagin-dual group
(note $BPU \sim B^2U(1)$ by stability of reps of PU).
G finite subgroup of $SU(2)$ so that $H^2_{\mathrm{Grp}}(G, U(1) = 0$.

The B-field twist – Inner local systems

On fixed loci (orbi-singularities)

$$\mathrm{X} /\!\!/ G \;\simeq\; \mathrm{X} \times * /\!\!/ G \;=\; \mathrm{X} \times \mathrm{B}G$$

the B-field twist induces *secondary* twists by "inner local systems":

$$\begin{aligned}
\overset{\textbf{stable twists over fixed locus}}{\mathrm{Maps}\big(\mathrm{X} \times * /\!\!/ G, \mathbf{B}^2\mathrm{U}(1)\big)} &\simeq \mathrm{Maps}\big(\mathrm{X} \times \mathbf{B}G, \mathbf{B}^2\mathrm{U}(1)\big) \\
&\simeq \mathrm{Maps}\big(\mathrm{X}, \mathrm{Maps}(\mathbf{B}G, \mathbf{B}^2\mathrm{U}(1))\big) \\
&\simeq \mathrm{Maps}\big(\mathrm{X}, \mathbf{B}G^* \times \mathbf{B}^2\mathrm{U}(1)\big) \\
&\simeq \underbrace{\mathrm{Maps}\big(\mathrm{X}, \mathbf{B}G^*\big)}_{\textbf{inner local systems}} \times \underbrace{\mathrm{Maps}\big(\mathrm{X}, \mathbf{B}^2\mathrm{U}(1)\big)}_{\textbf{bundle gerbes}}
\end{aligned}$$

Here we are assuming $G \underset{\text{fin}}{\subset} \mathrm{SU}(2)$ so that $H^2_{\mathrm{Grp}}\big(G, \mathrm{U}(1)\big) = 0$.

$G^* := \mathrm{Hom}\big(G, \mathrm{U}(1)\big)$ denotes the Pontrjagin-dual group.

Combining with the B-field twist

Combining CPT-quantum symmetries (more later) and the *twisting by a B-field*:

The homotopy fiber sequence of 2-stacks (the hat means correct stacky cofibrant resolution)

$$\mathbf{B}\mathrm{U}(\mathcal{H}) \longrightarrow \mathbf{B}\big(\mathrm{U}(\mathcal{H})/\mathrm{U}(1)\big) \xrightarrow[]{\overset{\text{universal Dixmier-Douady class}}{\mathrm{DD}}} \mathbf{B}^2\mathrm{U}(1)$$

induces a surjection of equivalence classes of equivariant higher bundles

$$\underset{\text{equivariant projective bundles}}{\pi_0 \;\mathrm{Maps}\Big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}\big(\mathrm{U}(\mathcal{H})/\mathrm{U}(1)\big)\Big)} \xrightarrow{\mathrm{DD}_*} \underset{\text{equivariant bundle gerbes}}{\pi_0 \,\mathrm{Maps}\Big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}^2\mathrm{U}(1)\Big)}$$

which has a natural section:

$$\underset{\text{equivariant bundle gerbes}}{\overset{\text{“stable twists”}}{\pi_0\mathrm{Maps}\big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}^2\mathrm{U}(1)\big)}} \hookrightarrow \underset{\text{full quantum-symmetry twists}}{\pi_0\,\mathrm{Maps}\bigg(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}\Big(\tfrac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \big(\{\mathrm{e},C\}\times\{\mathrm{e},P\}\big)\Big)\bigg)}$$

- The LHS is what is done in the literature (and what we focus on).
- The RHS should be richer ... yet unstable ... perhaps beyond K-theory.

The B-field twist – Inner local systems – Chern character.

One aspect of these twistings becomes transparent under the Chern character:

$$\underset{\text{complex K-theory}}{\mathrm{KU}^0(\mathrm{X})} \xrightarrow[\text{Chern character}]{} \mathrm{KU}^0(\mathrm{X};\, \mathbb{C}) \;\simeq\; \underbrace{\bigoplus_{d \in \mathbb{N}} H^{2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\, \mathbb{C}),\, \mathrm{d}\Big)}_{\text{periodic de Rham cohomology}}$$

(1) For twist by B-field $\widehat{B}_2$ there is a closed differential 3-form H_3 such that:

$$\underset{\substack{\text{plain B-field}\\ \text{-twisted K-theory}}}{\mathrm{KU}^{n+\widehat{B}_2}(\mathrm{X})} \xrightarrow[\text{Chern character}]{\text{twisted}} \mathrm{KU}^{\widehat{B}_2}(\mathrm{X};\, \mathbb{C}) \;\simeq\; \underbrace{\bigoplus_{d \in \mathbb{Z}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\, \mathbb{C}),\, \mathrm{d} + H_3 \wedge\Big)}_{\text{3-twisted periodic de Rham cohomology}}$$

(2) For twist by inner $\mathbb{Z}_\kappa$-local system, there is closed 1-form ω_1 with holonomy in $\mathbb{Z}_\kappa \subset \mathrm{U}(1)$ such that:

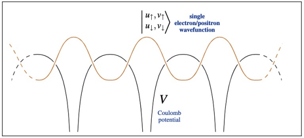
$$\underset{\substack{\text{inner local system}\\ \text{-twisted K-theory}\\ \text{of A-type singularity}}}{\mathrm{KU}^{n+[\omega_1]}_{C_\kappa}(\mathrm{X})} \xrightarrow[\text{Chern character}]{\text{twisted equivariant}} \underbrace{\bigoplus_{\substack{d \in \mathbb{Z}\\ 1 \leq r \leq \kappa}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\, \mathbb{C}),\, \mathrm{d} + r \cdot \omega_1 \wedge\Big)}_{\text{1-twisted periodic de Rham cohomology}}$$

This is the hidden 1-twisting in TED-K – will be related to anyons [talk by Urs]

Vacua of electron/positron field in Coulomb background

Q: Why K-theory in condensed matter? State-antistate, natural for e^- / e^+

Fact [Klaus-Scharf77][Carey-Hurst-O'Brien82]
The vacua of the free Dirac field in a classical Coulomb background are characterized by Fredholm operators



$$
\overset{\text{finite-dimensional kernel}}{\ker(F)} \hookrightarrow H \xrightarrow[\text{bounded linear}]{\overset{\text{Fredholm operator}}{F}} H \twoheadrightarrow \overset{\text{finite-dimensional cokernel}}{\operatorname{coker}(F)}
$$

$$
\ker(F) = \big\{ \psi \in H \,\big|\, \forall_\phi \ \langle \phi | F | \psi \rangle = 0 \big\} \qquad \operatorname{coker}(F) = \big\{ \psi \in H \,\big|\, \forall_\phi \ \langle \psi | F | \phi \rangle = 0 \big\}
$$

on the single-electron/positron Hilbert space:

$$
\begin{tikzcd}
\underset{\text{electron states in dressed vacuum}}{\ker(F)} \arrow[r, hook] & \overset{\text{single electron Hilbert space}}{H} \arrow[dr, "F"] & H \\
& \oplus & \oplus \\
& \underset{\text{single positron Hilbert space}}{H} \arrow[ur, "F^*"'] & H \arrow[r, two heads] & \underset{\text{positron states in dressed vacuum}}{\operatorname{coker}(F)}
\end{tikzcd}
$$

dressed vacuum Fredholm operator

$$
\begin{aligned}
\overset{\text{total charge in dressed vacuum}}{\operatorname{ind}(F)} &= \overset{\text{number of electrons in dressed vacuum state}}{\dim\big(\ker(F)\big)} - \overset{\text{number of positrons in dressed vacuum state}}{\dim\big(\operatorname{coker}(F)\big)} \\
&= \dim\big(\operatorname{coker}(F^*)\big) - \dim\big(\ker(F^*)\big)
\end{aligned}
$$

"Total charge"

Quantum symmetries

- What symmetry operators act on these?
- On these dressed vacua of electron/positron states, the following *CPT-twisted projective group* (graded Fredhom, so we need two copies)

$$\underbrace{\overbrace{\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}}^{\text{even projective unitary group}} \rtimes \Big(\underbrace{\overset{\text{grading involution}}{\mathbb{Z}_2}}_{\{\mathrm{e},P\}} \times \underbrace{\overset{\text{complex conjugation}}{\mathbb{Z}_2}}_{\{\mathrm{e},T\}} \Big)}_{\text{group of quantum symmetries}}$$

- The following explains CPT for the Dirac field:

$$C := PT\,, \quad P\cdot\big[U_+,\,U_-\big] := \big[U_-,\,U_+\big]\cdot P\,, \quad T\cdot\big[U_+,\,U_-\big] := \big[\overline{U}_+,\,\overline{U}_-\big]\cdot T$$

naturally acts by conjugation:

$$\begin{array}{rcccl}
[U_+,U_-] & : & F & \longmapsto & U_+^{-1}\circ F\circ U_- \\
C\cdot[U_+,U_-] & : & F & \longmapsto & U_-^{-1}\circ F^t\circ U_+ \\
P\cdot[U_+,U_-] & : & F & \longmapsto & U_-^{-1}\circ F^*\circ U_+ \\
T\cdot[U_+,U_-] & : & F & \longmapsto & U_+^{-1}\circ \overline{F}\circ U_-
\end{array}$$

These happen to be the group of quantum symmetries in condensed matter.

[Karoubi 70]: $\{X \longrightarrow \mathrm{Fred}^p_{\mathbb{K}}\}/_\sim \;=\; \begin{cases} \mathrm{KU}^p(X)=\mathrm{KU}^{p+2}(X) & \mid\; \mathbb{K}=\mathbb{C} \\ \mathrm{KO}^p(X)=\mathrm{KO}^{p+8}(X) & \mid\; \mathbb{K}=\mathbb{R} \end{cases}$

Group actions

$$
\begin{tikzcd}
 & 1 \arrow[d] & 1 \arrow[d] & \\
\substack{\text{Crystal}\\ \text{lattice}\\ \text{(full)}} & \Lambda \arrow[r, hook] \arrow[d, hook] & \mathbb{R}^d \arrow[d, hook] & \substack{\text{Euclidean}\\ \text{translation}\\ \text{group}} \\
\substack{\text{Crystallographic}\\ \text{group}\\ \text{(discrete)}} & G_{\mathrm{cr}} \arrow[r, hook] \arrow[d, two heads] & \mathbb{R}^d \rtimes \mathrm{O}(d) \arrow[d, two heads] & \substack{\text{Euclidean}\\ \text{isometry}\\ \text{group}} \\
\substack{\text{Point}\\ \text{group}\\ \text{(finite)}} & G_{\mathrm{pt}} \arrow[r, hook] \arrow[d] & \mathrm{O}(d) \arrow[d] & \substack{\text{Euclidean}\\ \text{rotation}\\ \text{group}} \\
 & 1 & 1 &
\end{tikzcd}
$$

$$
\textbf{BT} \quad \overset{\substack{\text{Brillouin}\\ \text{torus}}}{\widehat{\mathbb{T}}^d} \;:=\; \overset{\substack{\text{all}\\ \text{Euclidean momenta}}}{\mathrm{Hom}\big(\Lambda, \mathbb{R}\big)} \Big/ \overset{\substack{\text{trivial}\\ \text{lattice momenta}}}{\mathrm{Hom}\big(\Lambda, \mathbb{Z}\big)} \;\simeq\; \overset{\text{Pontrjagin dual group}}{\mathrm{Hom}\big(\Lambda, \mathrm{U}(1)\big)}
$$

$$
G_{\mathrm{pt}} \circlearrowright \widehat{\mathbb{T}}^d \;=\; \mathrm{Hom}\big(G_{\mathrm{pt}} \circlearrowright \Lambda, \mathrm{U}(1)\big)
$$

$$
\overset{\substack{\text{external}\\ \text{symmetry}}}{G_{\mathrm{ext}}} \;:=\; \overset{\substack{\text{crystallographic}\\ \text{point symmetry}}}{\mathrm{G}_{\mathrm{pt}}} \rtimes \overset{\text{CP-symmetries}}{\big(\{\mathrm{e}, T\} \times \{\mathrm{e}, C\}\big)} \qquad \vdash \qquad \overset{\substack{\text{crystallographic}\\ \text{orbi-orientifold}}}{X /\!\!/ G_{\mathrm{ext}}}\,.
$$

Group actions 2

Symmetries G		
External G_{ext}		**Internal** G_{int}
Crystallographic G_{pt}, e.g. $\{\mathrm{e}, I\}$	**Time-reversal** $\{\mathrm{e}, T\}$, $\{\mathrm{e}, C\}$	**On-site** e.g. $\{\mathrm{e}, P\}$, $\{\mathrm{e}, S\}$

$$\overbrace{\underbrace{G_{\text{pt}} \rtimes \{\mathrm{e}, T/C\}}_{G_{\text{ext}}} \times \underbrace{\{\mathrm{e}, P\} \times \{\mathrm{e}, S\}}_{G_{\text{int}}}}^{G}$$

orbi-orienti-folded Brillouin torus

$$\widehat{\mathbb{T}}^d /\!\!/ G \simeq \widehat{\mathbb{T}}^d /\!\!/ G_{\text{ext}} \times * /\!\!/ G_{\text{int}}$$

Symmetry name		Action
G_{pt}	*Crystallographic point transformation*	Orthogonal transformation on BT
I	*Inversion*	Point reflection on BT
T	*Time reversal*	Point reflection on BT & complex conj. on obs.
C	*Charge conjugation*	Point reflection on BT & complex conj. + deg. flip on obs.
P	*Parity reversal*	No action on BT & degree flip on obs.
S	*Spin flip*	No action on BT & some projective action on obs.

Corresponding description

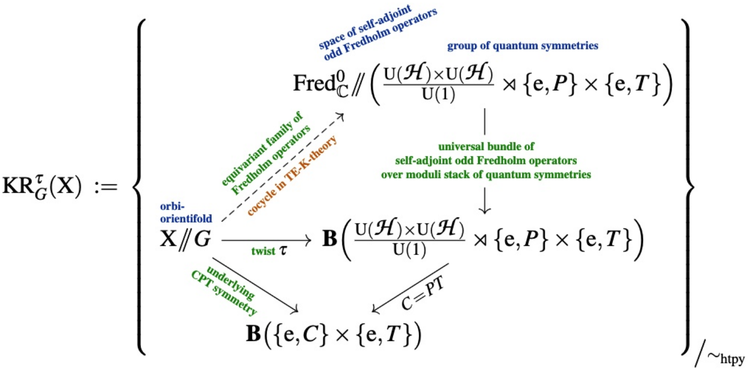
Twisted equivariance	Sector of TED-K	Type of symmetry protection
Projective involutions	KR/KU/KO-theory	Quantum CPT-symmetries
Orbifolding	Orbifold K-theory	Crystallographic symmetries
Orbi-singularity	Fixed point theory	Internal symmetries
Orbi-singularity with Inner local system	Twisted differential Fixed point theory	"fictitious" gauge symmetries (anyonic braiding phases)

$$
\left\{ \begin{array}{c} G_{\mathrm{ext}}\text{-SPT/SET} \\ \text{crystalline insulator phases} \end{array} \right\} = \coprod_{[\tau]} \mathrm{KR}^{\tau}\big(\widehat{\mathbb{T}}^d /\!\!/ G_{\mathrm{ext}}\big) \simeq \coprod_{[\tau]} \left\{ \begin{tikzcd} & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, P\} \times \{\mathrm{e}, T\} \right) \arrow[d] \\ \widehat{\mathbb{T}}^d /\!\!/ G_{\mathrm{ext}} \arrow[ur, dashed] \arrow[r, "\tau"'] & \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, P\} \times \{\mathrm{e}, T\} \right) \end{tikzcd} \right\}
$$

SPT=symmetry protected phases: G-equivariantly nontrivial, in that it cannot be adiabatically deformed (while repsecting given G-symmetry) to a topologically trivial phase.

SET=symmetry enhanced phases: one in addition requires that the underlying topological phase (i.e., forgetting the quantum symmetry) is trivial.

Homotopy classes of quantum-symmetry equivariant families of self-adjoint odd Fredholm operators constitute *twisted equivariant* KR-*cohomology*:



Concrete setting: Free topological phases of matter

$\Rightarrow$ Idea: *Single*-particle valence bundle of electrons in crystalline insulator classified by topological K-theory of Brillouin torus equivariant wrt quantum symmetries [Kitaev 09] [FreedMoore12]

Single particle valence bundle

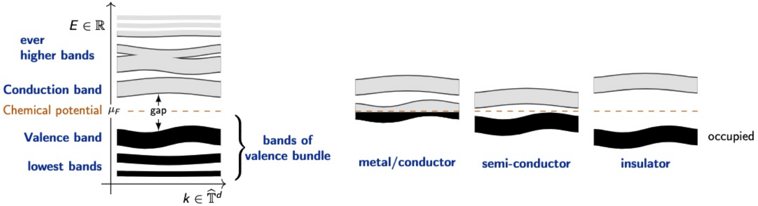
$$V = \left\{ k \in \widehat{\mathbb{T}}^d, |\psi\rangle \in H \oplus H \;\middle|\; \left|\langle\psi|H_k|\psi\rangle\right| \le \mu_F \right\} \subset B$$

Bundle of all relativistic Bloch states

$$\downarrow$$

Brillouin torus of momenta in crystal

$$\widehat{\mathbb{T}}^d$$



[Mathai-Thiang] use T-duality (in analogy with D-branes) to understand such systems.

Example – Orientifold KR-theory

Let I be Inversion action on 2-torus $\widehat{\mathbb{T}}^2 \simeq \mathbb{R}^2/\mathbb{Z}^2$ and trivial action on observables

$$\begin{array}{ccc} \mathbb{T}^2 & \xrightarrow{\;I\;} & \mathbb{T}^2 \\ k & \longmapsto & -k\,, \end{array} \qquad \begin{array}{ccc} \mathrm{Fred}^0_{\mathbb{C}} & \xrightarrow{\;I\;} & \mathrm{Fred}^0_{\mathbb{C}} \\ F & \longmapsto & F\,. \end{array}$$

If T acts as I on $\mathbb{T}^2$, then $\mathrm{KR}^{\widehat{T}^2 = +1}$ is *Atiyah's Real K-theory* aka *orienti-fold* K-theory: (the hat is the image on Fredholm operators)
(so this gives an interpretation of KR-theory as the time-reversal-equivariant topological phases).

$$\mathbf{KR}\Big(\widehat{\mathbb{T}}^{0,2}\Big) \simeq \left\{ \begin{tikzcd} & & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[d] \\ \mathbb{T}^2 /\!\!/ \{\mathrm{e}, I\} \arrow[urr, dashed] \arrow[rr, "\widehat{T}^2 = +1"] \arrow[dr, "\text{inversion of space}", "I \mapsto T"'] & & \mathbf{B}\big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[dl, "\text{complex conj. of observables}"'] \\ & \mathbf{B}\{\mathrm{e}, T\} \arrow[u, phantom, "\text{combined with}"] & \end{tikzcd} \right\} \Big/ {\sim_{\mathrm{htpy}}}$$

But what happens on I-fixed loci, i.e., on "orientifolds" ?

CPT Quantum symmetries – 10 global choices

[FreedMoore12]

Equivariance group	$G =$	$\{e\}$	$\{e,P\}$	$\{e,T\}$		$\{e,C\}$		$\{e,T\}\times\{e,C\}$			
Realization as quantum symmetry	$\tau : \widehat{T}^2 =$			$+1$	-1			$+1$	-1	-1	$+1$
	$\widehat{C}^2 =$					$+1$	-1	$+1$	$+1$	-1	-1
Maximal induced Clifford action anticommuting with all G-invariant odd Fredholm operators	$E_{-3} =$								$\mathrm{i}\widehat{T}\widehat{C}\beta$		
	$E_{-2} =$					$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$		
	$E_{-1} =$		$\widehat{P}\beta$			$\widehat{C}\beta$		$\widehat{C}\beta$	$\widehat{C}\beta$		
	$E_{+0} =$	β	β	β	$\begin{pmatrix}\beta & 0\\ 0 & -\beta\end{pmatrix}$	β	β	β	β	β	β
	$E_{+1} =$				$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$		$\widehat{C}\beta$			$\widehat{C}\beta$	$\widehat{C}\beta$
	$E_{+2} =$				$\begin{pmatrix}0 & \mathrm{i}\\ \mathrm{i} & 0\end{pmatrix}$		$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$	
	$E_{+3} =$				$\begin{pmatrix}0 & -\widehat{T}\\ \widehat{T} & 0\end{pmatrix}$					$\mathrm{i}\widehat{T}\widehat{C}\beta$	
	$E_{+4} =$				$\begin{pmatrix}0 & \mathrm{i}\widehat{T}\\ \mathrm{i}\widehat{T} & 0\end{pmatrix}$						
τ-twisted G-equivariant KR-theory of fixed loci	$\mathrm{KR}^{\tau} =$	KU^0	KU^1	KO^0	KO^4	KO^2	KO^6	KO^1	KO^3	KO^5	KO^7

Example – TI-equivariant KR-theory is KO^0-theory

The combination $T \cdot I$ acts trivially on the domain space and by complex conjugation on observables.

Hence $(T \cdot I)$-equivariant $(\widehat{T}^2 = +1)$-twisted KR-theory is KO^0-theory:

$$
\mathrm{KO}^0(X) \;\simeq\; \left\{
\begin{tikzcd}
 & & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[d] \\
X \times * /\!\!/ \{\mathrm{e}, TI\} \arrow[urr, dashed] \arrow[rr, "\widehat{T}^2 = +1"] \arrow[dr, "\substack{\text{no action}\\ \text{on space}}", "TI \mapsto T"'] \arrow[r, phantom, "\substack{\text{combined}\\ \text{with}}" near end, shift right=6ex] & & \mathbf{B}\big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[dl, "\substack{\text{complex conj.}\\ \text{of observables}}"'] \\
 & \mathbf{B}\{\mathrm{e}, T\} &
\end{tikzcd}
\right\}_{\big/ \sim_{\mathrm{htpy}}}
$$

$n =$	0	1	2	3	4	5	6	7	8	9	$\cdots$
$\mathrm{KO}^0(S^n_*) =$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\cdots$

Interacting enhancement via Hypothesis H

- Interacting n-electron wavefunctions are functions on the space of n points in Brillioun torus
- Pauli exclusion $\Rightarrow$ these span vector bundle away from the locus of coinciding points:

$$
\begin{array}{ccl}
\text{Slater-Bloch valence bundle of interacting } n\text{-electron states} & \mathscr{V}_n \subset & \displaystyle\coprod_{(k^1,\cdots,k^n)} \mathrm{Span}\overbrace{\Big\{\Psi_{i_1,\cdots,i_n}\Big(\big(k^1,s^1\big),\cdots,\big(k^n,s^n\big)\Big)\Big\}}^{\text{Slater determinants of Bloch states}}_{\substack{(i_1,\cdots,i_n)\\(s^1,\cdots,s^n)}} \\
& \Big\downarrow & \\
\substack{\text{configuration space of}\\ n \text{ “probe” points}} & \displaystyle\operatorname*{Conf}_{\{1,\cdots,n\}}\Big(\underbrace{\widehat{\mathbb{T}}^d \setminus \{k_1,\cdots,k_N\}}_{\substack{\text{in complement of } N \text{ “nodal”}\\ \text{points inside the Brillouin torus}}}\Big) & = \left\{ (k^1,\cdots,k^n) \in \big(\widehat{\mathbb{T}}^d\big)^n \,\middle|\, \underbrace{\forall_{i \neq j}\, k^i \neq k^j}_{\substack{\text{Pauli}\\ \text{exclusion}}} \;\text{ and }\; \underbrace{\forall_{i,I}\, k^i \neq k_I}_{\substack{\text{nodal}\\ \text{singularities}}} \right\}
\end{array}
$$

This locus is known as the **configuration space of n points**.

Idea: Do not avoid these points, but embrace them (as we did for singularities).

Deep theorems (Hopf, Pontrjagin, Segal) relate configurations of points to *Cohomotopy* theory – a *non-abelian* generalized cohomology theory:

$$
\begin{array}{lccl}
\text{Cohomotopy} & \pi^n(X) & = & \mathrm{Map}\big(X, \overbrace{S^n}^{\text{sphere}}\big)/_{\mathrm{htpy}} \\
 & & & \quad\big\downarrow {\scriptstyle \left(S^n \to K(\mathbb{Z},n)\right)_*} \\
\text{ordinary cohomology} & H^n(X;\mathbb{Z}) & = & \mathrm{Map}\big(X, \underbrace{K(\mathbb{Z},n)}_{\text{E.-M.-space}}\big)/_{\mathrm{htpy}}
\end{array}
$$

Hypothesis H: 4-Cohomotopy is a flux quantization law for C-field in 11d super-gravity:

$$
\begin{array}{ccc}
\pi^4(X) & \xrightarrow[\mathrm{ch}^n_\pi]{\text{gen. character map}} & \left\{ \begin{pmatrix} G_7 \\ G_4 \end{pmatrix} \in \Omega^\bullet_{\mathrm{dR}}(X) \;\middle|\; \overbrace{\begin{array}{l} \mathrm{d}\,G_7 = -\tfrac{1}{2} G_4 \wedge G_4 \\ \mathrm{d}\,G_4 = 0 \end{array}}^{\text{Bianchi identity for supergravity C-field}} \right\}\Big/_{\mathrm{cncrd}} \\
\big\downarrow {\scriptstyle \left(S^4 \to K(\mathbb{Z},4)\right)_*} & & \big\downarrow {\scriptstyle \begin{array}{l} G_7 \mapsto 0 \\ G_4 \mapsto G_4 \end{array}} \\
H^4(X;\mathbb{Z}) & \longrightarrow & \left\{ G_4 \in \Omega^\bullet_{\mathrm{dR}}(X) \;\middle|\; \mathrm{d}\,G_4 = 0 \right\}\Big/_{\mathrm{cncrd}}
\end{array}
$$

In fact, tangentially twisted 4-Cohomotopy, coupling this to the spacetime metric, implies a list of subtle topological conditions expected to hold in M-theory.

Twisted Cohomotopy theory [FSS]

To include nontrivial spacetime topology (needed for anomaly cancellation), we involve structures arising from the tangent bundle.
In degree $d-1$, there is a canonical twisting on Riemannian d-manifolds, given by the unit sphere bundle in the orthogonal tangent bundle:

$$
\underset{\text{Cohomotopy theory}}{\overset{\text{J-twisted}}{}}\ \pi^{TX^d}(X^d) := \left\{
\begin{tikzcd}
 & & \overset{\text{tangent unit sphere bundle}}{S(TX^d)} \arrow[r] \arrow[d, "p"] & \overset{\text{universal tangent unit sphere bundle}}{S^{d-1} /\!\!/ \mathrm{O}(d)} \arrow[d] \\
X \arrow[urr, dashed, bend left, "\substack{\text{continuous section}\\ = \text{twisted cocycle}}"] \arrow[rr, equal] & & X \arrow[r, "TX^d", "\substack{\text{classifying map of}\\ \text{tangent/frame bundle}}"'] & BO(d)
\end{tikzcd}
\right\} \Big/ \sim_{\frac{\text{homotopy}}{BO(d)}}
$$

$$
\simeq \left\{
\begin{tikzcd}
X \arrow[rr, dashed, "\text{continuous function}"] \arrow[dr, "\substack{TX^d\\ \text{twist}}"'] & \arrow[d, phantom, "\Swarrow\ \text{homotopy}"] & S^{d-1} /\!\!/ \mathrm{O}(d) \arrow[dl] \\
 & BO(d) &
\end{tikzcd}
\right\} \Big/ \sim_{\frac{\text{homotopy}}{BO(d)}}
$$

Since the canonical morphism $\mathrm{O}(d) \longrightarrow \mathrm{Aut}(S^{d-1})$ is known as the *J-homomorphism*, we may call this *J-twisted Cohomotopy theory*, for short.

Hypothesis H: *The C-field 4-flux & 7-flux forms in M-theory are subject to charge quantization in J-twisted Cohomotopy cohomology theory in that they are in the image of the non-abelian Chern character map from J-twisted Cohomotopy theory.*

Framed M-branes and topological invariants	[arX:1310.1060]
ADE-Equivariant Cohomotopy and M-branes	[arX:1805.05987]
The rational higher structure of M-theory	[arX:1903.02834]
Cohomotopy implies M-theory anom. canc.	[arX:1904.10207]
Cohomotopy implies M5-brane WZ term	[arX:1906.07417]
Cohomotopy implies tadpole cancellation	[arX:1909.12277]
Cohomotopy implies intersecting brane obs.	[arX:1912.10425]
Cohomotopy implies M5-brane anom. canc.	[arX:2002.07737]
Cohomotopy implies String structure on M5	[arX:2002.11093]
Cohomotopy implies GS-mechanism	[arX:2008.08544]
Cohomotopy implies GS-mechanism on M5	[arX:2011.06533]
M/F-Theory as Mf-theory	[arX:2103.01877]

$\Rightarrow$ Cancellation of main anomalies:

Half-integral flux quantization	$\big[\underbrace{G_4 + \tfrac{1}{4}p_1}_{=:\,\widetilde{G}_4 \text{ integral flux}}\big] \in H^4(X,\mathbb{Z})$
Background charge	$\underbrace{q(\widetilde{G}_4)}_{\text{quadratic form}} = \widetilde{G}_4\big(\widetilde{G}_4 - \underbrace{\tfrac{1}{2}p_1}_{=(\widetilde{G}_4)_0}\big)$
DMW-anomaly cancellation	$W_7(TX) = 0$
Integral equation of motion	$\underbrace{\mathrm{Sq}^3}_{=\beta\mathrm{Sq}^2}(\widetilde{G}_4) = 0$
M5-brane anomaly cancellation	$\underbrace{I^{\mathrm{M5}}_{\mathrm{ferm}}}_{\text{chiral fermion}} + \underbrace{I^{\mathrm{M5}}_{\mathrm{sd}}}_{\text{self-dual 3-flux}} + \underbrace{I^{\mathrm{bulk}}_{\mathrm{infl}}}_{\text{bulk inflow}} = 0$
M2-brane tadpole cancellation	$\underbrace{N_{\mathrm{M2}}}_{\text{number of M2-branes}} + q(\widetilde{G}_4) = \underbrace{I_8}_{\text{One loop polynomial}}$

The perspective

- Single electron: old
- Multiple electrons $\to$ interacting ground states.
- The beauty of this is that K-theory still applies and is physically motivated.

	Single electron	**Multiple electrons interacting**
Classical	X	cohomotopy moduli involving $\mathrm{Maps}(X, S^4)$
Quantum	$K(X)$	$K\,(\text{cohomotopy moduli})$

Quantum field theory of Defect branes via TED-K

- Correlators of some Euclidean QTFs are encoded in the de Rham cohomology of a configuration space of points ([Berghoff14]).

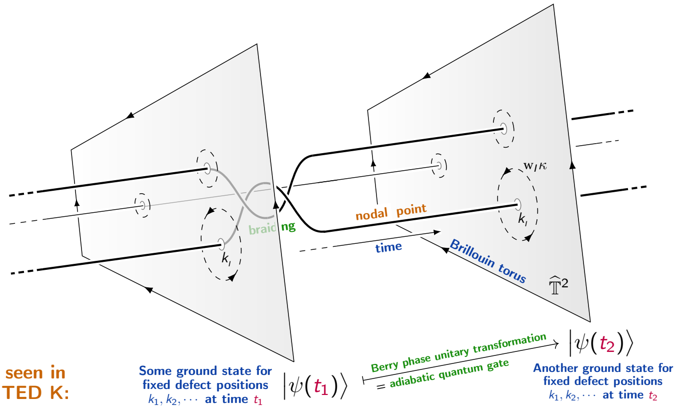
$$\boxed{\text{Correlators} = H^*_{\mathrm{dR}}(\text{Configuration space})}$$

- Example: for 3d Chern-Simons theory [AxelrodSinger94], leads to Kontsevich's graph complexes;
- The evident suggestion that therefore the generalized cohomology (such as the K-theory) of configuration spaces might reflect yet more details of quantum field theory.

$$\boxed{E^*(\text{Configuration space})}$$

- We find a fair bit of deep structure in QFT is reflected in the (twisted, equivariant, differential, generalized, ...) cohomology of configuration spaces of points.

- Realize the picture below physically and mathematically.
- Previously: no derivation of topological phases.
- Applying K-theory to configuration spaces makes the anyons.



Precise proposal for interacting enhancement via "Hypothesis H"

- Evaluate TED K-cohomology not on Brillouin torus/spacetime-orbifold itself, but on its configuration space of points, and generally: on its Cohomotopy moduli.
- The TED K-cohomology of n-point configurations in Brillouin torus classifies valence bundle of n-electron interacting states.
- Moreover, generalized cohomology *of* intersecting Cohomotopy moduli spaces reflects intersecting brane observables expected from non-abelian DBI action.

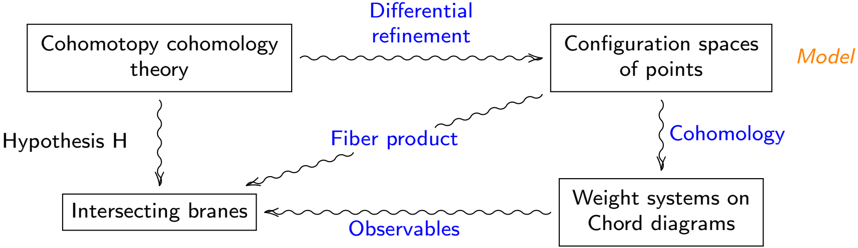
$\rightsquigarrow$ **Hypothesis H**:

Quantum observables on non-perturbative interacting ground states are in generalized cohomology (e.g. TED K-theory) of twisted Cohomotopy moduli.

Differential cohomotopy and D-brane gauge theories

Zoom in beyond foundational/structural M-theoretic considerations [S.-Schreiber]:

1. A differential refinement of Cohomotopy cohomology theory is given by *un-ordered configuration spaces of points*.
2. The fiber product of such differentially refined Cohomotopy cocycle spaces describing $\mathrm{D6} \perp \mathrm{D8}$-brane intersections is homotopy-equivalent to the *ordered configuration space of points* in the transversal space.
3. The higher observables on this moduli space are equivalently weight systems on horizontal chord diagrams.



Combining the above seemingly distinct mathematical areas reflect a multitude of effects expected on brane intersections in string theory. So aside from structural utility for M-theory, Hypothesis H implies:

- *M-theoretic observables on* $\mathrm{D6} \perp \mathrm{D8}$*-configurations* (cf. parametrized).
- *Chan-Paton observables.*
- *String topology operations.*
- *Multi-trace observables of BMN matrix model.*
- *Hanany-Witten states.*
- *BLG 3-Algebra observables.*
- *Bulk Wilson loop observables.*
- *Single-trace observables*
- *of SYK & BMN model.*
- *Fuzzy funnel observables.*
- *Supersymmetric indices.*
- *'t Hooft string amplitudes.*

Top-down M-theory via Hypothesis H: knowledge about gauge field theory and perturbative string theory is not used in deriving the algebras of observables of M-theory, but only to interpret them.

M-brane realization

Quantum states of branes as cohomology of Cohomotopy cocycle spaces according to Hypothesis H.

	Hanany-Witten theory of codim=3 branes	**Seiberg-Witten theory of defect codim=2 branes**
Intersecting branes	$\mathrm{NS5} = \mathrm{D6} \perp \mathrm{D8}$	$\text{“M3''} = \mathrm{M5} \perp \mathrm{M5}$
Charge quantization law	4-Cohomotopy	3-Cohomotopy
Cocycle space / configuration space	$\coprod_{N_{\mathrm{f}} \in \mathbb{N}} \underset{\{1, \cdots, N_{\mathrm{f}}\}}{\mathrm{Conf}} (\mathbb{R}^3)$	$\coprod_{N \in \mathbb{N}} \underset{\{1, \cdots, N\}}{\mathrm{Conf}} (\mathbb{R}^2)$
(Twisted, fiberwise) (Co)Homology / quantum states + observables	Weight systems/ Chord diagrams	$\widehat{\mathfrak{sl}}_2^k$-Conformal blocks/ Braid group reps

→ talk by Urs

Thank you!