A Non-Lorentzian View of the M5-brane

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Plan

- $\diamond$ Introduction
- $\diamond$ Part I: 5D Liftshitz Gauge Theories and Schrödinger Symmetry
- $\diamond$ Part II: Omega Deformation and $OSP(6|4)$
- $\diamond$ Conclusions

Introduction

CFT's are a central topic in String/M-Theory. Of particular note are those above 4D where there are no standard techniques available in general.

Various approaches: Bootstrap, Deconstruction, appeal to String Theory ... DLCQ

On the one hand we have a rather formal abstract CFT's: *c.f.* a telephone directory

On the other hand there are reported sightings of a non-abelian 2-form in 6D [everyone here?]

How can we define these theories and relate them to the well-known and loved geometrical structures of gauge theory?

In particular we know that a simple reduction on a circle leads to a familiar 5D Yang-Mills description.

A key problem is the lack of a Lagrangian.

Even without asking for fancy symmetries or non-abelian 2-form connections there is no good ‘playground’ of UV complete Lagrangians

i.e. what would such a theory look like? (*c.f.* [Saemann,Schmidt,Wolf,...],[NL])

5D SYM is naively non-renormalizable (but perhaps is non-perturbatively well-defined without new UV degrees of freedom [Douglas],[NL,Papageorgakis, Schmidt-Sommerfeld])

Yeah maybe but not so useful

Reduction on a null circle leads to a DLCQ description given by quantum mechanics on instanton moduli space [Aharony,Berkooz,Kachru,Seiberg, Silverstein]

Not very physical or satisfactory.

In either case instanton number corresponds to the KK momentum of an emergent dimension but the non-compact 6D theory is reached only in the limit of infinite coupling:

$$g^2 = 4\pi^2 R \to \infty$$

The null reduction of a Lorentzian theory gives a theory with Schrödinger symmetry: translations in space and time, rotations, boosts a Lifshitz scaling and a special conformal transformation.

Familiar from condensed matter physics, *e.g.* [Son,...], where the KK momentum is particle number.

The important point is that there exist UV complete 5D gauge theories which flow in the IR to theories with Super-Schrödinger Symmetry.

But:

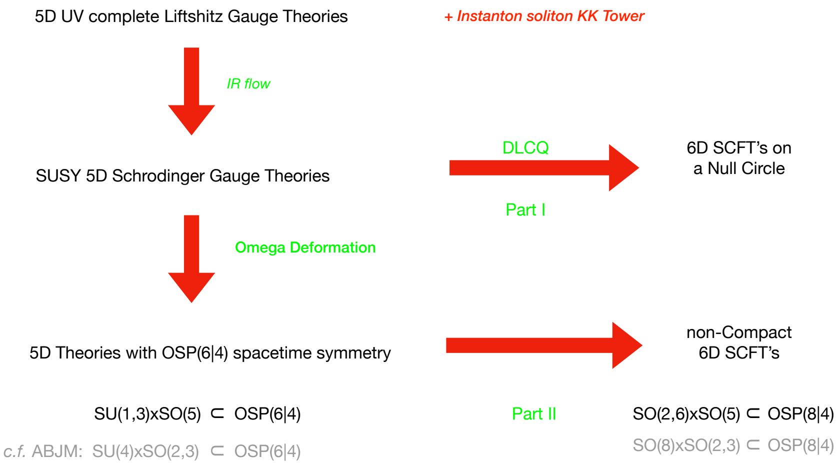
- Correlation functions are not suppressed for spatial separation.
- Reconstruction of the 6D theory is formal at best: first sum over all instanton sectors and then take the limit $R \to \infty$

Next we consider instead an Omega-deformed version with $SU(1,3)$ spacetime symmetry

- Better behaved correlators
- Can reconstruct (in principle) non-compact 6D correlators
- A coupling constant $1/k$ appears and the 6D theory is at $k = 1$.
- 5D analog of ABJM with $OSP(6|4)$ symmetry where non-perturbative effects enhance $SU(1,3) \times SO(5) \to SO(2,6) \times SO(5)$
- Weird and wonderful playground away from the familiar homeland of Lorentzian geometry.

In short lots of fun!

The Big Picture



Part I

Consider a 5D Bosonic gauge theory of the form

$$S = \frac{1}{2g^2}\mathrm{tr}\int dtd^4x\ (F_{ti})^2 - \lambda^2(D_iF_{ij})^2 + \ldots$$

One can consiter another gradient term $(D_iF_{jk})^2$ but this is equivalent to the one above up to total derivatives and a cubic term $F_{ij}[F_{jk}, F_{ki}]$

This has a lifshitz scaling symmetry

$$t \to \omega^{-2}t \qquad x^i \to \omega^{-1}x^i$$

It is perturbatively renormalisable as the A_i propagator takes the form *e.g.* [Horava][Iengo,Russo,Serone]

$$G \sim \frac{1}{E^2 - \lambda^2 p^4}$$

One can also imagine adding scalar and Fermions:

$$S_X = \frac{1}{g^2}\mathrm{Tr}\int D_t X^\dagger D_t X - \lambda_X^2 D^2 X^\dagger D^2 X + \dots$$
$$S_\psi = \frac{1}{g^2}\mathrm{Tr}\int \psi^\dagger D_t \psi - \lambda_\psi^2 D_i \psi^\dagger D_i \psi + \dots$$

Possibly in other representations of the gauge group.

Furthermore the β-function for g is negative [Horava][NL,Smith]

$$\beta(g) = -\frac{3}{2}\frac{C_2(G)g^3}{(4\pi)^2\lambda} + \frac{T_{scalar}(R)}{2(4\pi)^2\lambda_X}$$
$$\beta(\lambda) = \frac{13}{3}\frac{C_2(G)g^2}{(4\pi)^2}$$

So UV complete (sometimes) but non-Lorentzian

We claim that these theories cannot be made supersymmetric or boost invariant (although theories exist with a scalar supersymmetry) [NL,Smith]

But we can consider an RG flow induced by adding

$$M^2(F_{ij} + \star F_{ij})^2$$

now $M^2 \to \infty$ in the IR

The IR dynamics is constrained to anti-self-dual gauge fields:

$$\begin{aligned} S_{IR} &= \frac{1}{2g^2}\mathrm{tr}\int dt d^4x \; (F_{ti})^2 - \lambda_1^2 (D_i F_{ij})^2 - \frac{1}{2}\lambda_2^2 (D_i F_{jk})^2 \\ &\qquad + G_{ij}(F_{ij} + \star F_{ij}) + \ldots \\ &\cong \frac{1}{2g^2}\mathrm{tr}\int dt d^4x \; (F_{ti})^2 + G_{ij}(F_{ij} + \star F_{ij}) + \ldots \end{aligned}$$

Similarly deforming scalars

$$S_X^{(M)} = \frac{1}{g^2}\mathrm{Tr}\int D_t X^\dagger D_t X - M^2 D_i X^\dagger D_i X - \lambda_X^2 D^2 X^\dagger D^2 X$$

In the IR we are pushed onto the surface

$$D_i X = 0$$

and as a result the scalars are frozen. However, in this case we can introduce a new scalar

$$\phi = MX$$

with Lifshitz dimension 2. Now the action is

$$S_X^{(M)} = \frac{1}{g^2}\mathrm{Tr}\int \left(\frac{1}{M^2} D_t\phi^\dagger D_t\phi - D_i\phi^\dagger D_i\phi - \frac{\lambda_X^2}{M^2} D^2\phi^\dagger D^2\phi\right)$$

we would then expect the IR theory to be described by

$$S_X^{(IR)} = -\frac{1}{g^2}\mathrm{Tr}\int D_i\phi^\dagger D_i\phi$$

Such theories admit a symmetry Boost:

$$t' = t \qquad x'^i = x^i + v^i t$$
$$\phi'(t', x') = \phi(t, x)$$
$$A'_i(t', x') = A_i(t, x)$$
$$A'_t(t', x') = A_t(t, x) - v^i A_i(t, x)$$
$$G'_{ij}(t', x') = G_{ij}(t, x) + 2v_{[i}F_{|t|j]}(t, x)$$

The action then changes by

$$S' = S + \frac{v^i}{2g^2}\int dtd^4x\epsilon_{ijkl}\mathrm{Tr}(F_{tj}F_{kl})$$
$$= S + \frac{1}{2g^2}\int v \wedge \mathrm{Tr}(F \wedge F)$$

There is also a special Conformal transformation $v^i = \omega x^i$

And can be made supersymmetric: super-Schrödinger

Integrate out $G_{ij} \Longrightarrow F_{ij} = -(\star_4 F)_{ij} \Longrightarrow A_i = A_i(x, m^\alpha)$

$$S = \frac{1}{2g^2} \int dx^- g_{\alpha\beta}(m) \partial_- m^\alpha \partial_- m^\beta + \dots$$

where m^α are moduli and $g_{\alpha\beta}$ is the metric on instanton moduli space:

$$g_{\alpha\beta} = \mathrm{tr} \int d^4x \delta_\alpha A_i \delta_\beta A_i$$

c.f. DLCQ proposal
[Aharony,Berkooz,Kachru,Seiberg,Silverstein]

So the physics of the IR theory reproduces the DLCQ description of 6D CFT's on a compact null circle.

Part II

Conformal Null Compactification

Let us start by writing 6D Minkowski space in weird coordinates:

$$ds_M^2 = -2d\hat{x}^+ d\hat{x}^- + d\hat{x}^i d\hat{x}^i$$
$$= \frac{-2dx^+(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j) + dx^i dx^i}{\cos^2(x^+/2R)}$$

where $\Omega = \star_4 \Omega$, $\Omega^2 = -R^{-2}\mathbb{I}$ and

$$\hat{x}^+ = 2R\tan(x^+/2R)$$
$$\hat{x}^- = x^- + \frac{1}{4R}x^i x^i \tan(x^+/2R)$$
$$\hat{x}^i = x^i - \tan(x^+/2R) R\Omega_{ij}x^j$$

and $x^+ \in [-\pi R/2, +\pi R/2]$. Conformally rescale to

$$ds^2 = -2dx^+(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j) + dx^i dx^i$$

Since x^+ is a compact interval P_+ is discrete (for certain boundary conditions) and we diagonalize it.

What happens to the $SO(2,6)$ conformal symmetry?

$$SO(2,6) \underbrace{\longrightarrow}_{\textit{commutes with } P_+} SU(1,3) \oplus \underbrace{U(1)}_{P_+}$$

where the generators are

$$P_+ = P_+^{(6d)} + \frac{1}{4}\Omega_{ij}M_{ij}^{(6d)} + \frac{1}{8R^2}K_-^{(6d)} \qquad P_- = P_-^{(6d)}$$

$$P_i = P_i^{(6d)} + \frac{1}{2}\Omega_{ij}M_{j-}^{(6d)} \qquad M_{i+} = M_{i+}^{(6d)} - \frac{1}{4}\Omega_{ij}K_j^{(6d)}$$

$$B = -\frac{1}{4}R\,\Omega_{ij}M_{ij}^{(6d)} \qquad C^I = \frac{1}{4}\eta^I_{ij}M_{ij}^{(6d)}$$

$$K_+ = K_+^{(6d)} \qquad T = D^{(6d)} - M_{+-}^{(6d)}$$

We also break $3/4$ of the (conformal)supersymmetries:

$$16 \oplus 16 \to 8 \oplus 16$$

$$\epsilon + \hat{x}^\mu\Gamma_\mu\eta \to \epsilon + \hat{x}^\mu\Gamma_\mu\eta_+ \qquad \Gamma_+\eta_+ = 0$$

Holographic Construction

[Pope,Sadrzadeh,Scuro] showed that AdS_7 can be written as a timelike $U(1)$ Hopf fibration over $\widetilde{\mathbb{C}P}^3$:

$$ds^2_{AdS_7} \; = \; -\frac{1}{4}\left(dx^+ + e^\phi\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right)\right)^2 + ds^2_{\widetilde{\mathbb{C}P}^3}$$

Where $\widetilde{\mathbb{C}P}^3$ is defined by the surface

$$-|Z_0|^2 + |Z_1|^2 + |Z_2|^2 + |Z_3|^2 = -1$$

in $\mathbb{C}^{1,3}$ and has a manifest $SU(1,3)$ symmetry.

Reducing on x^+ the boundary metric at $\phi \to \infty$ is

$$ds^2_{bdry} = \frac{e^\phi}{4}\left[-2dx^+\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right) + dx^i dx^i\right]$$

Putting M5-branes at $\phi = const$ and reducing on x^+ leads to the action we had before when $\phi \to \infty$.

Action

$$S = \frac{k}{4\pi^2 R}\mathrm{tr}\int d^5x \Big\{ \frac{1}{2}F_{-i}F_{-i} - \frac{1}{2}\hat{D}_i X^I \hat{D}_i X^I + \frac{1}{2}\mathcal{F}_{ij}G_{ij} - \frac{i}{2}\bar{\Psi}\Gamma_+ D_- \Psi + \frac{i}{2}\bar{\Psi}\Gamma_i \hat{D}_i \Psi - \frac{1}{2}\bar{\Psi}\Gamma_+\Gamma^I[X^I, \Psi] \Big\}$$

With all fields in the adjoint of the gauge group and

$$\hat{D}_i = D_i - \tfrac{1}{2}\Omega_{ij}x^j D_-$$
$$\mathcal{F}_{ij} = F_{ij} - \frac{1}{2}\Omega_{ik}x^k F_{-j} + \frac{1}{2}\Omega_{jk}x^k F_{-i}$$
$$G_{ij} = (\star_4 G)_{ij}$$

Note that the hatted derivative has torsion: $[\hat{D}_i, \hat{D}_j] = \Omega_{ij}D_-$

Symmetries: $SU(1,3)$, $SO(5)$ R-Symmetry and 24 supercharges: $Osp(6|4)$ and a topological $U(1)$ $J \sim \star\mathrm{tr}(F \wedge F)$.

$(1,0)$ versions of these actions also exist.

Correlation Functions

$SU(1,3) \oplus U(1)$ places non-trivial constraints on correlation functions:

$$\begin{aligned}\langle \mathcal{O}^{(1)}(x_1^-, x_1^i) \dots \mathcal{O}^{(N)}(x_N^-, x_N^i) \rangle = \; & \delta_{0,p_1+\dots+p_N} \\ & \times \left[\prod_{a<b}^{N} (z_{ab}\bar{z}_{ab})^{-\alpha_{ab}/2} \left(\frac{z_{ab}}{\bar{z}_{ab}} \right)^{(p_a-p_b)R/N} \right] \\ & \times H\left(\frac{|z_{ab}||z_{cd}|}{|z_{ac}||z_{bd}|}, \frac{z_{ab}z_{bc}z_{ca}}{\bar{z}_{ab}\bar{z}_{bc}\bar{z}_{ca}} \right)\end{aligned}$$

Here α_{ab} are constants determined by the Lifshitz scaling dimensions Δ_a, p_a/R are the P_+ eigenvalues and

$$z_{ab} = x_a^- - x_b^- + \tfrac{1}{2}\Omega_{ij}x_a^i x_b^j + \frac{i}{4R}(x_a^i - x_b^i)(x_a^i - x_b^i)$$

H is an undetermined function that appears at 3-points.

Instanton Operators

So far we have cheated a bit as none of the fields in the classical Lagrangian carry any P_+ charge.

Key observation: $SU(1,3)$ is only a symmetry up to boundary terms which are non-zero if the instanton number changes.

Let us introduce local instanton operators:

$$\int D\varphi\, \mathcal{I}_{n_1}(x_1)\mathcal{I}_{n_2}(x_2)\ldots\mathcal{I}_{n_N}(x_N)\Big(\ldots\Big) = \int_{\{(x_a,n_a)\}} D\varphi\Big(\ldots\Big)$$

where $\{(x_a, n_a)\}$ indicates that we integrate over all field configurations where an instanton with instanton number $-n_a$ is created at the point $x_a = (x_a^-, x_a^i)$.

One can show that $SU(1,3) \oplus U(1)$ is restored as a symmetry of the quantum theory and $\mathcal{I}_n(x)$ have charge n/R under P_+.

Symmetry Enhancement

Note $SU(1,3) \oplus SO(5)$ is a Wick rotation of $SU(4) \oplus SO(2,3)$:

$$\underbrace{U(1)}_{topological} \oplus \underbrace{SU(1,3)}_{Spacetime} \oplus \underbrace{SO(5)}_{R} \longleftrightarrow \underbrace{U(1)}_{topological} \oplus \underbrace{SU(4)}_{R} \oplus \underbrace{SO(2,3)}_{Spacetime}$$

In fact combined with the supercharges we find $OSp(6|4)$ which is the same supergroup as ABJM for M2-branes.

In ABJM

$$\underbrace{U(1)}_{monopoles} \oplus \underbrace{SU(4)}_{R} \longrightarrow \underbrace{SO(8)}_{R}$$

And we want

$$\underbrace{U(1)}_{instantons} \oplus \underbrace{SU(1,3)}_{Spacetime} \longrightarrow \underbrace{SO(2,6)}_{Spacetime}$$

So that in both cases

$$OSp(6|4) \longrightarrow OSp(8|4)$$

Reconstructing 6D

We now want to see if somehow we can reconstruct operators and correlations functions which are those of a 6D theory with $SO(2,6)$.

Consider the following scalar operator

$$\mathcal{O}^{6d}(x^+, x^-, x^i) = \sum_{n\in\mathbb{Z}} \underbrace{f(n)}_{arbitrary} \; e^{-inx^+} \mathcal{I}_n(x^-, x^i) \; \underbrace{\mathcal{O}(x^-, x^i)}_{5D\ Operator}$$

So that

$$[P_+, \mathcal{O}^{6d}] = -i \frac{\partial}{\partial x^+} \mathcal{O}^{6d}$$

($f(n)$ is a fudge factor since we don't know enough about $\mathcal{I}_n$)

We want $\hat{\mathcal{O}}$ to have well-defined 6D scaling dimension *w.r.t.*:

$$\begin{aligned} D^{6d} &= \hat{x}^+\hat{\partial}_+ + \hat{x}^-\hat{\partial}_- + \hat{x}^i\hat{\partial}_i \\ &= \sin(x^+/R)\partial_+ + \left(x^- - \tfrac{1}{4}\sin(x^+/R)|\vec{x}|^2\right)\partial_- \\ &\qquad + \tfrac{1}{2}\Big(\big(1+\cos(x^+/R)\big)\, x^i + \sin(x^+/R)\Omega_{ij}x^j\Big)\partial_i \end{aligned}$$

Now we know

$$\begin{aligned} \left\langle \mathcal{I}_{n_1}(x_1)\mathcal{O}^{(1)}(x_1)\mathcal{I}_{n_2}(x_2)\mathcal{O}^{(2)}(x_2)\right\rangle &= \delta_{\Delta_1,\Delta_2}\delta_{0,n_1+n_2}d(\Delta_1,n_1) \\ &\quad \times \frac{1}{(z_{12}\bar{z}_{12})^{\Delta_1/2}}\left(\frac{z_{12}}{\bar{z}_{12}}\right)^{n_1} \end{aligned}$$

Demanding $\langle\hat{\mathcal{O}}^{(1)}(x_1^+,x_1^-,x_1^i)\hat{\mathcal{O}}^{(2)}(x_2^+,x_2^-,x_2^i)\rangle$ scales appropriately tells us

$$\left(n+\frac{\Delta}{2}\right)\hat{d}(\Delta,n) = \left(n-\frac{\Delta}{2}+1\right)\hat{d}(\Delta,n+1)$$

where $\hat{d}(\Delta,n) = f(n)f(-n)d(\Delta,n)$.

At least for $\Delta/2$ integer the solution is

$$\hat{d}(\Delta, n) = \begin{cases} C_{12}\binom{n+\frac{\Delta}{2}-1}{n-\frac{\Delta}{2}} & n \geq \Delta/2 \\ 0 & n < \Delta/2 \end{cases}$$

for an unknown constant C_{12}. Use this to choose the coefficients $f(n)$.

We can now compute the 2-point function

$$\langle \hat{\mathcal{O}}^{(1)}(\hat{x}_1^+, \hat{x}_1^-, \hat{x}_1^i)\hat{\mathcal{O}}^{(2)}(\hat{x}_2^+, \hat{x}_2^-, \hat{x}_2^i)\rangle$$

where the 6D Minkowski space operator is

$$\hat{\mathcal{O}}(x^+, x^-, x^i) = \underbrace{\cos^{\Delta/2}(x^+/2R)}_{conformal\ transformation} \mathcal{O}^{6d}(x^+, x^-, x^i)$$

$$
\begin{aligned}
&\langle \hat{\mathcal{O}}^{(1)}(\hat{x}_1)\hat{\mathcal{O}}^{(2)}(\hat{x}_2)\rangle \\
&= \cos^{\Delta}\left(\frac{x_1^+}{2R}\right)\cos^{\Delta}\left(\frac{x_2^+}{2R}\right)\sum_{n=\Delta/2}^{\infty} e^{-inx_{12}^+/R}\left\langle \mathcal{O}_n(x_1^-,x_1^i)\mathcal{O}_{-n}(x_2^-,x_2^i)\right\rangle \\
&= \cos^{\Delta}\left(\frac{x_1^+}{2R}\right)\cos^{\Delta}\left(\frac{x_2^+}{2R}\right)(z_{12}\bar{z}_{12})^{-\frac{\Delta}{2}}\sum_{n}^{\infty} e^{-inx_{12}^+/R}\hat{d}(\Delta,n)\left(\frac{z_{12}}{\bar{z}_{12}}\right)^{n} \\
&= C'_{12}\left[\frac{2R\,i\left(\bar{z}_{12}e^{ix_{12}^+/2R}-z_{12}e^{-ix_{12}^+/2R}\right)}{\cos\left(\frac{x_1^+}{2R}\right)\cos\left(\frac{x_2^+}{2R}\right)}\right]^{-\Delta} \\
&= C'_{12}|-2\hat{x}_{12}^+\hat{x}_{12}^- + \hat{x}_{12}^i\hat{x}_{12}^i|^{-\Delta} \\
&= C'_{12}|\hat{x}_{12}|^{-2\Delta}
\end{aligned}
$$

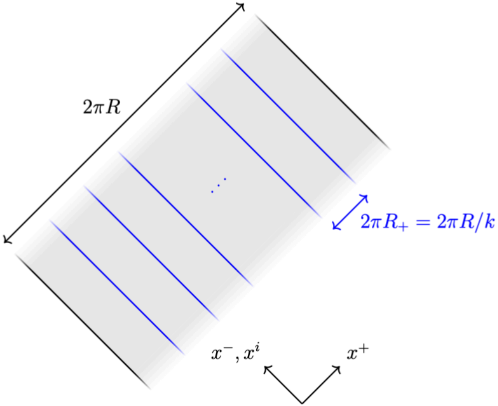
Which is the correct 6D Minkowskian $SO(2,6)$ correlator (also works for 3-point correlators and specific choices of H)

DLCQ Limit

We can take a limit $k \to \infty$ with $R/k = R_+$ fixed.

Since $\Omega_{ij} \to 0$ we have $x^+ \in (-\infty, \infty)$. But we still only capture discrete Fourier modes: ‘Ordinary’ null reduction

$$x^+ \sim x^+ + 2\pi R_+$$



The Action reduces to

$$S = \frac{1}{2\pi R_+}\mathrm{tr}\int \Big\{ \frac{1}{2}F_{-i}F_{-i} - \frac{1}{2}D_iX^ID_iX^I + \frac{1}{2}F_{ij}G_{ij} - \frac{i}{2}\bar{\Psi}\Gamma_+D_-\Psi + \frac{i}{2}\bar{\Psi}\Gamma_iD_i\Psi - \frac{1}{2}\bar{\Psi}\Gamma_+\Gamma^I[X^I,\Psi]\Big\}$$

Still has 24 supersymmetries a topological $U(1)$ but no $SU(1,3)$ (all rotations, translations, boosts, and a Liftshitz scaling): Super-Schrödinger Symmetry.

Integrate out $G_{ij} \Longrightarrow F_{ij} = -(\star_4 F)_{ij} \Longrightarrow A_i = A_i(x, m^\alpha)$

$$S = \frac{1}{4\pi R_+}\int dx^- g_{\alpha\beta}(m)\partial_- m^\alpha \partial_- m^\beta + \dots$$

where m^α are moduli. *c.f.* DLCQ proposal [Aharony,Berkooz,Kachru,Seiberg,Silverstein]

The correlation functions degenerate to Schrödinger ones: no fall off at large spatial separation, *e.g.*:

$$\langle \mathcal{O}_n^{(1)} \mathcal{O}_{-n}^{(2)} \rangle \sim \left(\frac{1}{x_{12}^-} \right)^{\Delta} \exp\left(\frac{in}{2R_+} \frac{|x_{12}^i|^2}{x_{12}^-} \right)$$

Our expression for higher point functions also reduces to known results in Schrödinger invariant theories:

$$\begin{aligned} \langle \mathcal{O}^{(1)}(x_1^-, x_1^i) \ldots \mathcal{O}^{(N)}(x_N^-, x_N^i) \rangle = \; & \delta_{0,p_1+\cdots+p_N} \\ & \times \left[\prod_{a<b}^{N} \left(\frac{1}{x_{ab}^-} \right)^{\alpha_{ab}} \exp\left(\frac{in}{2NR_+} (p_a - p_b) \xi_{ab} \right) \right] \\ & \times H\left(\frac{x_{ab}^- x_{cd}^-}{x_{ac}^- x_{bd}^-}, \xi_{ab} + \xi_{bc} + \xi_{ca} \right) \end{aligned}$$

where

$$\xi_{ab} = \frac{|x_{ab}^i|^2}{x_{ab}^-}$$

Three-Point Functions

Two-point functions are fixed by $SU(1,3)$ and hence agree with those found by reduction of a 6D SCFT.

In Schrödinger theories the functional form of the three-point function, $i.e.\ H$, can be determined if one operator has dimension $\Delta = d/2 = 2$ [Golkar,Son].

This is not interesting for us as the only operators with $\Delta = 2$ are free: *e.g.* $\mathrm{Tr}(X^I)$ and hence the three-point function vanishes.

But following their argument we find something else.

Expand the OPE as

$$\mathcal{O}_2(x)\mathcal{O}_3(0) = C_0(x)\mathcal{O}_1(0) + C_1^i \partial_i \mathcal{O}_1(0) + C_2 \partial_- \mathcal{O}_1(0) + \dots$$

Commute both sides with M_{i+}, re-expand and read off $\mathcal{O}_1(0)$ coefficient:

$$-(M_{i+})_\partial C_0 - \frac{1}{2}\Delta_3 \Omega_{ij} x^j C_0 + i p_3 x^i C_0 = -i\left(p_1 \delta_{ij} - \frac{i}{2}\Delta_1 \Omega_{ij}\right) C_1^j$$

where

$$\begin{aligned}(M_{i+})_\partial \ &= \ \left(\tfrac{1}{2}\Omega_{ij} x^- x^j - \tfrac{1}{8} R^{-2} x^j x^j x^i\right)\partial_- + x^- \partial_i \\ &\quad + \tfrac{1}{4}(2\Omega_{ik} x^k x^j + 2\Omega_{jk} x^k x^i - \Omega_{ij} x^k x^k)\partial_j\end{aligned}$$

But for $p_1 = \pm \Delta_1 / 2R$

$$\left(p_1 \delta_{ij} - \frac{i}{2}\Delta_1 \Omega_{ij}\right)\left(p_1 \delta_{ij} + \frac{i}{2}\Delta_1 \Omega_{ij}\right) = 0$$

Thus we find a single differential equation for C_0 alone:

$$\left(\delta_{ij} \pm i\frac{1}{2}\Omega_{ij}\right)\left((M_{j+})_\partial C_0 + \frac{1}{2}\Delta_3\Omega_{jk}x^k C_0 - ip_3 x^j C_0\right) = 0$$

This is solved by

$$C_0 = z^{-\frac{1}{2}\Delta_3 - Rp_3}\tilde{C}_0(\bar{z})$$

For any anti-holomorphic function of $z = x^- + \frac{i}{4R}x^i x^i$.

Furthermore scaling symmetry tells us

$$C_0(\lambda^2 x^-, \lambda x^i) = \lambda^{\Delta_1 - \Delta_2 - \Delta_3} C_0(x^-, x^i)$$

which fixes

$$C_0 = C\left(\frac{1}{z\bar{z}}\right)^{\frac{-\Delta_1+\Delta_2+\Delta_3}{4}}\left(\frac{z}{\bar{z}}\right)^{-Rp_3 - \frac{1}{4}(\Delta_1 - \Delta_3 + \Delta_3)}$$

So What: Look at 3-point function

$$\lim_{x\to 0}\langle \mathcal{O}_1^{(-p_2-p_3)}(y)\mathcal{O}_2^{(p_2)}(x)\mathcal{O}_3^{(p_3)}(0)\rangle$$

$$= \lim_{x\to 0} C_0(z_{x0},\bar{z}_{x0})\langle \mathcal{O}_1^{(-p_2-p_3)}(y)\mathcal{O}_1^{(p_2+p_3)}(0)\rangle$$

$$= C_{11}\lim_{x\to 0} C_0(z_{x0},\bar{z}_{x0})\left(\frac{1}{z_{y0}\bar{z}_{y0}}\right)^{\frac{\Delta_1}{2}}\left(\frac{z_{y0}}{\bar{z}_{y0}}\right)^{-R(p_2+p_3)}$$

$$= C_{11}C\lim_{x\to 0}\left(\frac{1}{z_{x0}\bar{z}_{x0}}\right)^{\frac{-\Delta_1+\Delta_2+\Delta_3}{4}}\left(\frac{1}{z_{y0}\bar{z}_{y0}}\right)^{\frac{\Delta_1}{2}}$$

$$\times\left(\frac{z_{x0}}{\bar{z}_{x0}}\right)^{-Rp_3-\frac{1}{4}(\Delta_1-\Delta_2+\Delta_3)}\left(\frac{z_{y0}}{\bar{z}_{y0}}\right)^{-R(p_2+p_3)}$$

On the other hand our general result tells us that:

$$\langle \mathcal{O}_1^{(-p_2-p_3)}(y)\mathcal{O}_2^{(p_2)}(x)\mathcal{O}_3^{(p_3)}(0)\rangle$$

$$= \left(\frac{1}{z_{yx}\bar{z}_{yx}}\right)^{\frac{\Delta_1+\Delta_2-\Delta_3}{4}} \left(\frac{1}{z_{x0}\bar{z}_{x0}}\right)^{\frac{-\Delta_1+\Delta_2+\Delta_3}{4}} \left(\frac{1}{z_{y0}\bar{z}_{y0}}\right)^{\frac{\Delta_1-\Delta_2+\Delta_3}{4}}$$

$$\left(\frac{z_{yx}}{\bar{z}_{yx}}\right)^{\frac{R}{3}(-2p_2-p_3)} \left(\frac{z_{x0}}{\bar{z}_{x0}}\right)^{\frac{R}{3}(p_2-p_3)} \left(\frac{z_{y0}}{\bar{z}_{y0}}\right)^{\frac{R}{3}(-p_2-2p_3)} H\left(\frac{z_{yx}z_{x0}z_{0y}}{\bar{z}_{yx}\bar{z}_{x0}\bar{z}_{0y}}\right)$$

$$\underset{x\to 0}{\longrightarrow} \left(\frac{1}{z_{x0}\bar{z}_{x0}}\right)^{\frac{-\Delta_1+\Delta_2+\Delta_3}{4}} \left(\frac{1}{z_{y0}\bar{z}_{y0}}\right)^{\frac{\Delta_1}{2}} \left(\frac{z_{x0}}{\bar{z}_{x0}}\right)^{\frac{R}{3}(p_2-p_3)}$$

$$\times \left(\frac{z_{y0}}{\bar{z}_{y0}}\right)^{-R(p_2+p_3)} H\left(\frac{z_{x0}}{\bar{z}_{x0}}\right)$$

Comparing the two we can read off that

$$H\left(\frac{z_{x0}}{\bar{z}_{x0}}\right) = C\left(\frac{z_{x0}}{\bar{z}_{x0}}\right)^{\frac{R}{3}(-p_2-2p_3)-\frac{1}{4}(\Delta_1-\Delta_2+\Delta_3)}$$

This agrees with the Fourier reduction of a 6D $SO(2,6)$ correlator:

$$\left\langle \hat{\mathcal{O}}^{(1)}(\hat{x}_1)\hat{\mathcal{O}}^{(2)}(\hat{x}_2)\hat{\mathcal{O}}^{(3)}(\hat{x}_3)\right\rangle$$
$$= \frac{\hat{C}_{123}}{|\hat{x}_1-\hat{x}_2|^{\Delta_1+\Delta_2-\Delta_3}\,|\hat{x}_2-\hat{x}_3|^{-\Delta_1+\Delta_2+\Delta_3}\,|\hat{x}_3-\hat{x}_1|^{\Delta_1-\Delta_2-\Delta_3}}$$

Thus all 5D 3-pt correlators containing one operator with $p_+ = \Delta/2R$ are those of a 6D theory.

Conclusions

In this talk we discussed a novel class offield theories in five-dimensions with a view to reconstructing 6D SCFT's

Part I

- non-Lorentzian, UV complete 5D gauge theories
- No Supersymmetry or Schrödinger Symmetry at weak coupling
- Can enhance to Super-Schrödinger in the IR

Part II

- Introduce an Ω-deformation and Super-Schrodinger $\to SU(1,3)$
- Notable control over correlation functions: including a spatial fall-off
- Can reconstruct non-compact Lorentzian 6D correlation functions
- Integrability? Formally a similar string background to ABJM: $AdS_4 \times \mathbb{C}P^3 \to S^4 \times \widetilde{\mathbb{C}P}^3$
- There is a 5D Yangian structure [Lipstein, Orchard]

Taking $R \to \infty$, $R_+ = R/k$ fixed we recover a DLCQ picture with Schrödinger symmetry.

- Loose the nice properties of correlation functions.
- Can't readily reconstruct the non-compact theory

But there are big issues:

- Can we compute something six-dimensional?
- Need a better understanding of instanton operators
- When do we really reconstruct 6D SCFTs? Not all actions we can write down should give such theories.

But there are also novel and interesting structures which we hope to report following the next Pandemic.

Just Before COVID

THANK YOU

Instanton Worldlines

G_{ij} appears as a Lagrange multiplier imposing

$$\mathcal{F}_{ij} = -(\star_4 \mathcal{F})_{ij}$$

For static configurations this is simply a well-known instanton: $F_{ij} = -(\star_4 F)_{ij}$.

But more generally we can solve this with an ’t Hooft ansatz

$$\hat{A}_i = -\frac{1}{2}\eta^I_{ij}\sigma^I \hat{\partial}_j \ln \Phi$$

provided

$$\hat{\partial}_i \hat{\partial}_i \Phi = 0$$

where $\hat{A}_i = A_i - \frac{1}{2}\Omega_{ij}x^j A_-$ and $\hat{\partial}_i = \partial_i - \frac{1}{2}\Omega_{ij}x^j\partial_-$.

Remarkably it still linearizes

Smooth spherically symmetric solutions take the form

$$\Phi = 1 + \int_{-\infty}^{\infty} \frac{\mu(\tau)}{|\tau - z|^2} d\tau \qquad z = x^- + \frac{i}{4R}|\vec{x}|^2$$

where $\mu(\tau) > 0$. Near $x^i = 0$ we find

$$\Phi = \frac{4\pi R}{|\vec{x}|^2}\mu(x^-) + finite$$

Leading to an instanton at the origin with size $\mu(x^-)$

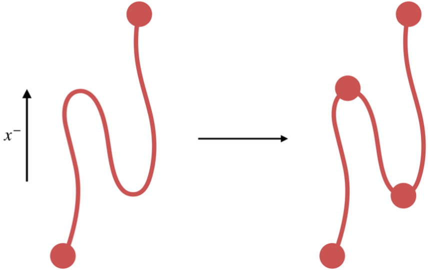
So we have a theory of instantons with dynamical sizes.

Whenever $\mu(x^-) = 0$ the instanton shrinks to zero size and the instanton number goes to zero.

But more generally we find solutions corresponding to a sum over an arbitrary number of instantons each following an arbitrary worldline $(y^-_A(\tau), y^i_A(\tau))$

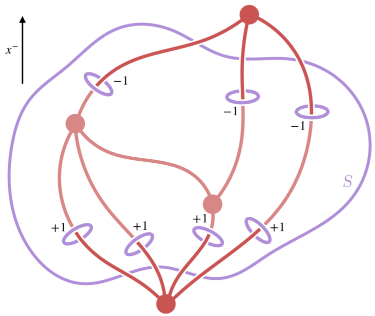
$$\Phi = 1 + \sum_{A=1}^{N} \int d\tau \frac{\mu_A(\tau)}{z(x, y_A(\tau_A))\bar{z}(x, y_A(\tau))}$$

These can be created and destroyed whenever $\mu_A(\tau) = 0$:



Turning points require $\mu_A(\tau_{turning}) = 0$ for finite action.

A typical contribution to the path integral involving instanton operators looks like:



Furthermore once instanton operators are included the action is only single-valued on configuration space if $2k \in \mathbb{Z}$