

D-Branes and Doubled Geometry

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Outline

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with Vincenzo Emilio Marotta

[arXiv: [1810.03953](#) , [1910.09997](#) , [2104.07774](#) , [2202.05680](#)]

Introduction

- ▶ Manifest T-duality invariance: Correct description involves algebroids and 'doubled geometry'
- ▶ **Generalized geometry:** $TM \longrightarrow \mathbb{T}M = TM \oplus T^*M$ with structure of (twisted) Courant algebroid (Hitchin '02; Gualtieri '04)
- ▶ **Double field theory (DFT):** $M \longrightarrow \mathcal{M} = M \times \tilde{M}$
Solving strong constraint (polarisation) reduces DFT structure to standard Courant algebroid (Siegel '93; Hull & Zwiebach '09; Hohm, Hull & Zwiebach '10; ...)
- ▶ In this talk: Global description of DFT provided by para-Hermitian geometry and metric algebroids
- ▶ Phenomena described by T-duality: What is a **D-brane** in this setting?
- ▶ Conformal boundary conditions for **Born sigma-model:** Covariant version of doubled sigma-models for duality-symmetric string theory (Duff '90; Tseytlin '90; Hull '05; Berman, Copland & Thompson '07; Hull & Reid-Edwards '09; Copland '11; Lee & Park '13 ...)
- ▶ Generalize previous treatments of D-branes and doubled geometry (Hull '04; Lawrence, Schulz & Wecht '06; Albertsson, Kimura & Reid-Edwards '08; Hull & Sz '19; Sakatani & Uehara '20)

Double Field Theory and Para-Hermitian Geometry

- ▶ **Para-Hermitian Geometry:** A “real version” of complex Hermitian geometry
- ▶ Addresses global issues of doubled geometry, provides simple elegant framework for generalized flux compactifications and non-geometric backgrounds
(Hull '04; Vaisman '12; Freidel, Rudolph & Svoboda '17; Chatzistavrakidis, Jonke, Khoo & Sz '18; Svoboda '18; Marotta & Sz '18; Mori, Sasaki & Shiozawa '19; Hassler, Lüst & Rudolph '19; Kimura, Sasaki & Shiozawa '22; ...)
- ▶ Other applications of para-Hermitian geometry:
 - ▶ Formulation of $\mathcal{N} = 2$ vector multiplets in Euclidean spacetimes
(Cortés, Mayer, Mohaupt & Saueressig '03; Cortés & Mohaupt '09)
 - ▶ Lagrangian and non-Lagrangian dynamical systems (Marotta & Sz '18)
 - ▶ 2D ‘twisted’ SUSY sigma-models (Abou-Zeid & Hull '99; Stojevic '09; Hu, Moraru & Svoboda '19)
- ▶ **Modern perspective:** Geometry on $\mathbb{T}M = TM \oplus T^*M \longleftrightarrow T\mathcal{M}$
- ▶ **Examples:** Fibre bundles (T^*M, TM, \dots) , Doubled Lie groups, Drinfel'd doubles, and quotients $(T^{2d}, \text{doubled twisted torus}, \dots)$

Para-Hermitian Manifolds

- ▶ **Para-complex structure** $K : T\mathcal{M} \rightarrow T\mathcal{M}$ on $2d$ -dim manifold \mathcal{M} with $K^2 = +\mathbb{1}$, whose ± 1 -eigenbundles L_{\pm} have same rank d
- ▶ Splits $T\mathcal{M} = L_+ \oplus L_-$, integrability of L_+ and L_- independent
- ▶ **Para-Hermitian structure** (K, η) : metric η with signature (d, d) satisfying compatibility $K^T \eta K = -\eta$
- ▶ **Fundamental 2-form** $\omega = \eta K$, $d\omega =$ 'generalized fluxes'
If symplectic ($d\omega = 0$) then (K, η) **para-Kähler structure**
- ▶ L_{\pm} maximally isotropic with respect to η and ω
- ▶ **Example:** $\mathcal{M} = T^*M \xrightarrow{\pi} M$ with canonical symplectic 2-form ω_0 ; para-Hermitian structures correspond to isotropic splittings of

$$0 \longrightarrow \ker(\pi_*) \longrightarrow T(T^*M) \longrightarrow \pi^*(TM) \longrightarrow 0$$

- ▶ **Para-Hermitian vector bundles:** $\mathbb{T}M = TM \oplus T^*M$, exact Courant algebroids, ...

Generalized Metrics & Born Geometry

- ▶ **B-transformation** of (K, η) on $T\mathcal{M} = L_+ \oplus L_-$:

$$e^B = \begin{pmatrix} \mathbb{1} & 0 \\ B & \mathbb{1} \end{pmatrix} \in \text{Aut}(T\mathcal{M}) \text{ where } B : L_+ \longrightarrow L_- \text{ with}$$
$$\eta(B(X), Y) = -\eta(X, B(Y)) =: b(X, Y)$$

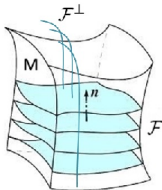
- ▶ $K \longrightarrow K_B = e^B K e^{-B}$ where (K_B, η) is another para-Hermitian structure with fundamental 2-form $\omega_B = \eta K_B = \omega + 2b$
- ▶ **Generalized metric** on a para-Hermitian manifold (\mathcal{M}, K, η) :
 $I \in \text{Aut}(T\mathcal{M})$ covering id_M with $I^2 = \mathbb{1}$
Defines Riemannian metric $\mathcal{H} = \eta I$ on \mathcal{M}
- ▶ \mathcal{H} defined by metric on L_+ and B -transformation (g, b)
- ▶ If $\mathcal{H}\omega^{-1}\mathcal{H} = -\omega$ then $(\eta, \omega, \mathcal{H})$ is a **Born geometry**
Specified by metric g on L_+
- ▶ **Generalized T-duality**: $O(T\mathcal{M}) \subset \text{Aut}(T\mathcal{M})$ isometries of η , preserve Born geometry structure: $K_\vartheta = \vartheta K \vartheta^{-1}$, $\mathcal{H}_\vartheta = \vartheta^*(\mathcal{H})$ for $\vartheta \in O(T\mathcal{M})$

Born Sigma-Model

$$S[\mathbb{X}] = \frac{1}{4} \int_{\Sigma_2} \mathcal{H}_{IJ} d\mathbb{X}^I \wedge \star d\mathbb{X}^J + \frac{1}{4} \int_{\Sigma_2} \mathbb{X}^*(\omega)$$

$\mathbb{X} : \Sigma_2 \rightarrow \mathcal{M}$, $(\eta, \omega, \mathcal{H}) = \text{Born geometry on } \mathcal{M}$

- Strong Constraint:** Assuming $L_- \subset T\mathcal{M}$ involutive selects physical spacetime as a **quotient** $M = \mathcal{M}/\mathcal{F}$ by action on leaves of foliation of \mathcal{M} by \mathcal{F} with $L_- = T\mathcal{F}$ (Hull & Reid-Edwards '09; Vaisman '12; Park '13; Lee, Strickland-Constable & Waldram '15)



Reduces Born sigma-model if there is a Riemannian submersion

$q : (\mathcal{M}, \mathcal{H}) \rightarrow (M, \bar{g})$ such that $g = q^* \bar{g}$ is leaf-invariant (Marotta & Sz '19)

- Killing Lie algebroid:** Lie algebroid A over Riemannian target space $(\mathcal{M}, \mathcal{H})$ such that $\nabla^A \mathcal{H} = 0$ for flat A -connection $\nabla^A : \Gamma(A) \times \Gamma(T\mathcal{M}) \rightarrow \Gamma(T\mathcal{M})$ (Kotov & Strobl '14; ...)

Gauging the Born Sigma-Model

- ▶ Apply to Killing Lie algebroid $\rho: T\mathcal{F} \hookrightarrow T\mathcal{M}$:
Born sigma-model can be gauged along foliation $\mathcal{F} \iff \mathcal{L}_V g = 0$ for all $V \in \Gamma(T\mathcal{F})$, where \mathcal{H} is determined by metric g on L_+
- ▶ If \mathcal{M}/\mathcal{F} is smooth, then there is a Riemannian submersion $q: (\mathcal{M}, \mathcal{H}) \rightarrow (\mathcal{M}/\mathcal{F}, \bar{g})$ such that $g = q^* \bar{g}$
- ▶ ω descends to 2-form \bar{b} on \mathcal{M}/\mathcal{F} if L_+ is locally spanned by projectable vector fields V_i : $[V_i, W] \in \Gamma(T\mathcal{F})$ for all $W \in \Gamma(T\mathcal{F})$, and $\mathcal{L}_W \eta = 0$
- ▶ $d\mathbb{X}' \rightarrow D^A \mathbb{X}' = d\mathbb{X}' - \rho^{lj} A_j$ for $T\mathcal{F}$ -valued connection 1-form A
- ▶ Euler-Lagrange equation for A gives 'self-duality constraint':

$$D^A \mathbb{X} = \eta^{-1} \mathcal{H} \star d\mathbb{X}$$

Reduces Born sigma-model to standard string sigma-model into physical spacetime $(\mathcal{M}/\mathcal{F}, \bar{g}, \bar{b})$

- ▶ Generalized T-duality $(\mathcal{M}, \eta, K, \mathcal{H}) \rightarrow (\mathcal{M}, \eta, K_\vartheta, \mathcal{H}_\vartheta)$ with $T\mathcal{M} = L_+^\vartheta \oplus L_-^\vartheta$; if $L_-^\vartheta = T\mathcal{F}^\vartheta$ then sigma-models for $(\mathcal{M}/\mathcal{F}, \bar{g}, \bar{b})$ and $(\mathcal{M}/\mathcal{F}^\vartheta, \bar{g}^\vartheta, \bar{b}^\vartheta)$ are T-dual

Boundary Conditions for the Born Sigma-Model

- ▶ (σ, τ) local coordinates for Σ , with boundary $\partial\Sigma$:

$$\left(-\frac{1}{2} \mathcal{H}_{IJ} \partial_\sigma \mathbb{X}^J d\sigma + \omega_{IJ} \partial_\tau \mathbb{X}^J d\tau \right) \Big|_{\partial\Sigma} = 0$$

- ▶ Solution given by subbundle $L \subset T\mathcal{M}$ ("tangent vectors"):

$$0 \longrightarrow L \longrightarrow T\mathcal{M} \longrightarrow T\mathcal{M}/L \longrightarrow 0$$

and orthogonal splitting $T\mathcal{M} = L \oplus L^\perp$ wrt generalized metric \mathcal{H} , with orthogonal projectors $\Pi : T\mathcal{M} \rightarrow L$ and $\Pi^\perp : T\mathcal{M} \rightarrow L^\perp$

- ▶ Together with self-duality constraint, conformal boundary conditions are solved by

$$\eta(\Pi(Z_I), \Pi(Z_J)) = 0 = \eta(\Pi^\perp(Z_I), \Pi^\perp(Z_J)) \quad , \quad \omega(\Pi(Z_I), \Pi(Z_J)) = 0$$

for a local frame $\{Z_I\}$ of $T\mathcal{M}$

- ▶ Thus L is maximally isotropic wrt both η, ω (but not necessarily integrable)

Born D-Branes

- ▶ **Def.:** A **Born D-brane** is a maximally isotropic subbundle $L_D \subset T\mathcal{M}$ such that $K(L_D) = L_D$
- ▶ **Examples:** Eigenbundles L_{\pm} of para-complex structure K
- ▶ If $W_D = L_+ \cap L_D$ has constant rank, then $L_D = W_D \oplus \eta^{\sharp}(\text{Ann}(W_D))$ with metric

$$\mathcal{H}_D = \begin{pmatrix} g^D & 0 \\ 0 & \eta g_D^{-1} \end{pmatrix}, \quad g^D = g|_{W_D}$$

- ▶ Generalized T-duality $\vartheta \in O(T\mathcal{M})$ sends D-brane L_D for Born sigma-model $S(\mathcal{H}, \omega)$ into (\mathcal{M}, K, η) to D-brane $L_D^{\vartheta} = \vartheta(L_D)$ for Born sigma-model $S(\mathcal{H}_{\vartheta}, \omega_{\vartheta})$ into $(\mathcal{M}, K_{\vartheta}, \eta)$
- ▶ Standard picture of D-branes as submanifolds when $L_D = T\mathcal{F}_D$ is integrable: Each leaf of foliation \mathcal{F}_D of \mathcal{M} is a d -dim submanifold of \mathcal{M} whose tangent vectors satisfy the boundary conditions
- ▶ Chan-Paton bundles induced by B -transformations (with suitable integrality)

Dirac Structures

- ▶ **Generalised submanifold** (\mathcal{W}, L) for an exact Courant algebroid $E \rightarrow \mathcal{M}$ with anchor $\rho: \mathcal{W} \subset \mathcal{M}$, $L \subset E$ maximally isotropic integrable with $\rho(L) = T\mathcal{W}$ (Gualtieri '04; Zambon '07)
- ▶ **Generalized para-complex D-brane** supported on $\mathcal{W} \subseteq \mathcal{M}$ for an exact Courant algebroid $E \rightarrow \mathcal{M}$ with anchor ρ and generalized para-complex structure \mathcal{K} : Generalized submanifold (\mathcal{W}, L) such that $\mathcal{K}(L) = L$
- ▶ Born sigma-model corresponds (up to B -transformations) to the 'large Courant algebroid' $\mathbb{T}\mathcal{M} = T\mathcal{M} \oplus T^*\mathcal{M}$ with generalized metric determined by \mathcal{H} (Alekseev & Strobl '04; Ševera '15)
- ▶ (η, K) gives generalized para-complex structure $\mathcal{K}_K = \begin{pmatrix} K & 0 \\ 0 & -K^\top \end{pmatrix}$ preserving splitting on $\mathbb{T}\mathcal{M}$ (Hu, Moraru & Svoboda '19)
- ▶ Born D-brane L_D defines **Dirac structure** $D = L_D \oplus \text{Ann}(L_D)$ on $\mathbb{T}\mathcal{M}$
- ▶ For each leaf \mathcal{W}_D of L_D , $(\mathcal{W}_D, D|_{\mathcal{W}_D})$ is a generalized para-complex brane: $\mathcal{K}_K(D|_{\mathcal{W}_D}) = D|_{\mathcal{W}_D}$ (since $K(L_D) = L_D$)

Metric Algebroids

- ▶ **Metric algebroid:** Anchored pseudo-Euclidean vector bundle $(E, \langle -, - \rangle_E, \rho)$ with bracket $[-, -]_E : \Gamma(E) \times \Gamma(E) \rightarrow \Gamma(E)$:
 - ▶ $\rho(e) \cdot \langle e_1, e_2 \rangle_E = \langle [e, e_2]_E, e_2 \rangle_E + \langle e_2, [e, e_2]_E \rangle_E$
 - ▶ $\langle [e, e]_E, e_1 \rangle_E = \frac{1}{2} \rho(e_1) \cdot \langle e, e \rangle_E$
- ▶ Any anchored pseudo-Euclidean vector bundle admits infinitely many metric algebroid structures (Vaisman '12)
- ▶ Metric algebroids \longleftrightarrow symplectic 2-algebroids (del Carpio-Marek '15; Marotta & Sz '21)
(aka 'symplectic nearly Lie 2-algebroids' (Bruce & Grabowski '16)
'symplectic pre-NQ-manifolds of degree 2' (Deser & Sämann '16)
'pre-QP-manifolds' (Heller, Ikeda & Watamura '16))
- ▶ Any para-Hermitian manifold (\mathcal{M}, K, η) admits a unique 'canonical' metric algebroid bracket $[-, -]_{T\mathcal{M}}$ preserving K , with anchor $\mathbb{1}_{T\mathcal{M}}$:
 L_{\pm} are involutive wrt $[-, -]_{T\mathcal{M}}$, and $[-, -]_{T\mathcal{M}}$ is compatible with Lie algebra of vector fields on $T\mathcal{M}$ (Freidel, Rudolph & Svoboda '17)

Related Algebroids

- ▶ **Pre-Courant algebroids:** $\rho : E \rightarrow T\mathcal{M}$ bracket morphism
(symplectic almost Lie 2-algebroids (Bruce & Grabowski '16))
- ▶ **Courant algebroids:** Jacobi identity for $[-, -]_E$
(symplectic Lie 2-algebroids (Ševera '98; Roytenberg '99))
- ▶ **DFT algebroid** on a para-Hermitian manifold (\mathcal{M}, K, η) :
 $\rho : (E, \langle -, - \rangle_E) \rightarrow (T\mathcal{M}, \eta)$ isomorphism of pseudo-Euclidean vector bundles with $\rho \rho^* = \eta^{-1}$ (Chatzistavrakidis, Jonke, Khoo & Sz '18; Svoboda '18; Hu, Moraru & Svoboda '19; Grewcoe & Jonke '20; Marotta & Sz '21)
- ▶ **Example:** Splitting and projection of large Courant algebroid $\mathbb{T}\mathcal{M}$ is a DFT algebroid isomorphic to canonical metric algebroid, reduces to standard Courant algebroid on physical spacetime \mathcal{M}/\mathcal{F} when $L_- = T\mathcal{F}$ — DFT algebroids lie “in between” two Courant algebroids
- ▶ **Note:** Generalised para-complex branes make sense for exact pre-Courant algebroids — extension to metric algebroids?

Adding a Wess-Zumino Term

- ▶ Difference between any two metric algebroid brackets on (\mathcal{M}, K, η) is a 3-form H_D on \mathcal{M}
- ▶ **Canonical 3-form H_{can}** : Choose canonical metric algebroid and reference bracket induced by Levi-Civita connection of η ($H_{\text{can}} = 0$ iff $d\omega = 0$)
- ▶ (\mathcal{M}, K, η) is **admissible** if $H_2(\mathcal{M}) = 0$ and $\frac{1}{4\pi} [H_{\text{can}}] \in H^3(\mathcal{M}; \mathbb{Z})$
- ▶ Defines Wess-Zumino term $\frac{1}{2} \int_V \mathbb{X}^*(H_{\text{can}})$, $\partial V = \Sigma$ for Born sigma-model, H_{can} represents \check{S} evera class of associated Courant algebroid
- ▶ For open strings, consider **relative maps** $\mathbb{X} : (\Sigma, \partial\Sigma) \rightarrow (\mathcal{M}, \mathcal{W})$ and **relative admissibility**: $H_2(\mathcal{M}, \mathcal{W}) = 0$, $\frac{1}{4\pi} [(H_{\text{can}}, B_{\text{can}})] \in H^3(\mathcal{M}, \mathcal{W}; \mathbb{Z})$ for some 2-form B_{can} on \mathcal{W}
- ▶ $L_{\mathcal{W}} := \text{im}(T\mathcal{W} \rightarrow T\mathcal{M})$ is a Born D-brane iff $\mathcal{W} \subset \mathcal{M}$ Lagrangian submanifold, $B_{\text{can}} = 0$ and $H_{\text{can}}|_{L_{\mathcal{W}}} = 0$ (orientation condition)
- ▶ Can only couple to *flat* Chan-Paton bundles — analogous to A-branes

Generalized Para-Complex Branes

- ▶ Generalized submanifolds (\mathcal{W}, L) on an exact Courant algebroid correspond to subbundles

$$L = L^F := \{X + \alpha \in T\mathcal{W} \oplus T^*\mathcal{M}|_{\mathcal{W}} \mid \alpha|_{\mathcal{W}} = \iota_X F\} \subset \mathbb{T}\mathcal{M}$$

for some 2-form F on \mathcal{W} with $dF + H_{\text{can}}|_{\mathcal{W}} = 0$

- ▶ **Example:** For a Born D-brane L_D and its Dirac structure $D = L_D \oplus \text{Ann}(L_D)$, $(\mathcal{W}_D, D|_{\mathcal{W}_D})$ is a generalized para-complex D-brane iff $H_{\text{can}}|_{\mathcal{W}_D} = 0$ (since $F = 0$)
- ▶ **Example:** For a Born D-brane L_D and given $F \in \Omega^2(\mathcal{W})$, (\mathcal{W}_D, L^F) is a generalized para-complex D-brane iff

$$K^\top(\iota_X F) + \iota_{K(X)} F \in \text{Ann}(T\mathcal{W}_D) \quad \forall X \in \Gamma(T\mathcal{W}_D)$$

If $H_{\text{can}}|_{\mathcal{W}_D} = 0$ (integrability), $F \in \Omega_{\mathbb{Z}}^2(\mathcal{W}_D)$ and K integrable, then F is the curvature of a para-holomorphic Chan-Paton bundle (C, ∇^C) on \mathcal{W}_D — analogous to B-branes (Lawn & Schäfer '05)

D-Branes on the Physical Spacetime

- ▶ D-branes are defined by “tangent vectors” – distributions on tangent bundle of target space, need to be integral to interpret leaves as D-brane worldvolumes
- ▶ When $L_- = T\mathcal{F}$ and $q : (M, \mathcal{H}) \rightarrow (M = \mathcal{M}/\mathcal{F}, \bar{g})$ is a Riemannian submersion, $dq|_{L_+} : L_+ \rightarrow TM$ fibrewise isomorphism
- ▶ Born D-brane $L_D = T\mathcal{F}_D \subset TM$ induces $dq(L_D) = T\mathcal{F}_D^q \subseteq TM$, leaves of foliation \mathcal{F}_D^q supported by physical D-branes in (M, \bar{g}, \bar{b})
- ▶ **Example:** $L_- = T\mathcal{F} \implies$ 0-branes on M (fully Dirichlet)
 L_+ integrable \implies spacetime-filling D-branes (fully Neumann)
- ▶ D-branes are associated with Dirac structures on Courant algebroid for corresponding sigma-model (Zabzine '04; Asakawa, Sasa & Watamura '12)
- ▶ Consider reduction of Born D-branes as reduction of Dirac structures, using techniques of Courant algebroid reduction (Bursztyn, Cavalcanti & Gualtieri '05; Zambon '07)

Dirac Reduction of Born D-Branes

- ▶ For $A = T\mathcal{F} \oplus \{0\} \subset \mathbb{T}\mathcal{M}$, A^\perp spanned by $Y + d(q^*f)$ for projectable $Y \in \Gamma(T\mathcal{M})$ and $f \in C^\infty(M)$, which are 'basic'
- ▶ Hence large Courant algebroid $\mathbb{T}\mathcal{M}$ reduces to standard Courant algebroid $\mathbb{T}M$ through pullback diagram

$$\begin{array}{ccc}
 \frac{(T\mathcal{F} \oplus \{0\})^\perp}{T\mathcal{F} \oplus \{0\}} & \longrightarrow & \mathbb{T}M \\
 \downarrow & & \downarrow \\
 \mathcal{M} & \xrightarrow{q} & M
 \end{array}$$

- ▶ For a Born D-brane $L_D = T\mathcal{F}_D \subset T\mathcal{M}$, $D = L_D \oplus \text{Ann}(L_D)$ is a Dirac structure for large Courant algebroid $\mathbb{T}\mathcal{M}$ such that $D \cap A^\perp$ still spanned by $Y + d(q^*f)$
- ▶ Hence if L_D admits a sub-bundle spanned by projectable vector fields, then D descends to a Dirac structure D_{red} on $M = \mathcal{M}/\mathcal{F}$

Example: D-Branes on Doubled Nilmanifolds

- ▶ $H = 3d$ Heisenberg group with Drinfel'd double $T^*H = H \ltimes \mathbb{R}^3$, basis $\{Z_i, \tilde{Z}^i\}_{i=x,y,z}$ of left-invariant vector fields on $T(T^*H)$
- ▶ (\mathcal{M}, K, η) : $\mathcal{M} = \Gamma_m \backslash T^*H$ for discrete cocompact subgroup Γ_m with $m \in \mathbb{Z}$, $K(Z_i) = +Z_i$, $K(\tilde{Z}^i) = -\tilde{Z}^i$, and η induced from duality pairing between $\text{Lie}(H)$ and \mathbb{R}^3
- ▶ **Nilmanifold**: Principal T^3 -bundle $\mathcal{M} \rightarrow N_m = \text{nilmanifold of degree } m$
 - ▶ $L_+ = \text{D3-brane filling } N_m$, Dirac structure $TN_m \subset \mathbb{T}N_m$
 - ▶ $L_D = \text{Span}(Z_x, Z_y, \tilde{Z}^z)$ reduces to Dirac structure associated with foliation of N_m with T^2 leaves wrapped by D2-branes
- ▶ T^3 with H -flux: T^3 -fibration $\mathcal{M} \rightarrow T^3$, $m = \text{DD class of gerbe on } T^3$
 B -transformation sends $\{Z_i, \tilde{Z}^i\} \rightarrow \{Z'_i, \tilde{Z}'^i\}$, $K \rightarrow K'$
 - ▶ $L_D = \text{Span}(Z'_x, \tilde{Z}'^y, \tilde{Z}'^z)$ yields Dirac structure associated with foliation of T^3 with S^1 leaves wrapped by D1-branes, T-dual to D0-branes on N_m from reducing Born D-brane L_-
 - ▶ $H_{\text{can}} = -\frac{3}{2}m dx \wedge dy \wedge dz$, $H_{\text{can}}(Z'_x, Z'_y, Z'_z) \neq 0$ forbids D3-branes wrapping T^3 for $m \neq 0$