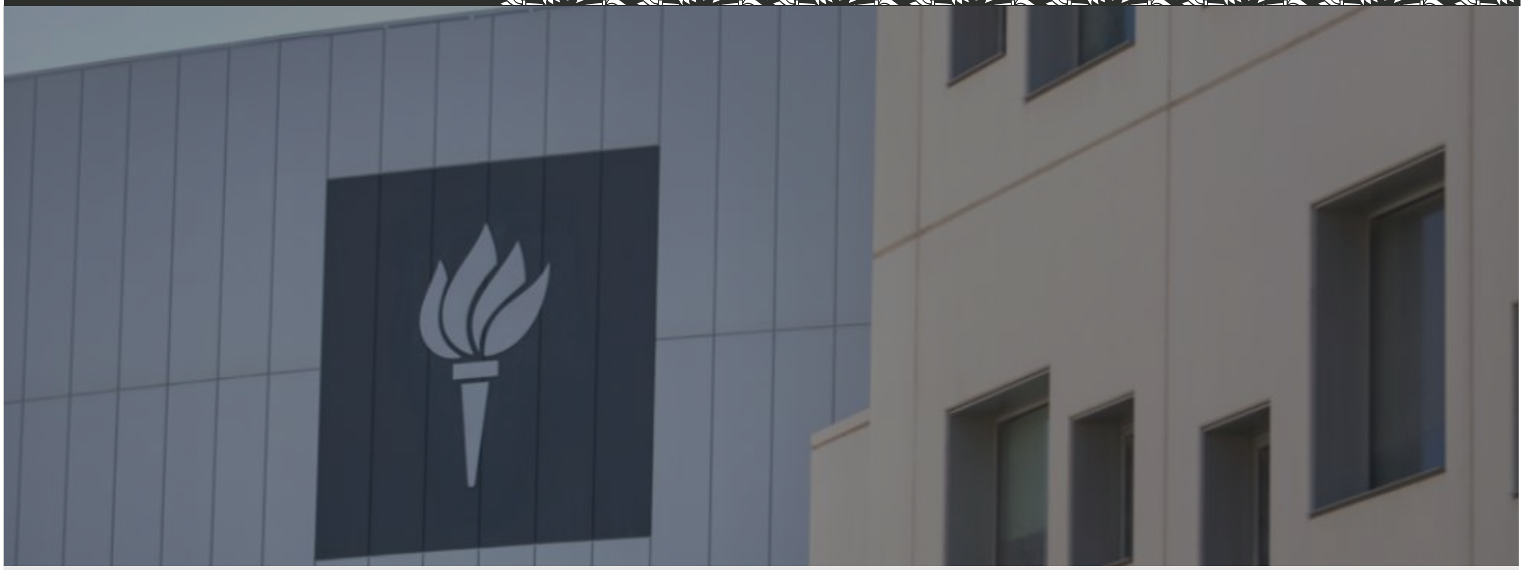

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The Home Market Effect in a Home-Biased Geography

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Abstract

A demand-side mechanism for international trade, the Home Market Effect (HME), predicts a more-than-proportional relationship between domestic expenditure and domestic production. Yet, since its inception in the 1980s by Paul Krugman, this theoretical result has only been shown to be generally valid in two-location models. I prove that the HME is maintained in an arbitrary number of locations provided the geography of trade is home-biased: the majority of domestic sales go to domestic consumers. Intuitively, without home bias, increasing domestic expenditure can actually benefit foreign production more, thus causing domestic production to rise by less, violating the more-than-proportional relationship. This result has been overlooked until now because in standard two location models all geographies are necessarily home-biased.

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1 Introduction

Why do countries trade with each other? Classically, this had been understood through supply-side comparative advantage --- such as in productivity (Ricardian) or factor endowments (Heckscher-Ohlin) --- with countries exporting the goods for which they have the comparative advantage in. Beginning in the 1980s, Paul Krugman formalized (though informally going back to Linder (1961)) a mechanism through demand-side comparative advantage. For an industry exhibiting i) scale economies and ii) transportation costs, its production is concentrated geographically due to i), and it is concentrated in the country with the larger consumer base to minimize ii). The result is that countries export products from industries for which they have the larger home market of demand (Krugman (1980)). This has become known as the Home Market Effect (HME).

This supply-demand dichotomy has gained much attention academically, with the HME being a central tenant in New Trade Theory and New Economic Geography (Krugman (1991)), and it has brought with it distinct policy implications, implying that import protection may be used as export promotion. Yet, since its inception, the canonical theoretical link between a country's demand and industrial specialization has only been shown to be valid in two-location models (Behrens et al. (2009)), or special cases of many-location models such as those with perfect geographic symmetry (Costinot et al. (2019)).[1] This is potentially problematic for the empirical relevance of the HME, as these cases are counterfactual.[2] The importance of this theoretical limitation on geography is acutely emphasized by leading scholars in the field (Thisse (2010)).[3]

In this paper, I generalize the conditions under which the HME is valid to a broad, empirically relevant class: a *home biased geography*, defined as the majority of the sales from a location going to consumers based in that location. In an otherwise arbitrary geography — with an arbitrary number of locations — I show that home bias is a sufficient condition for the HME when stated in derivatives: an increase in \$1 of domestic expenditure causes, to first order, an increase of more than \$1 of domestic production. This more-than-proportional relationship, characteristic of HME models, implies domestic net exports increase following the expenditure increase.[4]

The HME in derivatives I use is one of a number of characterizations of the HME in the

[1] Alternatively, researchers have considered more generalized demand determinants of production in many location models, such as market access (Bartelme (2015),Matsuyama (2017)) and quality (Dingel (2016)).

[2] Empirical tests of the theory usually apply the two location predictions to settings with many locations (Davis and Weinstein (2003)). Authors acknowledge their specifications are misspecified.

[3] "...it should be clear that accounting explicitly for a multiregional economy with different trade costs should rank high on the research agenda" – Thisse (2010) pg 294.

[4] The strong home market effect, as termed by Costinot et al. (2019).

literature. These are all equivalent in symmetric two-location models (Ottaviano and Thisse (2004) pg 2582), but distinct more generally (Behrens et al. (2009) pg 264). I choose this characterization as, in contrast to alternatives, I show it permits sharp theoretical results, while maintaining the tight connection with net exports.[5]

I prove my results in the canonical HME model (Helpman and Krugman (1985)) extended to an arbitrary geography by Behrens et al. (2009). Formally, I prove that a home biased geography is a sufficient condition for the HME to hold on average across countries, and I provide simulation evidence showing this appears to be sufficient for countries individually. In proving this, I make, to my knowledge, a general contribution to the linear algebra literature: the real component of each eigenvalue for the inverse of a diagonally dominant markov matrix is bounded below by one.

The intuition behind the sufficiency of a home biased geography for the HME can be explained intuitively, as follows. In a model where the geography is not home biased, then the largest consumer base of demand for a location's producers is not the domestic consumers. Thus, increasing domestic expenditure in such a geography can actually benefit producers in other locations relatively more. This reduces competitiveness of the domestic producers, causing domestic production to rise less than proportionally, or even contract. I provide a simple example demonstrating this in a perfectly geographically symmetric four location model.

This intimate connection between the HME and a home biased geography is hitherto overlooked in the literature. This is perhaps because, in two location models, all geographies are home biased when foreign trade is more costly than domestic trade — the standard parameterization. My results therefore provide a non-trivial qualitative difference of modeling many-locations vs only two locations. The latter being a convenient simplification often used in theoretical frameworks.[6]

The outline of the paper is as follows. In section 2, I present the model. In section 3, I give my theoretical results. In section 4, I provide intuition on the results, I show why this is overlooked in two location models, and provide a simple example. In section 5, I conclude. All my proofs and derivations are presented in the appendix.

[5]Behrens et al. (2009) considers a ranking in levels, and, separately, a derivative using logs of shares. Suedekum (2007) considers derivatives in the level of shares in three location models.

[6]In the same spirit, Fabinger (2011) provides a distinct qualitative difference between models with small vs large number of locations.

2 Model

For presentational clarity, I present a partial equilibrium model that is isomorphic to the canonical HME model extended to an arbitrary geography (which is general equilibrium and microfounded). See appendix B.2 for the isomorphism.

The economy consists of N locations $i \in \{1, ..., N\}$. Each location has a representative consumer and producer, and shipment of goods between locations i and j incur iceberg transport cost $\tau_{ij} \geq 1$. No arbitrage implies that the price in j of consuming a good produced in i is $P_{ij} = P_i \tau_{ij}$, where P_i is the production price in i.

Goods are differentiated by location of production and consumers have preferences with constant elasticity of substitution $\sigma > 1$ between them. Goods produced in i therefore face total quantity demanded

$$Q_i^D = \sum_j \underbrace{\tau_{ij} \frac{P_{ij}^{-\sigma}}{\sum_k {P_{kj}}^{1-\sigma}} E_j}_{=Q_{ij}} \tag{1}$$

where Q_{ij} is the quantity of goods produced in i demanded by j, and E_j is total exogenous expenditure by consumers in j. An exogenous increase in E_i can be interpreted as an increase in location population, or shifting expenditure from goods in other industries — both of these are abstracted from in the partial equilibrium model (see appendix B.2.3).

The producer in i has supply curve

$$Q_i^S \equiv a_i P_i^{-\sigma} \tag{2}$$

where $a_i > 0$ is an exogenous supply shifter. The elasticity of supply is equal to negative of the demand elasticity of substitution, $-\sigma$; an emergent symmetry of the canonical HME model. It being negative reflects the presence of increasing returns to scale in production: production is more efficient at greater scales therefore price decreases as output increases.

Product market clearing occurs in all locations

$$\forall i: \quad Q_i^S = Q_i^D \tag{3}$$

Some useful objects I use. I denote dollar output $Y_i \equiv P_i Q_i$, export share matrix $\gamma_{ij} \equiv \frac{P_{ij} Q_{ij}}{Y_i}$ and import share matrix $\lambda_{ij} \equiv \frac{P_{ij} Q_{ij}}{E_j}$. The "trade freeness" matrix, $\Phi_{ij} = \tau_{ij}^{1-\sigma} \in (0, 1]$. This is a non-linear transformation of the trade costs matrix, with a larger value indicating freer trade. Notably, $\Phi_{ij} = 0$ when trade costs are infinite $\tau_{ij} = \infty$, and $\Phi_{ij} = 1$ when trade is free $\tau_{ij} = 1$

Throughout, I assume the following regularity conditions. Note that these conditions are on the equilibrium objects Q_i, γ_{ii} which are implicit restrictions on the structural parameters through their equilibrium solutions — see equations (4) and (5).[7]

Assumption 1. Regularity Conditions.

1. *The equilibrium is interior,* $\forall i : Q_i > 0$.
2. *No global autarky* $\exists i : \gamma_{ii} \neq 1$
3. *Export share matrix* γ *is full rank and diagonalizable.*

Assumption 1 is needed for my HME propositions that follow in section 3, but they can be well-justified. Assumption 1.1 follows the precedent in the HME literature: the HME is defined about an interior equilibria, ignoring the equilibria where at least one location does not produce any of the good (Behrens et al. (2009) pg 262). Assumption 1.2 rules out global autarky, which is a special case where the HME is violated because $Y_i = E_i$ always. 1.3 is not of practical concern as the export share matrices ruled out are of measure zero.[8] Full rank is needed otherwise the perturbed equilibrium isn't unique. Diagonalizability is needed for the method of proof I use.

2.1 Equilibrium Solution

An equilibrium is such that the demand equation (1), supply equation (2) and market clearing equation (3) hold. This system of equations can be solved analytically, giving output

$$Y_i = \sum_j \frac{\Phi_{ji}^{-1} a_i}{\sum_k \Phi_{jk}^{-1} a_k} E_j \tag{4}$$

and bilateral export share

$$\gamma_{ij} = a_i^{-1} \Phi_{ij} \sum_k \left\{ \Phi^{-1} \right\}_{jk} a_k \tag{5}$$

which are both only functions of the exogenous parameters. See appendix B.1 for the derivation.

[7]Note that $Q_i = a_i^{\frac{-1}{\sigma-1}} Y_i^{\frac{\sigma}{\sigma-1}}$, by using the supply equation (2).

[8]In either case, a full rank or diagonalizble matrix can be attained by an arbitrarily small perturbation of the elements, so that the eigenvalues become non-zero or distinct, respectively.

2.2 Home Market Effect Definition

I define the HME to be present in i if a differential increase in total i expenditure E_i causes a more-than-proportional increase in i dollar output Y_i. I also define an average HME, as this is easier to characterize in my main theoretical results.

Definition 1. Home Market Effect. *Is present in location i iff*

$$\frac{\partial Y_i}{\partial E_i} > 1 \tag{6}$$

and on average across all locations iff

$$\frac{1}{N} \sum_{i=1}^{N} \frac{\partial Y_i}{\partial E_i} > 1 \tag{7}$$

Definition 1 characterizes questions of comparative statics. It is silent on questions regarding levels in a given equilibrium. Both sets of questions are important, with interest depending on the research question.

The average HME can be understood as follows. Consider an experiment where expenditure is only increased in location 1. Output in location 1 is measured, and $\frac{\partial Y_1}{\partial E_1}$ is determined. Then, redo the experiment but with expenditure only increased in location 2 and $\frac{\partial Y_2}{\partial E_2}$ is determined. This is repeated for each location. Although in each experiment, the output-expenditure relation may not be more-than-proportional, i.e. $\frac{\partial Y_i}{\partial E_i} \gtrless 1$, if we take the average across all experiments, the relation is more-than-proportional, $\frac{1}{N} \sum_{i=1}^{N} \frac{\partial Y_i}{\partial E_i} > 1$.

Definition 1 is not the unique characterization of the HME in the literature. Alternative forms considered are derivatives in shares or logs, or rankings in levels.[9] In the symmetric two location model, all these alternatives are equivalent (Ottaviano and Thisse (2004) pg 2582) but not necessarily so more generally.

I use definition 1 not only as it allows for precise theoretical results, whereas other definitions do not (Behrens et al. (2009)), but because it maintains desirable implications for trade patterns.

In particular, iff equation (6) holds, an increase in in the home market of i, E_i, causes i to increase its net exports and therefore improve its trade balance, $TB_i \equiv Y_i - E_i$. That is

$$\frac{\partial Y_i}{\partial E_i} > 1 \quad \Longleftrightarrow \quad \frac{\partial TB_i}{\partial E_i} > 0 \tag{8}$$

[9] Respectively: $\frac{\partial\left(Y_i/\sum_j Y_j\right)}{\partial\left(E_i/\sum_j E_j\right)} \geq 1, \frac{\partial \ln Y_i}{\partial \ln E_i} \geq 1$, and $E_1 \geq E_2 \geq ... \implies \frac{Y_1}{E_1} > \frac{Y_2}{E_2} > ....$ See Behrens et al. (2009) appendix C for a discussion.

using $\frac{\partial TB_i}{\partial E_i} = \frac{\partial Y_i}{\partial E_i} - \frac{\partial E_i}{\partial E_i} = \frac{\partial Y_i}{\partial E_i} - 1 > 0$. A definition in terms of derivatives in logs or shares does not have this property generally, except in the symmetric two location model.[10]

3 Theoretical Results

In proposition 1, I derive the endogenous change in output Y_i with respect to exogenous shifts in expenditure $\{E_j\}_{j=1}^N$.

Proposition 1. Equilibrium Comparative Statics. *Let equations (1), (2) and (3) hold. Moreover, assume γ is full rank. Then,*

$$dY_i = \sum_j \left\{ \gamma'^{-1} \right\}_{ij} dE_j \tag{9}$$

Proof. See appendix A.1. □

The relationship takes a very succinct form. Notably, the export share matrix γ is a sufficient statistic for the equilibrium change in output. This result is reminiscent of the exact-hat algebra (Dekle et al. (2008)), but with a particular simple form, in part, due to the emergent symmetry of the canonical HME model: the supply elasticity is equal to negative of the demand elasticity of substitution.

The necessary and sufficient condition on the export share matrix for the HME to hold in a given location is a simple corollary of proposition 1.

Corollary 1. Home Market Effect. *The Home Market Effect (definition 6) in location i is present iff*

$$\left\{ \gamma'^{-1} \right\}_{ii} > 1 \tag{10}$$

Although terse, equation (10) is abstruse: under what geography of trade, γ, is equation (10) satisfied?[11]

With matrix dimensions greater than 2×2, the relation between a matrix and its inverse is generally very complicated. Therefore, I derive a simple and intuitive sufficient condition on the export share matrix γ such that the HME holds. This condition is a home biased

[10] The change in trade balance in these alternatives depends on the initial trade balance. In logs: $\frac{\partial \ln Y_i}{\partial \ln E_i} > 1$, which is equivalent to $\frac{\partial Y_i}{\partial E_i} > \frac{Y_i}{E_i}$ implies $\frac{\partial TB_i}{\partial E_i} > \frac{Y_i}{E_i} - 1$. In shares, $s_i^Z \equiv \frac{Z_i}{\sum_j Z_j}$: $\frac{\partial s_i^Y}{\partial s_i^E} > 1$, which is equivalent to $\frac{\partial Y_i}{\partial E_i} > \frac{1-s_i^E}{1-s^{Y_i}}$, implies $\frac{\partial TB_i}{\partial E_i} > \frac{1-s_i^E}{1-s^{Y_i}} - 1 = \frac{s_i^Y - s_i^E}{1-s_i^Y}$. Both of these can be negative given a sufficiently negative trade balance, $Y_i < E_i$.

[11] Equation (10) puts an implicit restriction on the structural parameters of the model through the equation (4).

geography (assumption 2): the majority of the sales from a location go to consumers in that location. This is equivalent to all the diagonal elements of γ being greater than 0.5.

Assumption 2. Home-Biased Geography. *Assume that each diagonal element of the export share matrix* γ *is greater than* 0.5

$$\forall i: \quad \gamma_{ii} > 0.5$$

This condition is also highly relevant empirically. In figure 1, I present γ_{ii} for the major world economies. As the HME model abstracts from intermediate good trade and assumes all trade is in final goods, I use the value-added content of trade flows estimated by Johnson and Noguera (2017) in the construction of γ_{ii}. The figure shows that the majority of countries i satisfy $\gamma_{ii} > 0.5$, and more so throughout history.

The sufficiency of a home biased geography for the HME is presented in proposition 2. Although I only prove that a home-biased geography is sufficient for the average HME — part i) in proposition 2 — in section 3.1 I provide simulation evidence suggesting it is also sufficient for the HME to hold in each location. I do prove sufficiency for the HME in each location if I add the additional assumption of a symmetric geography — part ii) in proposition 2 — but the simulation evidence also implies this additional assumption is not necessary.

Proposition 2. Home Market Effect in a Home Biased Geography . *i) Assume the geography is home-biased (assumption 2), then the HME is present on average*

$$\frac{1}{N}\sum_{i=1}^{N}\frac{\partial Y_i}{\partial E_i} > 1$$

ii) Furthermore, assume the geography is symmetric, $\gamma = \gamma'$*, then the HME is present in each location*

$$\forall i: \quad \frac{\partial Y_i}{\partial E_i} > 1$$

Proof. See appendix A.2 for i), appendix A.3 for ii). □

In proving proposition 2.i), I make, to my knowledge, a general contribution to linear algebra literature. For any matrix M that is diagonally dominant and a markov matrix, I prove that the real component of the inverse of all its eigenvalues are greater than one. This is proposition 3.

Proposition 3. (Eigenvalues of the inverse of a row-diagonally dominant, markov matrix have real component greater than one) . *Let a full-rank and diagonalizble matrix* M *with* *dimensions $N \times N$ be strictly row-diagonally dominant*

$$\forall i: \quad M_{ii} > \sum_{j \neq i} M_{ij}$$

and a (row) markov matrix

$$\begin{aligned} \forall i, j: \quad & M_{ij} \geq 0 \\ \forall i: \quad & \sum_{j} M_{ij} = 1 \end{aligned}$$

Then, all the eigenvalues of M^{-1} have real component greater than or equal to one.

If we also assume that $M \neq I$, where I is the identity matrix, then at least one eigenvalue of M^{-1} has real component strictly greater than one.

Proof. See appendix A.4. □

Proposition 3 is key to proving sufficiency of a home biased geography in proposition 2. The export matrix γ is full-rank and diagonalizable by regularity condition 1.3. The home-biased assumption 2 implies diagonal dominance. It is a markov matrix as trade flows cannot be negative, and the rows add up to one as element is a share. Hence, proposition 3 applies to γ, and with the equality strict as the no global autarky regularity assumption 1.2 implies $\gamma \neq I$.

To see where proposition 3 is used in proposition 2, write

$$\begin{aligned} \frac{1}{N} \sum_{i=1}^{N} \frac{\partial Y_i}{\partial E_i} &= \frac{1}{N} \sum_{i=1}^{N} \left\{ \gamma'^{-1} \right\}_{ii} \\ &= \frac{1}{N} \sum_{i=1}^{N} eigenvalue_i \left(\gamma'^{-1} \right) \\ &> 1 \end{aligned}$$

The first equality follows from the comparative statics equation (9). The second equality follows from the trace of a matrix equalling the sum of its eigenvalues. Noting that the imaginary components of the eigenvalues in the sum cancel out as γ (and therefore γ'^{-1}) is real,[12] the third inequality follows from proposition 3, and proposition 2 is proved.

[12] γ'^{-1} is real therefore the complex conjugate of any eigenvalue is also an eigenvalue. Thus, the imaginary components exactly cancel when all eigenvalues are summed together.

3.1 Simulation Evidence

Given I do not prove sufficiency of a home-biased geography for the HME to hold separately in each location — equation (6) — I use simulations to show that this nonetheless seems to be true.

To do this, I randomly simulate export share matrices γ that do and do not satisfy home bias 1,000,000 times each. The simulation process in detail for each is:

- **Non-home biased export share matrix,** γ. I randomly generate an $N \times N$ matrix M with each element distributed uniform between zero and one, $\forall i, j : M_{ij} \sim U[0, 1]$. I then scale each row by the row sum so that the resulting matrix is a row markov matrix, $\gamma_{ij} = \frac{M_{ij}}{\sum_k M_{ik}}$. I keep this γ in the distribution if it is not diagonally dominant $\gamma_{ii} \leq 0.5$. I repeat this 1,000,000 times.

- **Home biased export share matrix**, γ. The share of γ that are diagonally dominant in the above process is increasingly small as N raises. Therefore, I modify the algorithm so that the randomly generated matrix is diagonally dominant by construction. I first randomly generate the diagonal elements of γ so that diagonal dominance is satisfied, $\forall i : \gamma_{ii} \sim U\left[\frac{1}{2}, 1\right]$. Second, I randomly generate $\forall i, j \neq i : M_{ij} \sim U[0, 1]$. The off-diagonal elements of γ_{ij} are the scaled M_{ij} so that their sum equals $1 - \gamma_{ii}$, thus being a markov matrix: $\forall i, j \neq i : \gamma_{ij} = (1 - \gamma_{ii}) \frac{X_{ij}}{\sum_{k \neq i} X_{ik}}$. I repeat this 1,000,000 times.

I do this simulation for the set of $N \in \{2, 3, 5, 10\}$. For each γ simulated, I calculate $\frac{\partial Y_i}{\partial E_i} = \left\{\gamma^{-1\prime}\right\}_{ii}$. If $\min_i \frac{\partial Y_i}{\partial E_i} > 1$, then the HME holds separately for each location, as this implies $\forall i : \frac{\partial Y_i}{\partial E_i} > 1$.

Figure 2 presents the results, with each sub-figure a different N. The distribution of $\min_i \frac{\partial Y_i}{\partial E_i}$ is displayed in each sub-figure (density truncated at 0.01). We see that $\min_i \frac{\mathrm{d}Y_i}{\mathrm{d}E_i} > 1$ holds for the entire distribution simulated when trade is home biased, whereas not necessarily when trade is non-home biased. Hence, a home-biased geography appears to be sufficient for the HME to hold separately for each location, not just on average.

4 Discussion

4.1 The role of a Home Biased Geography

Intuition to the role of home bias in the theoretical results can be attained by looking at textbook supply and demand curves in $P_i - Y_i$ space in a single location i, and considering the comparative statics (to first order) of increasing expenditure in i, $\mathrm{d}E_i > 0, \{\mathrm{d}E_j = 0\}_{j \neq i}$.

Figure 3 illustrates the comparative static graphically in a home-biased and sufficiently non-home-biased geography. The supply curve for producers in location i is given by multiplying equation (2) by P_i and differentiating to give

$$\mathrm{d}Y_i^S = \underbrace{-(\sigma-1)\,Q_{ii}}_{\text{Supply gradient in } i}\,\mathrm{d}P_i \tag{11}$$

which is downward sloping due to the supply elasticity being negative (equal to $-\sigma < -1$). The demand curve facing producers in location i is given by multiplying equation (1) by P_i and differentiating to give[13]

$$\mathrm{d}Y_i^D = \underbrace{-(\sigma-1)\left(1-\{\lambda\gamma'\}_{ii}\right)Q_{ii}}_{\text{Demand gradient facing } i}\,\mathrm{d}P_i + \underbrace{(\sigma-1)\cdot\sum_{k\neq i}\{\lambda\gamma'\}_{ik}\cdot Q_k\mathrm{d}P_k + \overbrace{\lambda_{ii}\mathrm{d}E_i}^{\text{Initial Shift}}}_{\text{Total Demand Shift}} \tag{12}$$

The term labeled "initial shift" is the shift in the demand curve due to an increase in expenditure $\mathrm{d}E_i$, holding prices in all other locations' fixed, $\mathrm{d}P_{j\neq i} = 0$. The initial shift is, to first order, simply the increase in domestic expenditure $\mathrm{d}E_i$ multiplied by the share of expenditure that is spent on domestic goods, i.e. the own-import share λ_{ii}. Because $0 \leq \lambda_{ii} \leq 1$, the initial shift always causes a weakly less-than-proportional change in output. Intuitively, the increase in expenditure of i consumers, $\mathrm{d}E_i$, is often not spent one-for-one on goods produced in i. i consumers consume from goods in all locations; they only spend λ_{ii} of their income locally.

In equilibrium, the total demand shift incorporates the change in demand facing i producers due to the change in other locations' prices. This total shift can be rewritten by substituting out the endogenous price changes $\mathrm{d}P_{j\neq i}$ using their solution in equilibrium, leaving the shift in terms of only the exogenous shock $\mathrm{d}E_i > 0, \{\mathrm{d}E_j = 0\}_{j\neq i}$[14]

$$\mathrm{d}Y_i^D = (\sigma-1)\left(1-\{\lambda\gamma'\}_{ii}\right)Q_{ii}\mathrm{d}P_i + \underbrace{\{\lambda\gamma'\}_{ii}\cdot\left\{\gamma'^{-1}\right\}_{ii}\cdot\mathrm{d}E_i}_{\text{Total Demand Shift}} \tag{13}$$

In contrast to the initial shift, where the shift in the demand curve was positive and weakly less-than-proportional, the total demand shift faces no such constraint. Pivotally, it depends on the value of $\left\{\gamma'^{-1}\right\}_{ii}$. Intuitively, the positive initial shift in i demand can be

[13] The demand elasticity is negative as $0 < \{\gamma\lambda'\}_{ii} + \sum_{j\neq i}\{\gamma\lambda'\}_{ij} = 1 \implies 0 < \{\gamma\lambda'\}_{ii} < 1$, using $\sum_{j\neq i}\{\gamma\lambda'\}_{ij} > 0$. Note also that the the demand curve is steeper than the supply curve.

[14] $\mathrm{d}P_i = -\frac{1}{(\sigma-1)Q_i}\mathrm{d}Y_i = -\frac{1}{(\sigma-1)Q_i}\left\{\gamma'^{-1}\right\}_{ii}\mathrm{d}E_i$ where the first equality uses equation (11) and the second uses equation (9).

amplified if $j \neq i$ prices increase in response to $\mathrm{d}E_i$, as this causes further substitution of consumption to goods produced in i (figure 3a).

However, it can be the case that $j \neq i$ prices decrease. This occurs if producers in j are highly exposed to consumers in i, so that demand facing j producers sufficiently increases following an expenditure increase in i. As a result, production in j expands and prices in j fall due to downward-sloping supply, causing substitution of consumption away from i. Thus a less-than-proportionate, even negative, equilibrium shift in i demand may result (figure 4b). Consequently, the HME effect is violated.

This is precisely what a home-biased geography rules out. j producers having high direct exposure to i consumers translates to a high sales share from j to i, γ_{ji}. j can be highly exposed indirectly to i through a third country, thus all off-diagonal elements of γ factor into this exposure. A home-biased geography implies that $\gamma_{jj} > 0.5$, therefore all off-diagonal elements are sufficiently small such that $\forall j \neq i$ locations are not highly exposed to i consumers. Thus, demand facing $j \neq i$ producers will not increase sufficiently in response to an expenditure increase in i, and the HME will not be violated.

4.2 Home Bias in Two Location Models

The results of section 3 on the sufficiency of a home biased geography for the home market effect provides a novel insight to a theory that is four decades old. Perhaps the reason for this being overlooked in the literature is because the theory has almost exclusively focused on models with only two locations. As I now show, a home biased geography is in fact implied under the standard parameter restrictions in two location models. Hence no explicit consideration of it has been necessary in two location models, and therefore no consideration of it has been taken into account when extending the theory to more than two locations.

The standard parameter restriction referred to is a very natural and empirically appropriate one: trade between locations is more costly than trade within a location (with geographic symmetry in all other parameters). This, quite intuitively, implies home bias in a two location model as it's cheaper to consume domestically relative to consuming from foreign. However, with more than two locations, this is not sufficient. Even though the relative cost of consuming from domestic producers is still less, there are now more foreign locations to buy from. Given the love variety in preferences (due to CES), the non-domestic consumption share rises, and a home biased geography is no longer guaranteed.

This result is formalized in proposition 4.

Proposition 4. Home Bias in Two Location Models. *Consider $N = 2$, with trade between locations being more costly than within a location, $\forall i, j \neq i : \tau_{ii} < \min\{\tau_{ij}, \tau_{ji}\}$, and sym-*

metric initial expenditures $E_1 = E_2$, productivities $a_1 = a_2$. Then, for all interior equilibria, assumption 2 is satisfied.

Proof: see appendix A.5.

4.3 Simple Example

I present a simple example demonstrating violation of the home market effect in more than two locations when trade between locations is more costly than trade within a location.

This is the "symmetric square geography" (see figure 4).[15] There are four identical locations, $N = 4$, with equal (initial) expenditure in all locations $\forall i : E_i = E$, and equal productivities in all locations $\forall i : a_i = a$. Trade within a location is free, $\tau_{ii} = 1$, is costly between adjacent vertices, incurring transport cost $\tau > 1$, and impossible between opposite vertices (infinite trade costs). Such a geography may characterize a perishable good which can only be shipped to nearby locations. Because all locations are symmetric, an interior equilibrium exists for all $\tau > 1$ and is symmetric.

The math is much cleaner if we use the trade freeness matrix Φ, which in this geography is

$$\Phi = \begin{pmatrix} 1 & \phi & 0 & \phi \\ \phi & 1 & \phi & 0 \\ 0 & \phi & 1 & \phi \\ \phi & 0 & \phi & 1 \end{pmatrix}$$

where $\phi \equiv \tau^{1-\sigma}$. Using equation (5), the export matrix and it's transposed inverse is

$$\gamma = \begin{pmatrix} \frac{1}{1+2\phi} & \frac{\phi}{1+2\phi} & 0 & \frac{\phi}{1+2\phi} \\ \frac{\phi}{1+2\phi} & \frac{1}{1+2\phi} & \frac{\phi}{1+2\phi} & 0 \\ 0 & \frac{\phi}{1+2\phi} & \frac{1}{1+2\phi} & \frac{\phi}{1+2\phi} \\ \frac{\phi}{1+2\phi} & 0 & \frac{\phi}{1+2\phi} & \frac{1}{1+2\phi} \end{pmatrix}, \quad \gamma^{-1\prime} = \begin{pmatrix} \frac{1-2\phi^2}{1-2\phi} & \frac{\phi}{2\phi-1} & \frac{2\phi^2}{1-2\phi} & \frac{\phi}{2\phi-1} \\ \frac{\phi}{2\phi-1} & \frac{1-2\phi^2}{1-2\phi} & \frac{\phi}{2\phi-1} & \frac{2\phi^2}{1-2\phi} \\ \frac{2\phi^2}{1-2\phi} & \frac{\phi}{2\phi-1} & \frac{1-2\phi^2}{1-2\phi} & \frac{\phi}{2\phi-1} \\ \frac{\phi}{2\phi-1} & \frac{2\phi^2}{1-2\phi} & \frac{\phi}{2\phi-1} & \frac{1-2\phi^2}{1-2\phi} \end{pmatrix}$$

By equation (9), the change in domestic output from a change in domestic expenditure in this geography is

$$\frac{\partial Y_i}{\partial E_i} = \left\{\gamma^{-1\prime}\right\}_{ii} = \frac{1-2\phi^2}{1-2\phi} \tag{14}$$

which is equal for all i due to geographic symmetry. Figure 6a displays $\frac{\partial Y_i}{\partial E_i}$ from equation (14) as a function of $\phi \in (0, 1]$. We see that the HME is present, $\frac{\partial Y_i}{\partial E_i} > 1$, only when trade costs are sufficiently high, $\phi \in \left(0, \frac{1}{2}\right)$. For $\phi \geq 0.5$, the HME is violated. For $\phi \in \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$,

[15] At least four locations is needed for this. In three locations, the analogous geography is a line, which leads to complete agglomeration at the central point.

the effect on output is negative, $\frac{\partial Y_i}{\partial E_i} \leq 0$. For $\phi \in \left(\frac{1}{\sqrt{2}}, 1\right]$, the relationship is positive but less than proportional, $\frac{\partial Y_i}{\partial E_i} \in (0, 1]$.

Why is the HME violated for $\phi \geq 0.5$? Insight can be gained by looking at the own-export share γ_{ii} as a function of ϕ, see figure 6b. We see that when $\phi \geq 0.5$, trade costs are sufficiently small that the geography is no longer home biased, $\gamma_{ii} < 0.5$. Thus, the HME is no longer guaranteed by proposition 2 (due to geographic symmetry, the average HME is equal to the individual HME). Intuitively, as explained in section 4.1, an increase in domestic expenditure causes a sufficient increase in the demand facing producers in other locations. This is because those producers are highly exposed due to low trade costs, materializing in a low own-export share. The expansion of production in other locations attenuates the domestic expansion in production, so violating the HME.

5 Conclusion

In this paper, I provide new results on a decades old theory: the Home Market Effect. The theory has gained much attention academically and in policy, yet its canonical theoretical implications have mostly been confined to two location models. This is a potentially important limitation as two locations is empirically counterfactual.

I extend the applicability of the HME to an arbitrary number of locations under the additional assumption of a home-biased geography: the majority of the sales from a location going to consumers based in that location. In proving sufficiency of this condition for the HME to hold on average, I contribute a new result to the linear algebra literature.

This assumption is both empirically relevant, and its connection with the HME simple to explain. When the geography is not home-biased, an increase in domestic expenditure causes production in foreign locations to expand sufficiently, reducing the expansion of domestic production. Thus, the more-than-proportional relationship between output and expenditure, characteristic of the HME, can be violated.

This result is vacuous, and therefore overlooked, in standard two location models as all geographies are home-biased. My results therefore provide a non-trivial qualitative difference of modeling many-locations vs only two locations. The latter being a convenient simplification often used in theoretical frameworks.

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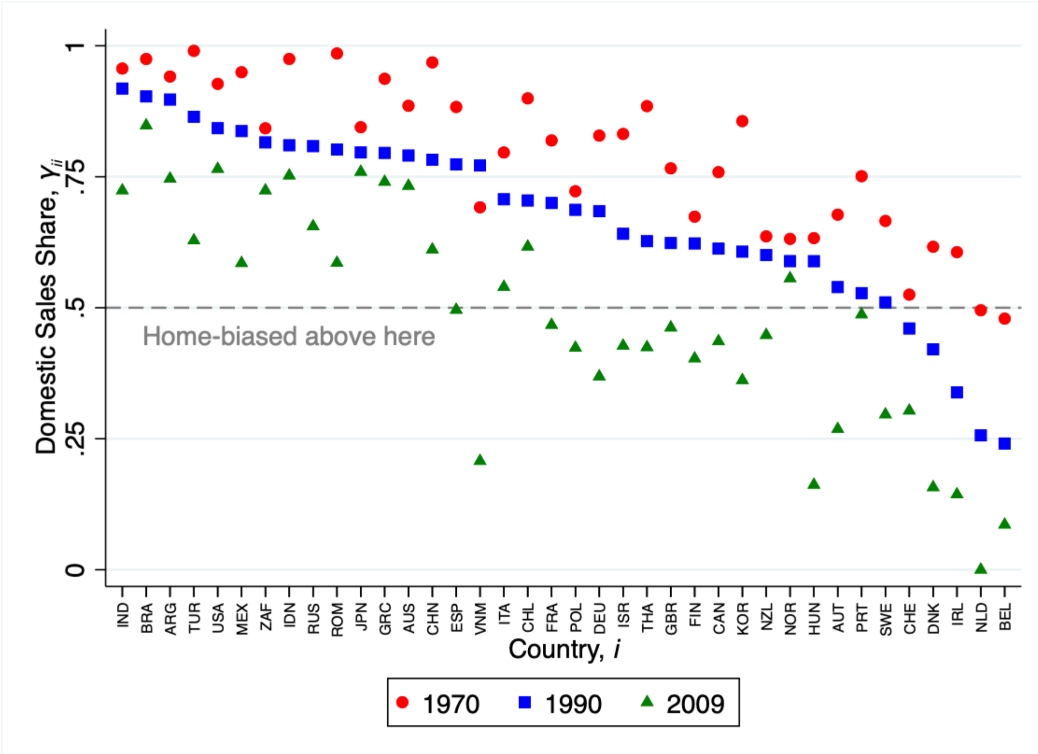
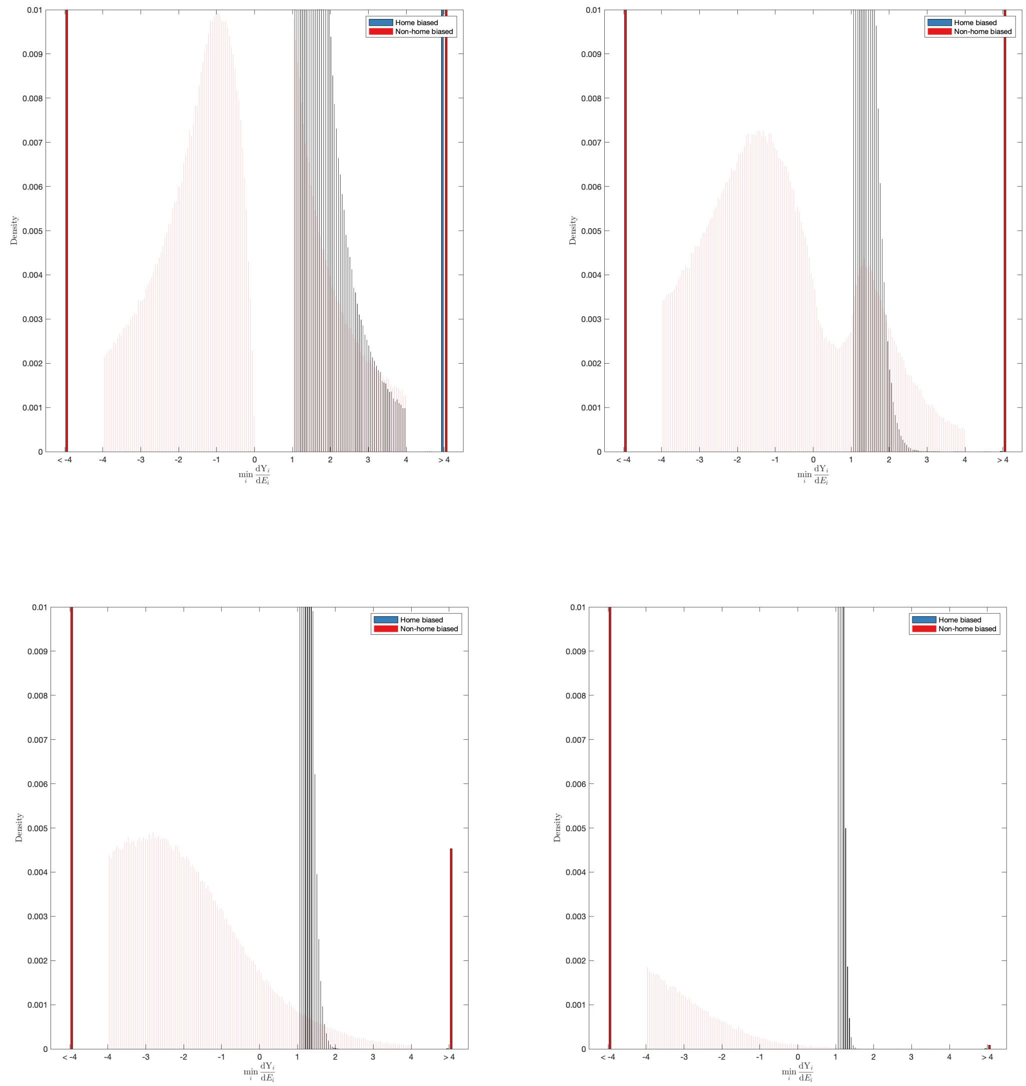
Figure 1: Observed Domestic Sales Share, γ_{ii}



Notes. The geography is home biased (assumption 2) if all countries are above the dashed line for a given year. Constructed using the estimated value-added content of international trade flows from Johnson and Noguera (2017). Included are the 42 OECD countries and major emerging markets, which account for around 90% of world GDP.

Figure 2: Sufficiency of Home Biased Geography for the HME in each i: Simulation Evidence

(a) $N = 2$ (b) $N = 3$

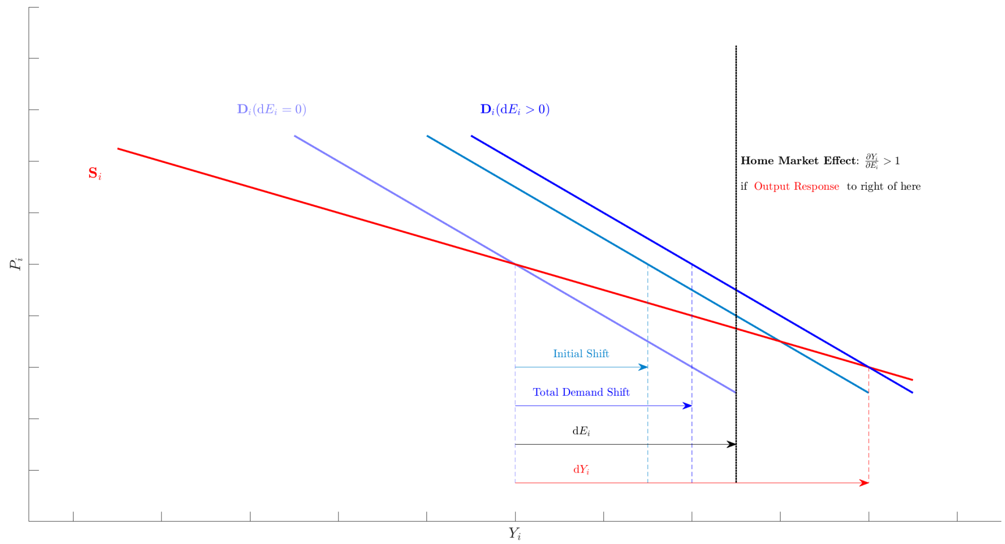


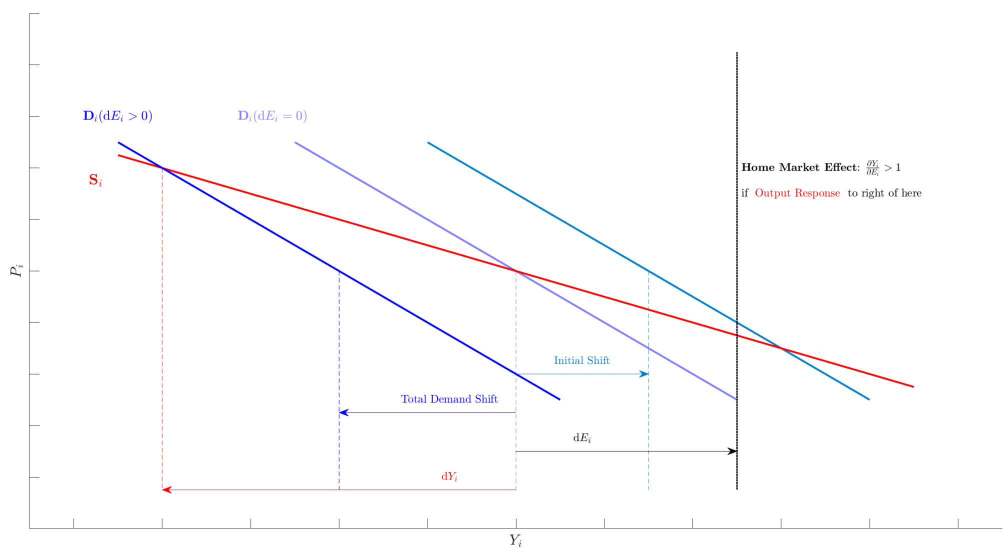
(c) $N = 5$ (d) $N = 10$

Notes. Export share matrices that satisfy home bias and not are separately randomly simulated. For each N, we see that $\min_i \frac{\mathrm{d}Y_i}{\mathrm{d}E_i} > 1$ holds for the entire distribution simulated when trade is home biased, whereas not necessarily when trade is not home biased. Hence, home bias appears sufficient for the HME to hold separately for each location. See section 3.1.

Figure 3: Home Market Effect dependence on Geography

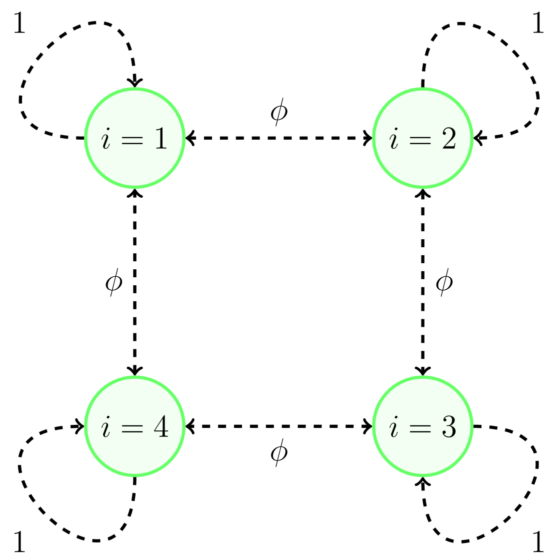
(a) Home-Biased Geography





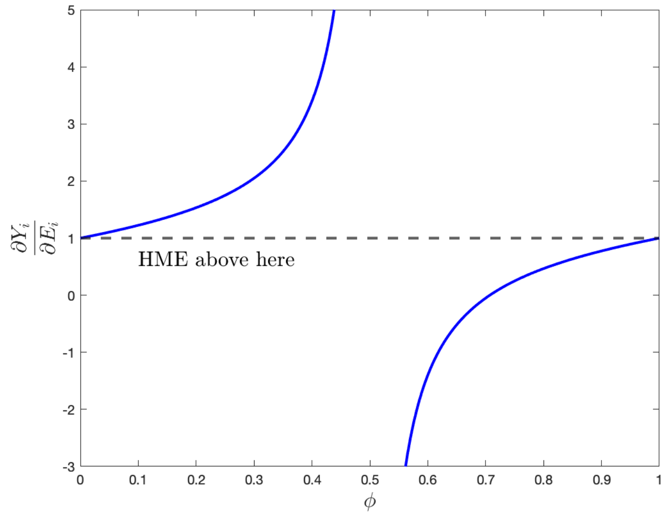
(b) Non-Home-Biased Geography

Figure 4: Symmetric Square Geography: Visualization

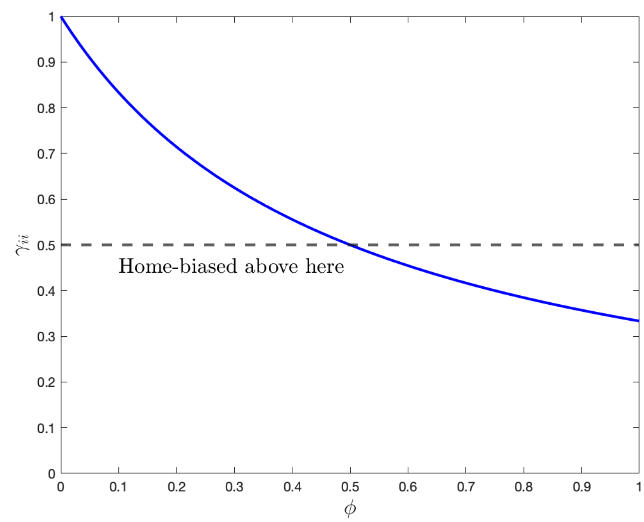


Notes. Trade is only permitted between locations i that are directly connected by a dashed line, and faces the labeled trade freeness parameter. Diagonally opposite locations are not able to trade with each other. See section 4.3.

Figure 5: Symmetric Square Geography: Equilibrium Variables



(a) Change in domestic output from a change in domestic expenditure, $\frac{\partial Y_i}{\partial E_i}$



(b) Own-export share, γ_{ii}

A Proposition Proofs

A.1 Proposition 1 (Equilibrium Comparative Statics)

Proof. Multiplying demand equation (1) by P_i and differentiating gives

$$\begin{aligned}
\mathrm{d}\ln Y &= (1-\sigma)\cdot \mathrm{d}\ln P + \gamma\cdot\left[-(1-\sigma)\cdot\lambda'\cdot \mathrm{d}\ln P + \mathrm{d}\ln E\right] \\
\mathrm{d}Y &= (1-\sigma)\left(I-\lambda\cdot\gamma'\right)\left(Q\circ \mathrm{d}P\right) + \lambda\cdot \mathrm{d}E
\end{aligned} \tag{15}$$

Multiplying supply equation (2) by P_i and differentiating gives

$$\begin{aligned}
\mathrm{d}\ln Y &= -\left(\sigma-1\right)\cdot \mathrm{d}\ln P \\
\mathrm{d}Y &= -\left(\sigma-1\right)\cdot\left(Q\circ \mathrm{d}P\right)
\end{aligned} \tag{16}$$

Substitute out $Q\circ \mathrm{d}P$ from equation (15) by using (16)

$$\mathrm{d}Y = \left(I-\lambda\cdot\gamma'\right)\mathrm{d}Y + \lambda\cdot \mathrm{d}E$$

Rearrange, using the fact that X, and therefore γ, λ are full rank, to give

$$\mathrm{d}Y = \gamma'^{\,-1}\,\mathrm{d}E$$

□

A.2 Proposition 2 i) (Average Home Market Effect under Home Biased Geography)

Proof. Using proposition 1

$$\begin{aligned}
\frac{1}{N}\sum_{i=1}^{N}\frac{\partial\ln Y_i}{\partial\ln E_i} &= \frac{1}{N}Trace\left(\gamma'^{-1}\right) \\
&= \frac{1}{N}\sum_{k=1}^{N} eig\left(\gamma'^{-1}\right)_k \\
&= \frac{1}{N}\sum_{k=1}^{N} Re\left\{eig\left(\gamma'^{-1}\right)_k\right\}
\end{aligned}$$

where $eig\left(\gamma'^{-1}\right)_k$ is the k^{th} eigenvalue of γ'^{-1}, and $Re\left\{eig\left(\gamma'^{-1}\right)_k\right\}$ its real component. The second line follows from the trace equalling the sum of the eigenvalues. The last line follows from the complex conjugate root theorem: for any real matrix, the complex conjugate of a complex eigenvalue is also an eigenvalue; therefore the imaginary components of $eig\left(\gamma'^{-1}\right)_k$ cancel in the sum.

To prove the proposition, it is sufficient to show that the real part of all eigenvalues of γ'^{-1} are greater than or equal to 1, with at least one eigenvalue strictly greater than one. To do this, note: γ is row-diagonally dominant (from the assumption of home biased geography); it is a markov matrix (because it is a share matrix with rows summing to one); and the eigenvalues of a matrix and it's transpose are the same (so that the eigenvalues of γ'^{-1} and γ^{-1} are the same). Hence, proposition 3 applies, $\forall k : Re\left\{eig\left(\gamma'^{-1}\right)_k\right\} \geq 1$. Furthermore, the inequality is strict for at least one k as $\gamma \neq I$ due to the no global autarky assumption 1.2. Thus,

$$\begin{aligned}\frac{1}{N}\sum_{i=1}^{N}\frac{\partial \ln Y_i}{\partial \ln E_i} &= \frac{1}{N}\sum_{k=1}^{N} Re\left\{eig\left(\gamma'^{-1}\right)_k\right\} \\ &> 1\end{aligned}$$

the second inequality is implied by proposition 3. $\square$

A.3 Proposition 2 ii) (Individual Home Market Effect in a Home-Biased and Symmetric Geography)

Proof. Because γ is symmetric, we can take the spectral decomposition $\gamma^{-1} = Q\Lambda Q'$ where Λ is the diagonal matrix of eigenvalues, $\Lambda_{kk} = eig\left(\gamma^{-1}\right)_k$, and Q has each column k equal to the corresponding orthonormal eigenvector x_k of γ. Writing this in component form

$$\begin{aligned}\left\{\gamma^{-1}\right\}_{ii} &= \sum_k \underbrace{eig\left(\gamma^{-1}\right)_k}_{>1}\underbrace{x_{ki}^2}_{>0} \\ &> \underbrace{\sum_k x_{ki}^2}_{=1} \\ &> 1\end{aligned}$$

where x_{ki} is the i^{th} element of eigenvector x_k.

In the first line, $eig\left(\gamma^{-1}\right)_k > 1$ follows from applying proposition 3 to a symmetric matrix: all eigenvalues are real for a symmetric matrix, therefore $eig\left(\gamma^{-1}\right)_k = Re\left\{eig\left(\gamma^{-1}\right)_k\right\} > 1$. In the second line, $\sum_k x_{ki}^2$ is the modulus of eigenvector x_k, which is equal to one as the eigenvectors are orthonormal. $\square$

A.4 Proposition 3 (Eigenvalues of the inverse of a row-diagonally dominant, markov matrix have real component greater than or equal to one)

Proof. There are two parts. In part A, I manipulate the matrix M into a form in which the Leontieff Inverse can be applied. In part B, I show that the real part of the eigenvalues of the resulting Leontieff inverse expansion can be bounded below by one.

Part A. Rewrite matrix M in Leontieff inverse form

$$\begin{aligned} M^{-1} &= \left(I - \tilde{M}\right)^{-1} \\ &= \sum_{n=0}^{\infty} \tilde{M}^n \end{aligned} \tag{17}$$

where

$$\begin{aligned} \tilde{M} &\equiv I - M \\ \tilde{M}_{ij} &= \begin{cases} \sum_{j \neq i} M_{ij} & i = j \\ -M_{ij} & i \neq j \end{cases} \end{aligned} \tag{18}$$

The expression for the diagonal elements in equation (18) used the fact that M is a (row) markov matrix $\tilde{M}_{ii} = 1 - M_{ii} = \sum_{j \neq i} M_{ij}$. For the Leontieff inverse expansion — equation (17) — to be valid, the eigenvalues of $\tilde{M}$, $eig\left(\tilde{M}\right)_k$, where $k \in \{1, ..., N\}$ indexes the eigenvalue, must each be less than one in absolute value. Using the Gershgorin Circle theorem, each eigenvalue of satisfies, for some i^*,

$$\begin{aligned} \left| eig\left(\tilde{M}\right)_k - \tilde{M}_{i^*i^*} \right| &\leq \sum_{j \neq i^*} \left| \tilde{M}_{i^*j} \right| \\ \left| eig\left(\tilde{M}\right)_k - \sum_{j \neq i^*} M_{i^*j} \right| &\leq \sum_{j \neq i^*} M_{i^*j} \end{aligned} \tag{19}$$

where the second line follows from the definition of $\tilde{M}$, equation (18), and uses that all elements of M are nonnegative. Equation (19) bounds each eigenvalue of $\tilde{M}$, $eig\left(\tilde{M}\right)_k$, within a circle in the complex plane of radius $\sum_{j \neq i^*} M_{i^*j}$ centered on $\sum_{j \neq i^*} M_{i^*j}$ (see the blue circle in figure 6, written in terms of M using $\tilde{M} \equiv I - M$).

Furthermore, $\sum_{j \neq i^*} M_{i^*j} < 0.5$ as

$$\begin{aligned} \forall i: \quad M_{ii} &> \sum_{j \neq i} M_{ij} \\ 1 - \sum_{j \neq i} M_{ij} &> \sum_{j \neq i} M_{ij} \\ 0.5 &> \sum_{j \neq i} M_{ij} \end{aligned} \tag{20}$$

where the first line follows as M is strictly row-diagonally dominant, and the second line from M being a markov matrix. Equations (19) and (20) imply $\left| eig\left(\tilde{M}\right)_k \right| < 1$ for all $k \in \{1, ..., N\}$ eigenvalues of $\tilde{M}$, therefore the Leontieff inverse expansion in equation (17) is valid.

Part B. The Leontieff Inverse summation in equation (17) can be be re-written using the spectral decomposition $\tilde{M} = Q\Lambda Q^{-1}$, where Λ is the diagonal matrix with the $k \in \{1, ..., N\}$ eigenvalues $eig\left(\tilde{M}\right)_k$ along its diagonal.

$$\begin{aligned} M^{-1} &= \sum_{n=0}^{\infty} \tilde{M}^n \\ &= Q \sum_{n=0}^{\infty} \Lambda^n Q^{-1} \end{aligned}$$

Thus, the matrix $\sum_{n=0}^{\infty} \Lambda^n$ is the diagonalized form of M^{-1}, and thus comprise its eigenvalues. That is,

$$eig\left(M^{-1}\right)_k = \sum_{n=0}^{\infty} eig\left(\tilde{M}\right)_k^n = \frac{1}{1 - eig\left(\tilde{M}\right)_k}$$

using that $\left| eig\left(\tilde{M}\right)_k \right| < 1$ for each k, as proved in part A. The real component

$$Re\left\{eig\left(M^{-1}\right)_k\right\} = \frac{1 - Re\left\{eig\left(\tilde{M}\right)_k\right\}}{\left(1 - Re\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2 + \left(Im\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2}$$

Now, this is greater than or equal to one iff

$$\frac{1 - Re\left\{eig\left(\tilde{M}\right)_k\right\}}{\left(1 - Re\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2 + \left(Im\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2} \geq 1$$

$$
\begin{aligned}
1 - Re\left\{eig\left(\tilde{M}\right)_k\right\} &\geq \left(1 - Re\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2 + \left(Im\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2 \\
1 - Re\left\{eig\left(\tilde{M}\right)_k\right\} &\geq 1 - 2\cdot Re\left\{eig\left(\tilde{M}\right)_k\right\} + Re\left\{eig\left(\tilde{M}\right)_k\right\}^2 + \left(Im\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2 \\
0.5^2 &\geq 0.5^2 - Re\left\{eig\left(\tilde{M}\right)_k\right\} + Re\left\{eig\left(\tilde{M}\right)_k\right\}^2 + \left(Im\left\{eig\left(\tilde{M}\right)_k\right\}\right)^2 \\
0.5^2 &\geq \left[Re\left\{eig\left(\tilde{M}\right)_k\right\} - 0.5\right]^2 + \left[Im\left\{eig\left(\tilde{M}\right)_k\right\}\right]^2 \qquad (21)
\end{aligned}
$$

Equation (21) is a circle in the complex plane centered at $eig\left(\tilde{M}\right)_k = 0.5$ with radius 0.5 (see the red circle in figure 6, written in terms of M using $\tilde{M} \equiv I - M$). That is, if $eig\left(\tilde{M}\right)_k$ is within this circle, then $Re\left\{eig\left(M^{-1}\right)_k\right\} \geq 1$.

The final step is to note that the circular bound of equation (19) is contained weakly within the circular bound of equation (21), because $0.5 > \sum_{j\neq i} M_{ij}$ from equation (20) (the blue circle is always weakly within the red circle in figure 6). Thus, M being diagonally dominant and a markov matrix implies $Re\left\{eig\left(M^{-1}\right)_k\right\} \geq 1$ for all eigenvalues k.

The additional result of the proposition on the strict inequality follows from noting that $Re\left\{eig\left(M^{-1}\right)_k\right\} = 1$ is true only when equation (21) holds with equality (on the boundary of the red circle in figure 6). The only value of $eig\left(\tilde{M}\right)_k$ satisfying (21) with equality, while satisfying equation (19), is $eig\left(\tilde{M}\right)_k = 0$ (the only point of overlap of the blue and the red circles in figure 6 is at the origin). This is equivalent to $Re\left\{eig\left(M\right)_k\right\} = 1, Im\left\{eig\left(M\right)_k\right\} = 0$. Hence, as long as at least one eigenvalue of M is not equal to one, then $Re\left\{eig\left(M^{-1}\right)_k\right\} > 1$ for some eigenvalue k.

The only matrix M with all eigenvalues equal to one is the identity matrix I. Thus, if we also assume $M \neq I$, then $\exists k : Re\left\{eig\left(M^{-1}\right)_k\right\} > 1$. □

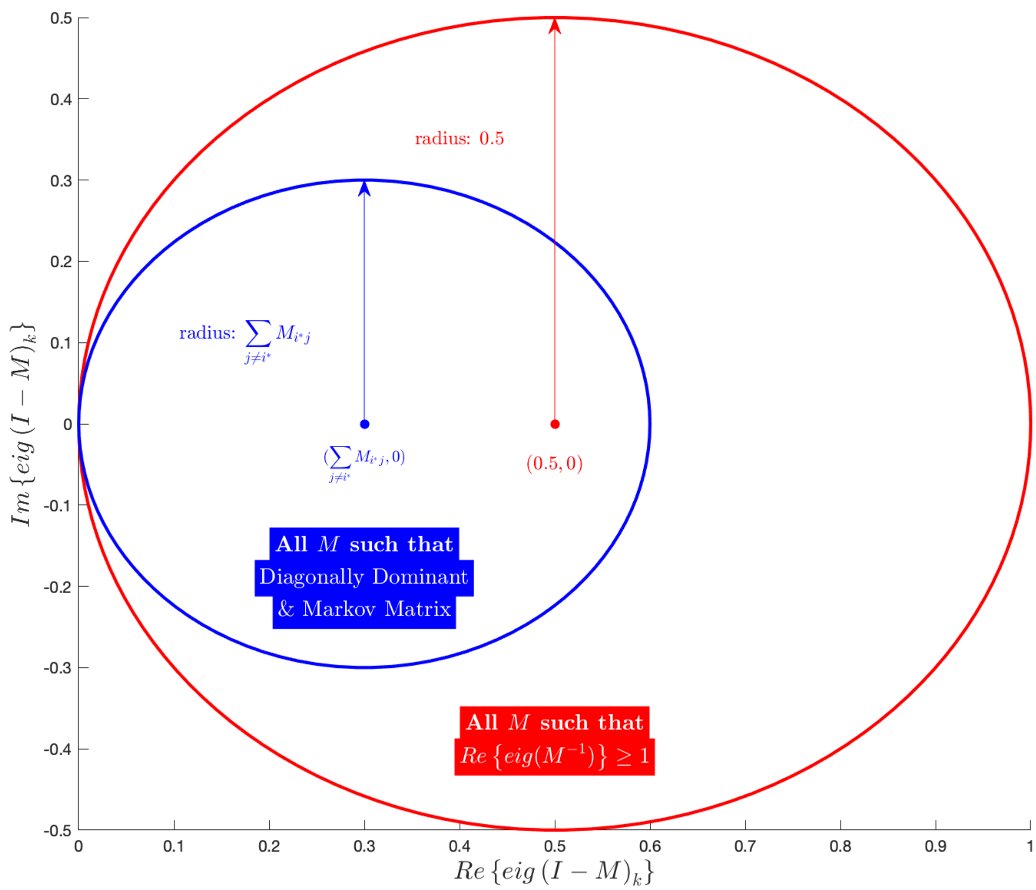


Figure 6: Eigenvalue bounds in proposition 3

Notes. The bounds hold for all eigenvalues, indexed by k.

A.5 Proposition 4 (Home Bias in Two Location Models)

Proof. To prove the proposition, I show that for all parameter values subject to the assumptions in the proposition, a home-biased geography is implied.

I will use the trade freeness matrix

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$$

where $\phi_{ij} = \tau_{ij}^{1-\sigma}$. The premise $\forall i, j \neq i : \tau_{ii} < \min\left\{\tau_{ij}, \tau_{ji}\right\}$ implies

$$\forall i, j \neq i : \phi_{ii} > \max\left\{\phi_{ij}, \phi_{ji}\right\} \tag{22}$$

I rewrite the interior equilibrium condition (assumption 1.1) as

$$\forall i: \quad Y_i > 0 \tag{23}$$

using $Y_i = P_i Q_i = a_i^{\frac{1}{\sigma}} Q_i^{\frac{\sigma-1}{\sigma}}$ from the supply equation (2). Using the equilibrium solution for Y_i in terms of the structural parameters — equation (26), and that $a_i = \overline{a}$, $E_i = \overline{E}$, in equation (23) for location 1 gives

$$\begin{aligned} Y_1 &> 0 \\ \frac{\phi_{22}}{\phi_{22} - \phi_{12}} \overline{E} - \frac{\phi_{21}}{\phi_{11} - \phi_{21}} \overline{E} &> 0 \\ \phi_{22} (\phi_{11} - \phi_{21}) &> \phi_{21} (\phi_{22} - \phi_{12}) \end{aligned} \tag{24}$$

and for location 2 gives

$$\begin{aligned} Y_2 &> 0 \\ \frac{\phi_{11}}{\phi_{11} - \phi_{21}} \overline{E} - \frac{\phi_{12}}{\phi_{22} - \phi_{12}} \overline{E} &> 0 \\ \phi_{11} (\phi_{22} - \phi_{12}) &> \phi_{12} (\phi_{11} - \phi_{21}) \end{aligned} \tag{25}$$

The home-biased geography assumption in a two location model is satisfied if $\gamma_{11} > \gamma_{12}$ and $\gamma_{22} > \gamma_{21}$. Using equation (27) to write export share matrix in terms of the structural parameters, and that $a_i = \overline{a}$, then we find

$$\begin{aligned} \gamma_{11} &= \frac{\phi_{11} (\phi_{22} - \phi_{12})}{\phi_{11}\phi_{22} - \phi_{12}\phi_{21}} \\ &> \frac{\phi_{12} (\phi_{11} - \phi_{21})}{\phi_{11}\phi_{22} - \phi_{12}\phi_{21}} \\ &= \gamma_{12} \end{aligned}$$

where the second line used the interior equilibrium condition for location 2, equation (25). Note that equation (22) implies $\phi_{11}\phi_{22} - \phi_{12}\phi_{21} > 0$. Also,

$$\begin{aligned} \gamma_{22} &= \frac{\phi_{22} (\phi_{11} - \phi_{21})}{\phi_{11}\phi_{22} - \phi_{12}\phi_{21}} \\ &> \frac{\phi_{21} (\phi_{22} - \phi_{12})}{\phi_{11}\phi_{22} - \phi_{12}\phi_{21}} \\ &= \gamma_{21} \end{aligned}$$

where the second line used the interior equilibrium condition for location 1, equation (24).

Hence $\gamma_{11} > \gamma_{12}$ and $\gamma_{22} > \gamma_{21}$ are both satisfied, and the geography is therefore home biased, for all parameter values subject to the assumptions in the proposition. □

B Model Derivations and Extensions

B.1 Equilibrium Solution in terms of Structural Parameters

B.1.1 Output

Output Y_i is solved for in equilibrium by combining demand equation (1), supply equation (2) and market clearing equation (3)

$$\begin{aligned} P_i Q_i^S &= P_i Q_i^D \\ Y_i &= \sum_j \frac{P_{ij}^{1-\sigma}}{\sum_k P_{kj}{}^{1-\sigma}} E_j \\ &= \sum_j \frac{a_i^{-1} Y_i \Phi_{ij}}{\sum_k a_k^{-1} Y_k \Phi_{kj}} E_j \end{aligned}$$

As shown by Behrens et al. (2009), this resulting set of equations can be solved explicitly for Y_i by converting to matrix form

$$\begin{aligned} Y &= diag\left[a\right]^{-1} diag\left[Y\right] \Phi diag\left[\Phi' diag\left[a\right]^{-1} Y\right]^{-1} E \\ \Phi^{-1} a &= diag\left[\Phi' diag\left[a\right]^{-1} Y\right]^{-1} E \\ diag\left[\Phi' diag\left[a\right]^{-1} Y\right] \Phi^{-1} a &= E \\ diag\left[\Phi^{-1} a\right] \Phi' diag\left[a\right]^{-1} Y &= E \\ Y &= diag\left[a\right] \Phi'^{-1} diag\left[\Phi^{-1} a\right]^{-1} E \end{aligned}$$

In component form

$$Y_i = \sum_j \frac{\Phi_{ji}^{-1} a_i}{\sum_k \Phi_{jk}^{-1} a_k} E_j \tag{26}$$

B.1.2 Export Matrix

$$
\begin{aligned}
\gamma_{ij} &\equiv \frac{P_{ij}Q_{ij}}{Y_i} \\
&= \frac{P_{ij}^{1-\sigma}}{\sum_k P_{kj}{}^{1-\sigma}} \frac{E_j}{Y_i} \\
&= \frac{\Phi_{ij}P_i^{1-\sigma}}{\sum_k \Phi_{kj}P_k{}^{1-\sigma}} \frac{E_j}{Y_i} \\
&= \frac{a_i^{-1}Y_i\Phi_{ij}}{\sum_k a_k^{-1}Y_k\Phi_{kj}} \frac{E_j}{Y_i} \\
&= a_i^{-1}\Phi_{ij} \frac{E_j}{\sum_k a_k^{-1}Y_k\Phi_{kj}} \\
&= a_i^{-1}\Phi_{ij} \sum_k \left\{\Phi^{-1}\right\}_{jk} a_k
\end{aligned} \tag{27}
$$

Where the second equality substituted out Q_{ij} using demand equation (1). The third equality used $P_{ij} = P_i\tau_{ij}$ and the definition of the trade freeness matrix $\Phi = \tau^{1-\sigma}$. The fourth equality used $Y_i = a_i P_i^{1-\sigma}$ from the supply equation (2) to substitute out P_i.

The sixth equality used equation (29), which follows from the equilibrium being interior, as derived here

$$
\begin{aligned}
P_iQ_i^S &= P_iQ_i^D \\
Y_i &= \sum_j \frac{P_{ij}^{1-\sigma}}{\sum_k P_{kj}{}^{1-\sigma}} E_j && (28) \\
&= \sum_j \frac{a_i^{-1}Y_i\Phi_{ij}}{\sum_k a_k^{-1}Y_k\Phi_{kj}} E_j \\
1 &= \sum_j \frac{a_i^{-1}\Phi_{ij}}{\sum_k a_k^{-1}Y_k\Phi_{kj}} E_j \\
\sum_i \left\{\Phi^{-1}\right\}_{ji} a_i &= \frac{E_j}{\sum_k a_k^{-1}Y_k\Phi_{kj}} && (29)
\end{aligned}
$$

B.1.3 Assumption 1.1 (Interior Equilibrium) in terms of the structural parameters

Here I rewrite the interior equilibrium condition 1.1, $\forall i : Q_i > 0$ in terms of the exogenous structural parameters of the model, rather than endogenous output Q_i.

I do this by first relating output in quantity units, Q_i, to output in dollar units, Y_i using the supply equation (2) to give

$$
\begin{aligned}
& Y_i = P_i Q_i = \left(a_i^{\frac{1}{\sigma}} Q_i^{-\frac{1}{\sigma}}\right) Q_i = a_i^{\frac{1}{\sigma}} Q_i^{\frac{\sigma-1}{\sigma}} \\
\implies \quad & Q_i = a_i^{\frac{-1}{\sigma-1}} Y_i^{\frac{\sigma}{\sigma-1}} \qquad (30)
\end{aligned}
$$

Next, I substitute out Y_i for using the equilibrium solution in equation (26)

$$
\begin{aligned}
Q_i &= a_i^{\frac{-1}{\sigma-1}} Y_i^{\frac{\sigma}{\sigma-1}} \\
&= a_i^{\frac{-1}{\sigma-1}} \left(\sum_j \frac{\Phi_{ji}^{-1} a_i}{\sum_k \Phi_{jk}^{-1} a_k} E_j\right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

Into the interior equilibrium condition

$$
\begin{aligned}
\forall i \quad Q_i &> 0 \\
a_i^{\frac{-1}{\sigma-1}} \left(\sum_j \frac{\Phi_{ji}^{-1} a_i}{\sum_k \Phi_{jk}^{-1} a_k} E_j\right)^{\frac{\sigma}{\sigma-1}} &> 0 \\
\sum_j \frac{\Phi_{ji}^{-1}}{\sum_k \Phi_{jk}^{-1} a_k} E_j &> 0
\end{aligned}
$$

B.2 General Equilibrium Isomorphism

B.2.1 The Model

This section replicates the extension of the canonical HME framework from Helpman and Krugman (1985) to an arbitrary geography by Behrens et al. (2009).

The economy consists of N locations indexed $i \in \{1, ..., N\}$. Location i hosts an exogenously given mass of $L_i > 0$ consumers, each of whom supplies one unit of labor inelastically. Hence, both the world population and the world labor endowment are given by $L = \sum_i L_i$. Labor is the only factor of production, is assumed to be geographically immobile, sectorally mobile, and its labor supply are traded in perfectly competitive local labor markets. Hence, the wages in all sectors within a location are the same, denoted w_i.

Preferences are defined over a homogenous agricultural good (A) and over a continuum of varieties of horizontally differentiated manufacturing goods (D). The preferences of the representative consumer of location i are represented by the following Cobb-Douglas utility function

$$U_i = X_i^{1-\mu_i} A_i^{\mu_i}$$

where parameter $\mu_i \in (0,1)$is the share of expenditure on manufacturing goods. The representative consumer in location i has income $w_i L_i$.

Agricultural goods are produced by perfectly competitive firms under constant returns to scale with z_i denoting the corresponding unit labor requirement in location i. Agricultural goods can be traded freely across locations, meaning that agricultural price, p_i^A, is equalized across all locations, and is chosen as the numéraire, $\forall i : p_i^A = 1$. Marginal cost pricing implies $p_i^A = z_i w_i$. Therefore, productivity-adjusted wages are equalized across all locations

$$w_i = 1/z_i \tag{31}$$

provided that some numéraire production takes place everywhere. Parameter values are restricted so that this is the case.

X is a CES subutility defined over the manufacturing varieties as follows

$$D_i = \left[\sum_{j=1}^{N}\left(\int_{\Omega_j} d_{ji}(\omega)^{\frac{\sigma-1}{\sigma}}\mathrm{d}\omega\right)\right]^{\frac{\sigma}{\sigma-1}} \tag{32}$$

where $d_{ji}(\omega)$ is the consumption in location i of variety ω produced in location j, and Ω_j is the set of varieties produced in location j. The parameter $\sigma > 1$ is the elasticity of substitution between any two varieties.

The production of any variety of the differentiated goods takes place under increasing returns to scale by a set of monopolistically competitive firms. This set is endogenously determined in equilibrium by free entry and exit.[16] n_i denotes the mass of firms located in location i.

Production of each variety requires a fixed and a constant marginal labor requirements, $f_i > 0$ and $c_i > 0$ respectively, which may be location-specific. The ratio $r_i \equiv f_i/c_i$ measures the intensity of increasing returns to scale in the manufacturing production technology. These are assumed common across locations, that is, $r_i = r$. Increasing returns to scale and costless product differentiation yield a one-to-one relationship between firms and varieties. Shipments of any variety between locations are subject to 'iceberg' trade costs: $\tau_{ji} \geq 1$ units have to be shipped from location j to location i for one unit to reach its destination.[17]

[16]Note that this is not the relevant source of scale economies for the HME in this model. Due to free entry, quantity produced per firm is fixed (see below) in equilibrium and therefore these scale economies (on the *intensive* margin) are not active in response to demand shocks. The relevant source of scale economies is the preference for diversity: as the number of products increases, the cost of achieving one unity of utility (the price index) decreases. This is a form of economies of scale (on the *extensive* margin): price decreases with scale of output.

[17]The remaining $\tau_{ji} - 1$ units "melts" along the way, hence the name iceberg trade costs.

In equilibrium, manufacturing firms differ only by their location. Accordingly, to simplify notation, the variety label ω is suppressed from here. Maximization of consumer utility subject to the budget constraint yields the following demand in location j for a variety produced in location i

$$d_{ij}^D = \frac{p_{ij}^{-\sigma}}{\mathcal{P}_j^{1-\sigma}} E_j, \quad \text{where} \quad \mathcal{P}_j = \left[\sum_k n_k p_{kj}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{33}$$

where p_{ij} is the consumption price of the variety,

$$\mathcal{P}_j = \left[\sum_k n_k p_{kj}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{34}$$

is the CES price index in location j, and

$$\begin{aligned} E_j &= \mu_j w_j L_j \\ &= \frac{\mu_j L_j}{z_i} \end{aligned} \tag{35}$$

is the aggregate expenditure from consumers in location j on manufacturing goods, where in the second equality I used equation (31) to substitute out w_i.

Firms set their production price p_i (assume no price discrimination by destination) and bilateral shipments q_{ij} to maximize profits

$$\Pi_i = \sum_{j=1}^{N} (p_i q_{ij} - w_i c_i q_{ij}) - w_i f_i \tag{36}$$

subject to demand $q_{ij} = \tau_{ij} d_{ij}^D$ (they must ship $\tau_{ij} - 1$ extra to cover the trade costs), the no arbitrage condition $p_{ij} = p_i \tau_{ij}$, and taking $\mathcal{P}_j$ as given (as they are infinitesimal in the market). The optimal production price is

$$p_i = \frac{\sigma}{\sigma - 1} \frac{c_i}{z_i} \equiv p_i^* \tag{37}$$

where I used equation (31) to substitute out w_i.

Due to free entry and exit, profits must be non-positive in equilibrium

$$\Pi_i \leq 0 \tag{38}$$

Using equations (36), (37) and (38) together imply that a firms' equilibrium scale of operation in location i must satisfy

$$q_i^S \equiv \sum_j q_{ij} \leq \frac{f_i(\sigma-1)}{c_i} \equiv q_i^* \tag{39}$$

with equality if $n_i > 0$. The product market clearing in the manufacturing sector is

$$\sum_j \tau_{ij} d_{ij}^D = q_i^S$$

Using the the product demand equation (33) for the LHS, and the free-entry condition (38) for the RHS, we can re-write this as

$$\sum_j \tau_{ij} \frac{p_{ij}^{-\sigma}}{\sum_k n_k p_{kj}^{1-\sigma}} E_j \leq q_i^*$$

$$\sum_j \frac{p_i^{*-\sigma} \tau_{ij}^{1-\sigma}}{\sum_k n_k p_k^{*1-\sigma} \tau_{kj}^{1-\sigma}} E_j \leq q_i^* \tag{40}$$

In an interior equilibrium, equation (**??**) holds with equality for all i. This system of equations solves for the equilibrium number of manufacturing firms in each location n_i. Note that p_i^*, q_i^* are both exogenous.

B.2.2 Isomorphism

I show the isomorphism in three parts. In part A I define a mapping from the variables in canonical model section B.2.1 to the variables in the main text model in section 2. In part B, I show the mapping implies the demand and supply equations (1) and (2) in the main text model. In part C, I show that the market clearing equation (3) imposed in the main text model gives the same equilibrium solution as the canonical model.

Part A. I map the following parameters

$$\underbrace{\left\{n_i, q_i^*, p_i^*, E_i, \{\tau_{ij}\}_{j=1}^N\right\}_{i=1}^N}_{\text{Canonical Model}} \mapsto \underbrace{\left\{P_i, Q_i, a_i, E_i, \{\tau_{ij}\}_{j=1}^N\right\}_{i=1}^N}_{\text{Main Text Model}}$$

Note that τ_{ij}, E_i are the same in both models. The mapping is as follows

$$P_i = n_i^{\frac{1}{1-\sigma}} p_i^* \tag{41}$$

$$Q_i = n_i^{\frac{\sigma}{\sigma-1}} q_i^* \tag{42}$$

$$a_i = p_i^{*\sigma} q_i^* \tag{43}$$

P_i can be interpreted as an effective aggregate production price for location i, which can be seen from inspecting the consumer price index in the canonical model equation (34)

$$\begin{aligned} \mathcal{P}_j &= \left[\sum_k n_k p_{kj}^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\ &= \left[\sum_k \tau_{kj}^{1-\sigma} \underbrace{n_k p_k^{*1-\sigma}}_{\equiv P_k^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} \end{aligned}$$

Q_i can be interpreted as effective aggregate production quantity for location i, because the product $P_i Q_i$ equals aggregate production revenue in the canonical model

$$\begin{aligned} P_i Q_i &= \left(n_i^{\frac{1}{1-\sigma}} p_i^* \right) \left(n_i^{\frac{\sigma}{\sigma-1}} q_i^* \right) \\ &= n_i p_i^* q_i^* \end{aligned}$$

Part B. The demand equation (1) is derived by defining the effective aggregate bilateral shipments, Q_{ij}, such that the product $P_i Q_{ij}$ is equal to total bilateral sales from location i to j in the canonical model

$$\begin{aligned} P_i Q_{ij} &= n_i p_i q_{ij} \\ n_i^{\frac{1}{1-\sigma}} p_i Q_{ij} &= n_i p_i q_{ij} \\ Q_{ij} &= n_i^{\frac{\sigma}{\sigma-1}} q_{ij} \\ &= n_i^{\frac{\sigma}{\sigma-1}} \tau_{ij} \frac{p_{ij}^{-\sigma}}{\sum_k n_k p_{kj}^{1-\sigma}} E_j \\ &= \tau_{ij} \frac{P_{ij}^{-\sigma}}{\sum_k P_{kj}^{1-\sigma}} E_j \\ &\equiv Q_{ij}^D \end{aligned}$$

The supply equation (2) is derived by using equations (41) and (42) to substitute out n_i

$$\begin{aligned}
Q_i &= n_i^{\frac{\sigma}{\sigma-1}} q_i^* \\
&= \left\{ \left(\frac{P_i}{p_i^*} \right)^{1-\sigma} \right\}^{\frac{\sigma}{\sigma-1}} q_i^* \\
&= P_i^{-\sigma} p_i^{\sigma} q_i^* \\
&= P_i^{-\sigma} a_i \\
&\equiv Q_i^S
\end{aligned}$$

where the second to last equality used equation (43).

Part C. Inserting the demand and supply equations (1) and (2) into the market clearing equation (3), and imposing the interior equilibrium assumption 1.1, $Q_i > 0$,

$$\begin{aligned}
Q_i^D &= Q_i^S \\
\sum_{ij} \tau_{ij}^{1-\sigma} \frac{P_i^{-\sigma}}{\sum_k P_{kj}^{1-\sigma}} E_j &= a_i P_i^{-\sigma} \\
\sum_{ij} \tau_{ij}^{1-\sigma} \frac{1}{\sum_k P_k^{1-\sigma} \tau_{kj}^{1-\sigma}} E_j &= q_i^* p_i^{*\sigma} \\
\sum_{ij} \tau_{ij}^{1-\sigma} \frac{p_i^{*\sigma}}{\sum_k n_k p_k^{*1-\sigma} \tau_{kj}^{1-\sigma}} E_j &= q_i^* \qquad (44)
\end{aligned}$$

Where the third equality used that $P_i^{-\sigma} > 0$ by the interior equilibrium assumption.

Equation (44) is exactly equation (40) with equality, i.e. under an interior equilibrium. Hence, the equilibrium solutions from the canonical model and the main text model are the same.

B.2.3 Exogenous Microfoundation for shift in Expenditure, E_i

Total expenditure E_i in the main text model of section (2) is assumed exogenous and given. It's in the this sense which the main text model is partial equilibrium. However, we can provide a microfoundation of E_i that is consistent with general equilibrium by using the canonical model of section B.2.1.

Equation (35), rewritten here for convenience

$$E_i = \frac{\mu_i L_i}{z_i}$$

gives E_i in terms of exogenous variables in the canonical model (section B.2.1).

Thus, through the lens of this microfoundation, a shift in the exogenous E_i in the partial equilibrium model of the main text can be consistently interpreted as a shift in the endogenous E_i in the general equilibrium canonical model arising from either a shock to population L_i, or to the share of expenditure on manufacturing μ_i.

This is consistent because μ_i, L_i are only appear in E_i and not the supply shifter a_i — see equation (43).[18] Thus a shock to μ_i, L_i only shifts E_i. Note that a shock to the agricultural unit labor requirement, z_i, is not consistent as the supply shifter a_i does depend on z_i.

[18]It may be surprising that supply doesn't depend on population L_i, which is also total labor, in the canonical model in section (B.2.1). This is because of the agricultural sector causing wages to be fixed in equilibrium. The manufacturing sector faces a perfectly elastic labor supply curve and therefore is independent of L_i.