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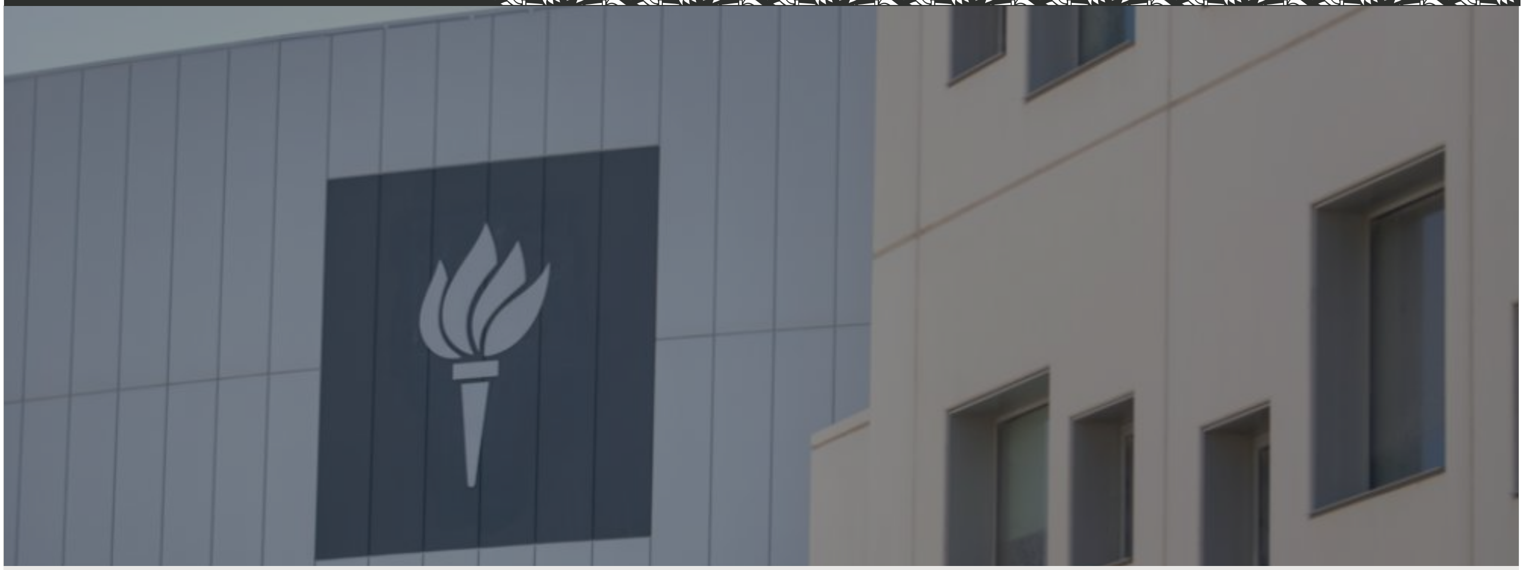

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Collaborative production networks among unequal actors

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Abstract

We develop a model of strategic network formation of collaborations to analyze the consequences of an understudied but consequential form of heterogeneity: differences between actors in the form of their production functions. We also address how this interacts with resource heterogeneity, as a way to measure the impact actors have as potential partners on a collaborative project. Some actors (e.g. start-up firms) may exhibit increasing returns to their investment into collaboration projects, while others (e.g. established firms) may face decreasing returns. Our model provides insights into how actor heterogeneity can help explain well-observed collaboration patterns. We show that if there is a direct relation between increasing returns and resources, start-ups exclude mature firms and networks become segregated by types of production function, portraying DOMINANT GROUP architectures. On the other hand, if there is an inverse relation between increasing returns and resources, networks portray CORE-PERIPHERY architectures, where the mature firms form a core and start-ups with low-resources link to them.

JEL Codes: D85, D03, C72

Keywords: Collaboration, Exchange, Inequality, heterogeneity

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1 Introduction

Collaboration is key to realize outcomes that are difficult to achieve individually. Examples of mutually beneficial collaboration can be found in joint ventures between firms (Goyal and Moraga-González 2001) as well as in scientific co-authorships (Jackson and Wolinsky 1996), among many other cases. A key question underlying collaboration choices is under which conditions engaging in a collaborative project with a specific partner becomes mutually beneficial, and how do such conditions affect choices in a network of collaborations where there are multiple partners and multiple projects at the same time. In this paper, we focus on how two characteristics of collaboration partners affect the way collaboration networks are shaped. Namely, we focus on the relation between the endowment of resources actors have and the production functions governing the way they can make use of such resources.

Resource endowments play a key role in how attractive actors are as potential collaborative partners (Blau 1964; Homans 1958; Cook and Emerson 1978; Molm 1994). Wealthier potential partners are more appealing than poorer ones to form alliances with (Cook et al. 1983; Emerson 1962). Yet, screening potential partners only for the size of their resource endowment neglects another key source of productivity: their ability to put those resources to productive use. This ability is captured by an actor's production function. An actor's *production function* can yield increasing or decreasing marginal returns to his investment into a collaborative project. Consequently, the relation of the production function and available resources represents the potential impact an actor can make on a collaborative project. That is, actors can potentially have a high impact on a collaboration either because they have large amounts of resources despite being less productive (i.e., decreasing marginal returns) or because they are more productive (i.e., increasing marginal returns) despite having smaller endowments.

Differences in production functions can arise from differences between actors in terms of skills, talents, or available technology (Collins 1990; Sellinger and Crease 2006). For example, a startup with an innovative technology that is in its early stages of development represents an actor whose production function generates increasing marginal returns, because further investments into it yield increasingly fast progress. An example of an actor whose production function generates decrease marginal returns would be a firm operating with a mature technology, for which investments into new technology do not yield significant productivity gains. For example, in the realm of inter-firm collaboration, consider Campbell Soup Co., which invested $125 million in January 2016 to finance food start-ups, hoping that this would allow them to keep up with small companies increasingly dominating the food trends in the United States.[1] A mature firm like Campbell has ample resources, and yet aimed for alliances with smaller partners, whose "start-up" production functions, promised higher returns on investment than collaboration with another equally large firm, or scaling up its own business. Notably, in a case such as this, having large available resources can compensate for having a decelerating production function, allowing large firms to occupy a central position in the collaboration network.

We propose a model of network formation to study the way individual heterogeneity in available resources and actors' production functions impact collaboration choices. Thus, the first aim of the paper is to formalize how the distribution of heterogeneous individuals in the population shapes the strategic formation of collaborations and the network architectures that emerge. Specifically, we model collaboration networks as weighted graphs were actors simultaneously choose with whom to collaborate and how much of their resources to allocate into each collaborative project. Actors can also keep resources to allocate into in-house production, for which they do not require any partners. To illustrate the strategies players follow, contingent on their type (production function), we provide a progressive characterization of equilibrium outcomes. We start with the simplest case of collaborations in a 2-person game, which allows us to look at all possible combinations of types of players and endowments of resources. We then move to the more general case of n-person games, where we focus on Nash as well as pairwise stable Nash equilibria.

The intuition of our main results is as follows: In terms of strategies, there are mixed effects of joint collaboration strategies with substantial differences between types of actors. Actors with production functions that yield decreasing marginal returns (DMR), e.g. mature firms, are better off diversifying their resources into multiple collaborations, while actors with increasing marginal returns (IMR), e.g. start-ups, are better off following an *all-or-nothing* strategy. This is so because IMR actors are only attracted to partners that can make a high impact on the collaborative project, otherwise they are better off investing all their resources into in-house production, while DMR actors benefit by establishing collaborative projects of different sizes.

Consequently, the way resource endowments are distributed between types of actors will impact the emerging patterns of collaborations. For instance, when resources are such that IMR actors can make a high impact into a collaborative project while DMR actors can only make a low impact, networks become segregated between types of players. These resulting networks resembling DOMINANT GROUP architectures, where IMR types only collaborate between them or stay isolated, while DMR actors end up excluded and forming multiple collaborations between them (see Figure 1a). On the other hand, when resources are distributed in such as way that DMR actors can make a

[1] "Campbell Invests $125 Million in Project to Fund Food Startups". The Wall Street Journal. February 17, 2016.

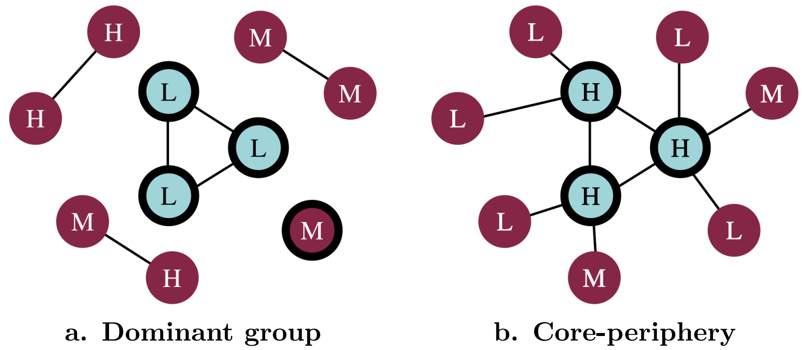

Figure 1. Examples of prominent collaboration networks.

Note: The color inside a node represents its type: DMR (light blue) or IMR (dark red). The letter inside the node represent the impact a node can have in the collaborative projects with his partners: **L**ow, **M**edium or **H**igh impact. Nodes with a thick black border allocate resources to in-house production. A link between two nodes represents a collaborative project.

high impact on the collaborative projects they form, while IMR actors can only make a low impact, networks resemble CORE-PERIPHERY architectures. Specifically, the well-endowed DMR actors form a core between them and also establish collaborations with IMR actors, who are unattractive to each other (see Figure 1b). Both of these classes of networks are prominent in the literature and provide evidence that our focus on variations of resources and production functions has valuable insights into real-world networks for different domains. This is further discussed in the following section.

In the remainder, we first highlight our contribution to the existing literature and then outline the model. Subsequently, we characterize equilibrium outcomes as a result of the interactions of actors with different resources and production functions. We then close the analysis by focusing on networks that are pairwise stable Nash equilibria. We conclude with a discussion of the implications and limitations of the study.

2 Relation to the literature

Our study draws on and contributes to the research on collaboration as well as on the literature on endogenous network formation.

First, its theoretical point of departure is the formation of collaboration projects, also referred to as strategic alliances (Belderbos et al. 2006) or productive exchanges (Molm 1994, 1997). Collaborations refer to interactions in which actors join their resources, aiming at outcomes greater than the aggregation of what each could have gotten separately (for a survey see Cook and Cheshire 2013). Notably, research on collaborations has singled out *resource heterogeneity* as a major antecedent of collaboration network structures: the larger an actor's resource endowment, the more attractive this actor becomes as a collaboration partner (Goyal and Joshi 2003; Galeotti et al. 2006).

Our work is closely related to Flache and Hegselmann (1999) and Hegselmann (1998) who study how heterogeneity in resources shapes social support network. Their main findings indicate that resource heterogeneity can result in exclusion and segregated networks. Specifically, they observe that resource rich actors need little help but can give a lot of help to those in need, while resource poor actors need a lot of help but have little to give. Resource rich actors seeking to optimize their collaborative relations prefer to form partnerships with other resource rich actors, thereby indirectly excluding resource poor actors from their collaboration choices. For the latter, only other resource poor actors remain as potential partners, leaving resource poor actors with less favorable collaboration opportunities (see also Flache 2001). An implicit assumption behind these resource heterogeneity approaches is that everyone has the same production function. Whereas in such cases resource rich actors may indeed be the most attractive collaborative partners. Our work extends this analysis by modeling heterogeneity of collaboration partners' *production functions*. The interaction between resources and production functions shows conditions under which resource rich actors do not acquire a central position.

A second indication from the empirical work on collaboration is that there seems to be a positive impact of the establishment of various collaborations, e.g., R&D ventures, on firm performance (see e.g., Goyal and Moraga-González 2001). In this sense, a key contribution of our work is to provide a framework that allows for differences in the production functions firms have. In this framework, we find that there are mixed effects of joint collaboration strategies with substantial differences between types of firms, due to their production functions. Namely, large firms benefit from diversification while smaller firms face diseconomies when pursuing multiple collaborations at the same time.[2] In this sense, our work is closely related to Belderbos et al. (2006), who observe empirical evidence showing that in many sectors and industries some firms diversify while others do not, and the main driver of these differences is the size, i.e., productive capacity, of the firms. Also to Baker et al. (2008) who found these patterns of "unstructured collaborations" in the pharmaceutical industry.

We model the collaboration strategies as resulting from actors strategically optimizing their investments across several collaborative projects. In this sense, we build on the literature on *endogenous network formation* (Jackson and Wolinsky 1996; Snijders and Doreian 2010), investigating

[2]There is a stream of literature in sociology looking at collaboration interactions. However, their focus is on exogenously imposed networks (Cook and Emerson 1978; Bienenstock and Bonacich 1992; Molm and Cook 1995; Dijkstra and van Assen 2006) or restricted to the activation of a single collaboration at a time (Willer 1999), which impedes the analysis of collaboration strategies.

which structures (i.e., patterns of relations) emerge from rational actors' attempts to optimize their exchange relations (Jackson and Wolinsky 1996; Jackson and Watts 2001; Buskens and van de Rijt 2008; Braun and Gautschi 2006; Dogan and van Assen 2009; Dogan et al. 2011; Doreian 2006; Hummon 2000; Raub et al. 2014). We specifically combine in a single choice network formation and endogenous effort and in this sense our work closely relates to some relevant work in economics (see e.g., Galeotti and Goyal 2010; Goyal and Joshi 2003; Jackson and Watts 2002). Most of this models, however, treat actors as homogenous and disregard differences in attributes, which is a main contribution of our work.

Our main findings are closely related to the results in Konig et al. (2014) and Belhaj et al. (2016), both of which identify that in settings of strategic complementarities, such as collaboration networks, the emerging pattern of strategic alliances resembles nested-split graphs. Two structures that are prominently observed: DOMINANT-GROUP and CORE-PERIPHERY architectures. Our model indicates how the relation between actors production functions and available resources may lead to either of these patterns of collaborations. The DOMINANT-GROUP architecture is observed when big firms have limited resources, which makes them unattractive for innovative firms, such as start-ups. The consequence of the dominant-group network is that the network segregates by types of firms. Another way interpret this segregated structures is that if firms do not manage to accumulate enough resources when they reach maturity, they are likely to be precluded from collaborating with innovative partners. On the other hand, the CORE-PERIPHERY architecture is observed when big firms have accumulated enough resources to become central while start-ups that have the potential to be innovative and productive do not have the capital to make it happen, and depend on the collaboration with mature firms.

In summary, our model contributes to the research on collaboration networks in two main ways: First, we study actor heterogeneity in terms of both resource endowments and production functions (e.g. expertise, skills, creativity, talent, or technology). This allows us to extend the analysis that has been widely focused only on differences in wealth, and to evaluate the effect of how actors' ability to use such wealth in collaborative projects makes them more or less attractive as potential partners. Second, we advance strategic network formation models by conceptualizing actors' investments as a continuous rather than a dichotomous variable. This allows us to study the problem of collaboration in weighted networks where the intensity of the interaction, and not just its existence, is evaluated. By means of this, we can show that the particular choice of a Cobb-Douglas payoff function for our model provides results in line with more general forms. But additionally, its specificity allows us to tackle a problem that is of utmost interest in the literature on networks, and specifically in the literature on collaborations: the relation of link existence and link

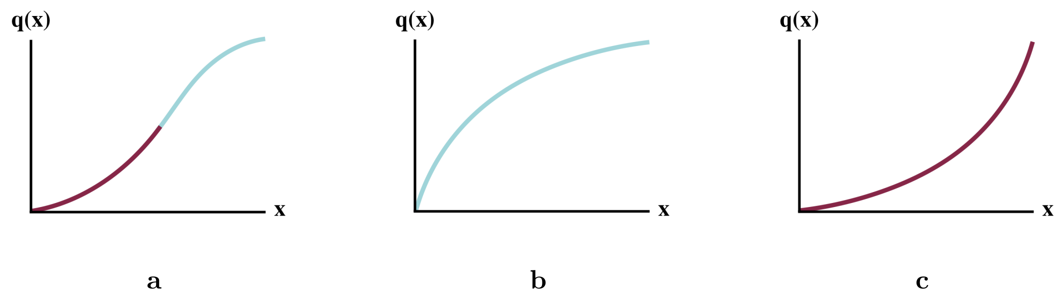

Figure 2. Production functions.

Note: The horizontal axis represents units of resources allocated by an actor to a collaborative project and the vertical axis represents levels of outputs (i.e., impact) achieved with these resources, given a fixed and strictly positive allocation by a collaborative partner.

intensity. That is, our specificity in the payoff function provides useful insights into the unexplored framework of weighted networks. As a result, we are able to show how some network structures that have been persistently observed in theoretical and empirical work, namely DOMINANT GROUP and CORE-PERIPHERY networks, arise as weighted networks, and how they depend on the relation between productive functions and available resources in the network.

3 The model

The model rests on two general assumptions. First, players differ in their *resource endowments* and in their *production functions*, which can yield *increasing* or *decreasing* marginal returns to investments. Second, players can form collaborations with others, in pairs, by pooling resources with their partners. They can establish multiple collaborative projects at a time, distributing their resources across partners. We elaborate on both assumptions below, proceeding to the game theoretic analysis thereafter.

3.1 Heterogeneity in resources and in production functions

Whether a collaborative project is mutually attractive to a pair of players depends on their resource endowments, their production functions, and the production functions and endowments of alternative collaborative partners. We distinguish production functions with *decreasing* or *increasing* marginal returns to their allocation of resources, which represents a player's type in the game. This is summarized in the definitions below.

Definition 1. ***Decreasing marginal returns to own investments*** *(*DMR*): A player has type* DMR *if his production function is such that for each extra unit of resources allocated to a collaborative* *project, the resulting output will be less valuable than that of the previous unit, keeping the allocation of the partner fixed.*

Definition 1 indicates that for a DMR player the first units of resources invested in a project have the greatest impact and subsequent units invested in the same project are less valuable, as illustrated in Figure 2.b.

Definition 2. ***Increasing marginal returns to own investments*** *(*IMR*): A player has type* IMR *if his production function is such that for an extra unit of resources allocated to a collaborative project, the resulting output will be more valuable than that of the previous unit, keeping the allocation of the partner fixed.*

Definition 2 indicates that the first units of resources invested by an IMR player into a collaborative project have negligible impact, and only after a certain amount of resources have been invested, the additional investments make a big difference, as illustrated in Figure 2.c.

The shapes of the production functions in the model can be understood as giving a snapshot of "short run" situations in which technology is fixed. Thus, firms may be in different stages of a more general production processes such that the usual s-shaped curve (see Figure 2.a) for marginal returns does not apply. Instead, firms production functions can be in the accelerating (IMR) or decelerating (DMR) part of the curve.[3]

3.2 Strategic link formation

Our model portrays collaboration networks as weighted graphs. A link in this graph represents a dyadic collaborative project. The weight of a link represents the output of the collaboration. The size of this output is determined by the joint impact of the parties involved. The impact is expressed as the partners' allocations to the relation and the combined effect of their production functions. That is, we integrate two choices actors make: with *whom* they connect and *how much of their resources* they allocate to each of their connections. These choices are decided simultaneously by the pair of allocation decisions made by two (potential) collaboration partners. If at least one of them allocates no resource to the collaborative project, the project does not take place. If both allocate resources to the project, the output of these allocations determines the link weights and

[3]The effects of production functions have been studied before, especially in Marwell and Oliver's work on critical mass in collective action (Marwell et al. 1985; Marwell and Oliver 1993). In their work, however, the shape of a production function is a property of the collective good, rather than a property of (potential) *individual* contributors, as in our study. In our approach, both partners' production functions jointly affect the output of the collaborative project.

the outcome of the collaboration for each partner. The total amount of resources an actor possesses puts a constraint on how much can be invested in a single project.

Decision making about link formation and resource allocations is modeled in terms of a one-shot non-cooperative game. The set $N = \{1, \ldots, n\}$, where $|N| \geq 2$, represents the players in the collaboration network game, denoted by Γ. Every player $i \in N$ is ex-ante and exogenously endowed with a fixed individual amount of resources $\Omega_i > 0$, that can vary across players i. Also, players are assigned a type expressed by his individual production function $\delta_i > 0$. A player has type DMR when his production function yields decreasing marginal returns to an additional unit of resources invested in a collaborative project, $\delta_i < 1$, and type IMR when the marginal returns are increasing, $\delta_i > 1$.[4]

Prior to the start of the game, players are informed about the size of the set of players, which is fixed throughout the analysis, and the endowments and types (i.e., production functions) of all players. We represent the network by the set of undirected links, g, denoting collaborative projects between connected players. A collaborative project between two players i and j is denoted by $ij \in g$, whereas $ij \notin g$ indicates that there is no collaboration. Resources not invested in collaborative projects are used by players for in-house production, denoted by the self-link $ii \in g$. The set of partners a player i has is $N_i(g) = \{j : ij \in g\}$, for all $j \in N$. The cardinality of $N_i(g)$ is n_i (the degree of node i in the network), and is endogenously determined through the simultaneous choices of all players.

Each player can form more than one collaboration simultaneously and at most $n - 1$. In addition, a player can establish a connection to himself (i.e., his in-house project). A player i simultaneously chooses whom to collaborate with and the amount of resources to allocate into each of his collaborative projects, expressed by the vector of allocations $x_i = \{x_{i1}, \ldots, x_{ii}, \ldots, x_{in}\}$, where Ω_i constrains the size of total investments player i can make. The allocation of resources by i can be made to two types of projects: in-house, x_{ii}, and collaboration with a partner j, x_{ij}. We denote $x_{(N_i(g))}$ as the vector of allocations made to i by i's partners. When a player j does not wish to collaborate with i he simply allocates no resources to i.

Payoffs in the game are determined by a Cobb-Douglas production function, $u_i(\Gamma)$, which depends on the allocation choices made by all players and the shapes of their individual production functions, i.e., their types, as follows:

[4] Following the functions in Figure 2, players with $\delta_i < 1$ are decelerating players, players with $\delta_i = 1$ are linear players, and players with $\delta_i > 1$ are accelerating players. We focus our analysis on accelerating and decelerating players. However, proofs account for linear players as well.

$$u_i(\delta_i, \delta_{-i}, x_i, x_{N_i(g)}) = \rho x_{ii}^{\delta_i} + \sum_{j \neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j} \quad (1)$$

where δ_{-i} is the vector of parameters of the production functions of players other than i, and $\rho > 0$ is a premium on individual production, weighting the relation between in-house and collaborative outputs.[5] Note how this production function captures the essential feature of productive collaborations, in which players cannot produce any value unless both partners to a collaborative project contribute.[6] We assume that players' payoffs are identical to the summed productiveness of their projects, $u_i(\Gamma)$. Note that, in our setup, one and the same player can be part of multiple collaborative projects without necessarily distributing his resources equally between them.[7]

As mentioned above, players can differ in the amount of resources they are endowed with, Ω_i, and in the shape of their production function, δ_i. We refer to the relation between resources and production functions as the potential impact a player can make as a partner in a collaborative project. Players' impact can be either high, medium or low. Formally, we assign to the set $H = \{i : \Omega_i^{\delta_i} > \rho\}$ those players whose impact is high because it is greater than the premium on individual production. Conversely, we assign to sets $M = \{i : \Omega_i^{\delta_i} = \rho\}$ and $L = \{i : \Omega_i^{\delta_i} < \rho\}$ those whose impact is medium or low, respectively. Given that potential impact is not contingent only on available resources, players of type DMR need a larger amount of resources than those of type IMR to have a high impact on a collaborative project.

We call the collection of allocation vectors of all players (one for each player) an *allocation profile*, and denote it by $(x_1, \ldots, x_n)$. When no player has incentives to unilaterally deviate from a given allocation profile $(x_1^*, \ldots, x_n^*)$, this profile is a Nash equilibrium. Formally:

$$u_i(\delta_i, \delta_{-i}, x_1^*, \ldots, x_n^*) \geq u_i(\delta_i, \delta_{-i}, x_1^*, \ldots, x_i', \ldots, x_n^*) \;\forall\; x_i^* \neq x_i', \quad i \in N.$$

The Nash equilibrium requirement can be seen as a minimal condition for a collaboration outcome to be consistent with the rational self-interest of the players involved. If the outcome is

[5] Note that players do not bargain or negotiate the exchange of resources but participate in reciprocal (and contingent) acts of giving resources (see e.g., Lawler 2001; Molm 1990, 1994).

[6] For two players i and j, if $x_{ij} > 0$ and $x_{ji} = 0$, no collaboration occurs between them and the resources invested by i in the failed project are lost. That is, the interaction is valuable if resources from i and j are used together, in coordinated fashion, as in Baker et al. (2008). However, the resources invested by a player in in-house production are multiplied by ρ. Coleman (1990), in his study of social exchange, assumes $\rho = 1$. In our case, by allowing for multiple values of the premium on individual production we cover a wider set of productive scenarios.

[7] This is a more general assumption than found in some existing models where every time a player forms a new link their resources are re-distributed symmetrically between all partners (e.g., Jackson and Wolinsky 1996).

not a Nash equilibrium, then at least some players could gain from reallocating their resources and would do so.

4 Equilibrium

In this section, we describe the Nash equilibria for the one-shot network game with complete information, $NE(\Gamma)$. We first define the set of strategies players have and discuss the 2-person game in Section 4.1. The 2-person game serves to explain which partners a player would prefer, given their potential impact, i.e., available resources and production functions, and illustrates the best response logic. This analysis is extended to the n-person case, for which we provide a characterization of the Nash equilibria in Section 4.2. Finally, in Section 4.3 we focus on the reduced set of equilibrium networks that are both Nash and pairwise stable.

4.1 Strategies

A player in the network game Γ chooses an allocation vector x_i. He either allocates his entire endowment into in-house production ($x_{ii} = \Omega_i; \sum_{j\neq i}^{n} x_{ij} = 0$), into collaborative projects with others ($x_{ii} = 0; \sum_{j\neq i}^{n} x_{ij} = \Omega_i$), or into a combination of both in-house and collaborative projects ($x_{ii} > 0; \sum_{j\neq i}^{n} x_{ij} > 0$), where always $x_{ii} + \sum_{j\neq i}^{n} x_{ij} = \Omega_i$. Lemma 1 describes the strategies players follow given their type, IMR or DMR, and their partner's impact to the collaborative project in a 2-person game, as follows:

Lemma 1. ***Optimal allocation in the 2-person game:*** *The optimal choice of a player i of type* IMR *($\delta_i > 1$) is to allocate all of his resources into in-house production if his partner's impact to the collaborative project is low, or to allocate all his resources in the joint collaboration if his partner's impact is medium or high. The optimal choice of a player i of type* DMR *($\delta_i < 1$) is to distribute his resources between in-house and collaborative projects, adjusted to his partner's impact.*

Proof. Lemma 1 describes the optimal allocations for the interaction between two players in the collaboration game Γ. Formally, in the proof we denote the set of resources a player i has as $\hat{\Omega}$, where $\hat{\Omega} \leq \Omega_i$. This means that we can generalize the proof for any proportion of resources considered from the entire endowment Ω_i. This is a useful consideration for the extension of the results to games of any size $n \geq 2$. However, we specifically use Ω_i when we want to make explicit that the entire endowment is allocated. Consider the optimization problem below, where a player i decides on the optimal way of allocating his resources between in-house and collaborative production:

$$\max_{x_{ii}} \quad u_i = \rho x_{ii}^{\delta_i} + (\hat{\Omega} - x_{ii})^{\delta_i} x_{ji}^{\delta_j}$$

Note that the maximization is phrased in terms of the resources i keeps for in-house production. The First Order Condition (FOC) implies:

$$\frac{\partial u_i}{\partial x_{ii}} = \rho\delta_i x_{ii}^{(\delta_i-1)} - \delta_i(\hat{\Omega} - x_{ii})^{(\delta_i-1)} x_{ji}^{\delta_j} = 0,$$

and the Second Order Condition (SOC) implies:

$$\frac{\partial^2 u_i}{\partial x_{ii}^2} = \rho\delta_i(\delta_i - 1)x_{ii}^{(\delta_i-2)} \mp \delta_i(\delta_i - 1)(\hat{\Omega} - x_{ii})^{(\delta_i-2)} x_{ji}^{\delta_j} \gtreqless 0$$

so that:

$$\begin{cases} u_i^{''} > 0 & \text{if } \delta_i > 1 : \nexists \text{ internal maximum} \\ u_i^{''} = 0 & \text{if } \delta_i = 1 : u_i' = \rho - x_{ji}^{\delta_j} \gtreqless 0 \\ u_i^{''} < 0 & \text{if } \delta_i < 1 : \text{internal maximum is feasible} \end{cases}$$

For the case of player i of type IMR, whose production function yields increasing marginal returns ($\delta_i > 1$), no interior point can be a local maximum, thus neither a global one. Therefore, only the corner solutions ($x_{ii} = 0; x_{ii} = \Omega_i$) are candidates for a global solution. The payoff functions for each are $u_i(x_{ii} = 0) = \Omega_i^{\delta_i} x_{ji}^{\delta_j}$ and $u_i(x_{ii} = \Omega_i) = \rho\Omega_i^{\delta_i}$, respectively. Thus, i's best response (BR) is:

$$BR = \begin{cases} x_{ii}^* = 0 & \text{if } x_{ji}^{\delta_j} \geq \rho \\ x_{ii}^* = \Omega_i & \text{if } x_{ji}^{\delta_j} < \rho \end{cases} \tag{2}$$

with indifference between the two possibilities if $x_{ji}^{\delta_j} = \rho$.

If a player i has a type that yields constant returns to scale ($\delta_i = 1$) it follows immediately from the FOC that:

$$BR = \begin{cases} x_{ii}^* = 0 & \text{iff } x_{ji}^{*\delta_j} > \rho \\ x_{ii}^* \in [0, \Omega_i] & \text{iff } x_{ji}^{*\delta_j} = \rho \\ x_{ii}^* = \Omega_i & \text{iff } x_{ji}^{*\delta_j} < \rho \end{cases} \tag{3}$$

If a player has type DMR, we know from the FOC that $\rho\delta_i x_{ii}^{\delta_i-1} = \delta_i(\hat{\Omega} - x_{ii})^{\delta_i-1} x_{ji}^{\delta_j}$, where $\rho x_{ii}^{\delta_i-1} = (\hat{\Omega} - x_{ii})^{\delta_i-1} x_{ji}^{\delta_j}$, so that $\hat{\Omega} = x_{ii}[1 + (\frac{1}{\rho})^{(\frac{1}{1-\delta_i})} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]$:

$$BR = \left\{ x_{ii}^* = \hat{\Omega}[1 + (\tfrac{1}{\rho})^{(\frac{1}{1-\delta_i})} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{-1} \quad \text{if } x_{ji}^{*\delta_j} \gtreqless \rho \right. \tag{4}$$

To ascertain that Eq. (4) leads to a global BR we compare it to the two corner solutions. Substituting Eq. (4), in u_i yields:

$$u_i(x_{ii}^*) = \rho(\Omega_i[1 + (\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{-1})^{\delta_i} + [\Omega_i - (\Omega_i[1 + (\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{-1})]^{\delta_i} x_{ji}^{\delta_j}$$

$$u_i(x_{ii}^*) = \frac{\rho\Omega_i^{\delta_i}}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}} + [\Omega_i - \frac{\Omega_i}{1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}}]^{\delta_i} x_{ji}^{\delta_j}$$

$$u_i(x_{ii}^*) = \frac{\rho\Omega_i^{\delta_i} + \Omega_i^{\delta_i}[(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i} x_{ji}^{\delta_j}}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}} = \frac{\Omega_i^{\delta_i}(\rho + \rho^{\frac{\delta_i}{\delta_i - 1}} x_{ji}^{\frac{\delta_j\delta_i}{1-\delta_i}+\delta_j})}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}} = \frac{\Omega_i^{\delta_i}\rho(1 + \rho^{\frac{\delta_i}{\delta_i - 1}} x_{ji}^{\frac{\delta_j}{1-\delta_i}})}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}}$$

$$u_i(x_{ii}^*) = \rho\Omega_i^{\delta_i}[1 + (\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{1-\delta_i}$$

Now, the question is when is $u_i(x_{ii}^*) \geq u_i(x_{ii} = \Omega_i)$. We say this condition is satisfied when:

$$\rho\Omega_i^{\delta_i}[1 + (\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{1-\delta_i} \geq \Omega_i^{\delta_i} x_{ji}^{\delta_j}$$

$$\rho^{\frac{1}{1-\delta_i}}[1 + (\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}] \geq x_{ji}^{\frac{\delta_j}{1-\delta_i}}$$

$$\rho^{\frac{1}{1-\delta_i}} \geq 0$$

which is always true. □

The proof for Lemma 1 formalizes how IMR and DMR players best respond to their partners in a dyadic interaction. The intuition of Lemma 1 is depicted in Table 1, where all possible matchings of 2-player games are summarized. Table 1 shows that IMR players have *all-or-nothing* best responses, as a function of their partner's impact. Thus IMR players have at most one collaborative project with a partner who has a medium or high impact on the collaboration. Moreover, if they have such as project, they dedicate all their resources to it (see Table 1b and Table 1c). Note that this is possible because a collaborative project is assumed to be always big enough to absorb all of a player's resources. Table 1 also shows how a player with type DMR is better off diversifying the use of his resources, by allocating positive *fractions* of his endowment into different projects. This, unlike with IMR players, is not impeded by his own or his partner's impact (see Tables 1a and 1c).

The intersections of the best responses presented in Lemma 1 result in the Nash equilibria of the 2-person game (which are not necessarily unique in terms of link intensity), as illustrated in Table 1. Specifically, the results of Lemma 1 generalize to n-person networks, given the solution to the optimization problem can be applied to any part $\hat{\Omega} \leq \Omega_i$ of i's resources, i's utility being additive across all projects he is engaged in (see Eq. 1). This is of particular importance for DMR players. Consider, for instance, a player i of type DMR involved in k collaborative projects. Since

Table 1. Equilibrium outcomes in the 2-person game

Note: Each cell reports the combination of allocations made by player 1 (rows) and player 2 (columns) in a 2-player game where both players have type IMR (Table 1a), both have type DMR (Table 1b), or one has type IMR and the other type DMR (Table 1c). Each table reports all combination of cases where a player can make a high (H), medium (M) or low (L) impact to a collaborative project. Allocations are reported as 1 if a player uses 100% of his endowment, 0 if he uses none, or + when he allocates some resources to the joint project and keep some for in-house production. The highlighted cells are the combination of players between whom a collaboration will take place.

IMG - IMG			
	H	M	L
H	(1,1)	(1,1)	(0,1)
M	(1,1)	(1,1)	(0,1)
L	(1,0)	(1,0)	(0,0)

(a)

DMG - DMG			
	H	M	L
H	(+, +)	(+, +)	(+, +)
M	(+, +)	(+, +)	(+, +)
L	(+, +)	(+, +)	(+, +)

(b)

IMG - DMG			
	H	M	L
H	(1,+)	(1,+)	(0,+)
M	(1,+)	(1,+)	(0,+)
L	(1,+)	(1,+)	(0,+)

(c)

the utility function of player i is additive in the k projects, we can consider any of the k projects as an independent 2-person game, conditional on the $k-1$ other projects. By Lemma 1, player i will best respond in any of the k projects according to Eq. (4). In particular, player i will have a non-zero self-allocation in any of the k projects, including in-house production. This is formally presented in the following section.

4.2 Nash equilibria

To describe the set of Nash equilibria, $NE(\Gamma)$, in terms of the resources players allocate, we consider the general problem of optimizing the payoff function $u_i(\Gamma)$, subject to the constraint $x_{ii} + \sum_{j\neq i}^{n} x_{ij} = \Omega_i$.

Proposition 1. ***Best Responses in*** Γ***:*** *For a collaboration network game, the proportion of resources player i allocates to a project is equal to the proportional productivity of the given project compared to his total productive output in equilibrium. Therefore, the best response of player i to the given allocations x_{ji} in terms of his allocation to in-house production, x^*_{ii}, must satisfy the condition:*

$$x^*_{ii} = \frac{\rho x_{ii}^{*\delta_i}}{\rho x_{ii}^{*\delta_i} + \sum_{j\neq i}^{n} x_{ij}^{*\delta_i} x_{ji}^{*\delta_j}} \Omega_i \tag{5}$$

The best response of player i in terms of his allocation to a collaborative project with j, x^*_{ij}, must satisfy the condition:

$$x^*_{ij} = \frac{x^{*\delta_i}_{ij} x^{\delta_j}_{ji}}{\rho x^{*\delta_i}_{ii} + \sum_{j\neq i}^{n} x^{*\delta_i}_{ij} x^{\delta_j}_{ji}} \Omega_i \tag{6}$$

Proof. Proposition 1 presents the best response functions in the general n-person productive exchange game. The proof is the solution to the optimization problem of the payoff function in Eq. (1):

$$\max_{x_{ii}} \quad u_i(\delta_i, \delta_j, x_i, x_{N_i}(g)) = \rho x^{\delta_i}_{ii} + \sum_{j\neq i}^{n} x^{\delta_i}_{ij} x^{\delta_j}_{ji} \tag{7}$$

$$\text{s.t.} \;\; x_{ii} + \sum_{j\neq i}^{n} x_{ij} \leq \Omega_i$$

The First Order Conditions (FOCs; Eq. 8 and Eq. 9) and the complementary slackness condition (C.S.C; Eq. 10) imply:

$$\frac{\partial L}{\partial x_{ii}} = \rho \delta_i x^{(\delta_i - 1)}_{ii} - \lambda = 0,$$

$$\rho \delta_i x^{\delta_i}_{ii} = \lambda x_{ii} \tag{8}$$

$$\frac{\partial L}{\partial x_{ij}} = \delta_i x^{(\delta_i - 1)}_{ij} x^{\delta_j}_{ji} - \lambda = 0,$$

$$\delta_i x^{\delta_i}_{ij} x^{\delta_j}_{ji} = \lambda x_{ij} \tag{9}$$

$$\lambda(x_{ii} + \sum_{j\neq i}^{n} x_{ij} - \Omega_i) = 0 \tag{10}$$

where L is the *Lagrange function* and $\lambda \geq 0$ is the *Lagrange multiplier*. From Eq. (8) and Eq. (9) it follows that $\lambda = 0$ implies $x_{ii} = 0$ and $x_{ij}x_{ji} = 0$ for all pairs i and j, yielding a total utility equal to zero. Since any player i can produce a strictly positive utility by working alone, this is never a best reply. So, we must have $\lambda > 0$ and according to Eq. (10) the constraint must be binding: $x_{ii} + \sum_{j=1}^{n_i} x_{ij} = \Omega_i$. Summing Eq. (9) in j:

$$\delta_i \sum_{j=1}^{n} x^{\delta_i}_{ij} x^{\delta_j}_{ji} = (\Omega - x_{ii})\lambda \tag{11}$$

Adding Eq. (8) and Eq. (11):

$$\delta_i(\rho x_{ii}^{\delta_i} + \sum_{j=1}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j}) = \lambda \Omega_i \tag{12}$$

Dividing Eq. (8) by Eq. (12), we obtain the best response of player i to the allocations of the other players, in terms of his allocation to an individual project, x_{ii}^*:

$$x_{ii}^* = \frac{\rho x_{ii}^{\delta_i}}{\rho x_{ii}^{\delta_i} + \sum_{j \neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j}} \Omega_i \tag{13}$$

Dividing Eq. (9) by Eq. (12), we obtain the best response of player i on his allocation to a combined project with j, x_{ij}^*:

$$x_{ij}^* = \frac{x_{ij}^{\delta_i} x_{ji}^{\delta_j}}{\rho x_{ii}^{\delta_i} + \sum_{j \neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j}} \Omega_i \tag{14}$$

□

The best response functions in Proposition 1 show that in the optimum the proportion of resources a player i invests in a collaborative project (or to in-house production) equals the proportional productivity of the given project compared to his total productive output. In other words, the greater the output of a productive project, the more resources i allocates to such project. This is a specification of the intensity of the links formed in the weighted networks, through the shares of resources players devote to each collaborative project.

The main takeaway from the equilibrium outcomes is that there are mixed effects of joint collaboration strategies with substantial differences between types of players. Players with DMR types perceive a positive impact by following diversification strategies, while players with IMR types are better off by specialization and focus on limited (i.e., a single) collaborative projects. The bottom line is that DMR players create multiple collaborations, in addition to in-house production, while IMR players create but a single project, either in-house or joint collaboration. As discussed before, our model provides consistent results to what has been observed in the literature on industrial organizations, where strategies to establish multiple collaborations can be either detrimental or beneficial depending on a firm's size (e.g., its production function). Large firms benefit from diversification while smaller firms face diseconomies when pursuing multiple collaborations at the same time (see e.g., Belderbos et al. 2006). Our work further these results by looking at how collaboration networks emerge given these strategies and different distributions of resources across types of players.

Since a Nash equilibrium is any combination of best responses, it is clear there will be very many different equilibria in any given network. An illustrative example is the empty network where each player allocates his entire endowment into in-house production. Such a network is a Nash equilibrium, given that unilateral deviations are not enough to establish collaborative projects. Moreover, we know, from Lemma 1, that it would be better for DMR players to use part of their endowment and form collaborative projects with others. Similarly, depending on their available resources, IMR players would also benefit by changing from in-house production to joint collaboration. Naturally, the *almost* empty network where a pair of players are involved in a collaborative project can be a Nash equilibrium, as well. But, as mentioned before, many of the unconnected players may be better off establishing different collaborations. Because of cases like these, in the following section we narrow down the set of network configurations that emerge in equilibrium by imposing a condition of stability to bilateral deviations. That is, by allowing those players who would be better off not staying isolated, for example, to jointly change their resources. This will conclude our analysis.

4.3 Pairwise stable Nash equilibria

Up until now, we have used Nash equilibrium as the solution concept. However, in social and economic settings such as the collaboration networks studied here, players can be expected to bilaterally form relationships that are mutually beneficial. To realign models of strategic network formation with this bilateral considerations Jackson and Wolinsky (1996) proposed pairwise stability as an alternative capturing mutual consent (see also Jackson and Watts 2001, 2002; Emerson 1972), where a network is said to be a pairwise stable Nash equilibrium (PNE) if it is Nash and pairwise stable.

Note that PNE has been widely used as a stability notion when links are either present or not. However, when studying weighted networks such as the collaboration networks we look at, players decide how much of their resources to devote to various collaborations, so that it is not only a matter of *whether* a connection exists, but also what its *intensity* (i.e., weight) is. Thus, we adapt the notion of pairwise stability as presented in Definition 3 below.

Definition 3. ***PNE in weighted collaboration networks:*** *A network is PNE if no player i would strictly benefit by any reallocation of his resources in vector x_i, and no pair of players i and j would both strictly benefit by a reallocation in x_i and x_j.*

In Proposition 2 we present the main result of the paper, which summarizes the entire analysis into specific network structures that conform the PNE set. Before presenting Proposition 2, we describe some network structures that facilitate its illustration. The networks described below can be grouped into the more general notion of *nested split graphs* (see e.g., Belhaj et al. 2016; Konig et al. 2014), which we adapt to our model of collaboration networks with heterogeneous players.

Specifically, while in nested-split graphs players are differentiated according to their degree, we differentiate players according to their type (i.e., production functions), which is one of the main variable of heterogeneity in our model. The first network of interest is the so-called DOMINANT-GROUP architecture (see Goyal and Joshi 2003), presented in the definition below:

Definition 4. ***Dominant-group architecture:*** *A network is a dominant-group architecture if players of one type form a main component while players of the other type are isolated from the main component.*

The second network of interest is the CORE-PERIPHERY architecture (see Galeotti and Goyal 2010). This is a generalization of the star network with various central players in the core. Specifically:

Definition 5. ***Core-periphery architecture:*** *A network is a core-periphery architecture if players of one type form a main core while players of the other type are linked to someone in the core.*

Now that the most relevant architectures have been introduced, we present the main result of our paper: PNE networks. Proposition 2 characterizes the PNE configurations in our model, taking into account the distribution of types of players in the population and the endowments assigned to them.

Proposition 2. ***Pairwise stable Nash equilibria:*** *The set of $PNE(\Gamma)$ is a subset of $NE(\Gamma)$, composed predominantly by two classes of networks: (i) if resources are such that* DMR *players can only make a low impact and* IMR *players can make medium or high impact,* DOMINANT-GROUP *architectures emerge where* DMR *players form the main component and* IMR *players stay isolated from the* DMR *players, or (ii) if resources are such that* DMR *players can make medium or high impact and* IMR *players can only make a low impact,* CORE-PERIPHERY *architectures emerge where* DMR *players form the core and* IMR *players are linked to them as peripherals.*

Proof. We present the proof for each class of PNE networks described in Proposition 2. If a network is PNE it is also a Nash equilibrium. Thus, it is straightforward that the set of $PNE(\Gamma)$ is a subset of $NE(\Gamma)$. Now we discuss the specific patterns of collaborative projects that emerge.

1. DOMINANT GROUP architectures: For this networks we show there are no links between types (point 1.1.), there is a main component formed by DMR players (point 1.2.), and IMR players stay mostly isolated (point 1.3.).

1.1. *No links between types*: Consider a player i of type $\delta_i > 1$ (IMR) and a player j of type $\delta_j < 1$ (DMR). Given that $\Omega_j^{\delta_j} < \rho$, we know from Lemma 1 that the impact each player can have on a collaborative project leads to no links between IMR and DMR players. This is pairwise stable because player i strictly prefers staying isolated and investing only into in-house production than creating a collaboration link with j, since $u_i(x_{ii} = \Omega_i) = \Omega_i^{\delta_i}\rho > \Omega_i^{\delta_i}\Omega_j^{\delta_j} = u_i(x_{ii} = 0)$. The same holds for every player i with type IMR in relation to any player j with type DMR.

1.2. *Links between* DMR *types*: Denote by $D = \{i \in N : \delta_i < 1\}$ the subset of DMR players in the population. From Lemma 1 we know that for players in D the empty network where each player only allocates resources into in-house production is not PNE, because any two players i and j in D could strictly increase their utility by forming a collaborative project. Now assume network g is a Nash network where some collaborative project between DMR players are formed. From the proof of Proposition 1, in particular from Eq. (14) we know that player i would increase an allocation to a new or existing project with a partner j by taking out resources from (at least) another project with a partner k. Given $x_{ij} > 0$ and $x_{ik} > 0$, we get $\frac{x_{ij}}{x_{ik}} = \frac{\rho x_{ij}^{\delta_i} x_{ji}^{\delta_j}}{x_{ik}^{\delta_i} x_{ki}^{\delta_k}}$. Then, $\frac{\partial x_{ik}}{\partial x_{ji}} \leq 0$, and $\frac{\partial x_{ij}}{\partial x_{ji}} \geq 0 \ \ \forall\, i \in N$ and $j, k \in N_i(g) : j \neq k, j \neq i,$ and $i \neq k$, which indicates that i would establish new collaborations up to the point where the marginal gains from it are equal to the marginal losses of reallocating resources from other projects. Moreover, we know from Eq. (4) that these re-allocation are always fractions of the endowment, for it is never a best response for i to use his entire endowment in a single project. Thus, the more resources the more collaborations DMR players can have, up to the point where they form a complete component.

1.3. *Links between* IMR *types*: Denote by $I = \{i \in N : \delta_i < 1\}$ the subset of IMR players in the population, with cardinality k. If the impact each player can make is low, $\Omega_i^{\delta_i} < \rho$, players respond by staying alone as shown in point 1.1. However, if the impact IMR players can make is medium or high, $\Omega_i^{\delta_i} \geq \rho$, rank and label all IMR players from 1 to k, such that $\Omega_1^{\delta_1} \geq \Omega_2^{\delta_2} \geq \Omega_3^{\delta_3} \geq \ldots \geq \Omega_{k-1}^{\delta_{k-1}} \geq \Omega_k^{\delta_k}$. Let pairs of players $\{1, 2\}, \{3, 4\}, \{5, 6\}$, etc., form collaborative projects in network g, where each invests his entire endowment. If k is uneven, player k is left without a partner. By Lemma 1 this is a Nash configuration. To see that it is pairwise stable first observe that Nash equilibrium guarantees that no player will individually want to reallocate resources. Second, consider non-existing links between players. Consider players i, j, l, m such that $\Omega_i^{\delta_i} \geq \Omega_j^{\delta_j} \geq \Omega_l^{\delta_l} \geq \Omega_m^{\delta_m}$, and $i, j \in g$ and $l, m \in g$. Suppose i proposes a link to player l, by allocating $x_{il}^{\delta_i} > \Omega_m^{\delta_m}$, then player l

is better off reciprocating i and allocating $x_{li}^{\delta_l} = \Omega_l^{\delta_l}$. However, following the construction of network g, $\Omega_j^{\delta_j} \geq \Omega_l^{\delta_l}$, which does not make i better off allocating any resources to player l. Notice this is also true if player l is the k^{th} player and is working alone, because $\Omega_j^{\delta_j} > \rho$. Moreover, it is also true when considering a player n such that $\Omega_n^{\delta_n} \leq \rho$. Thus, network g is PNE.[8]

Note that the patterns of interactions of IMR and DMR players would be the same, even if the population was homogenous such that all players were either in set D or set I.

2. CORE-PERIPHERY architectures: For this networks we show that a main core is formed by DMR players (point 2.1.), there are no links between IMR players (point 2.2.), and IMR players only connect to DMR players (point 2.3.).

   2.1. *Links between* DMR *types*: Given DMR players have endowments that allow them to make a high impact into the projects they are involved in, players with DMR types collaborate with both IMR and DMR partners. Thus, forming a core where DMR players are connected between them. The proof follows the arguments from point 1.2.

   2.2. *Links between* IMR *types*: There are no links between IMR players given each can only make a low impact on their collaborative projects. The proof follows from point 1.1.

   2.3. *Links between types*: Given players with type DMR have enough resources to make a high impact in the collaborative projects and players type IMR only have resources enough to make a low impact, IMR players will only form collaborative projects with DMR partners. The way this links are formed follows the same matching process presented in point 1.3.

□

In terms of efficiency in PNE networks, we know that bilateral deviations allow IMR players to pair in such a way that the most productive partners are matched, resulting in the highest output possible. This is evident in the ranking and matching of players by their impact, as described in point 1.3. in the proof. However, with respect to DMR players, the the pairwise stable outcomes are not always efficient, given there can be identical networks in terms of link presence but varying with respect to link intensity.

Finally, the intuition from Proposition 2 can be illustrated going back to our example of how firms can decide on the R&D collaborations. The first case, DOMINAT GROUP networks, would mean

[8]Note that since some players in I might have identical levels of production functions, g is not a unique *network*, but a unique *configuration*. In other words, if two players have identical production functions they are interchangeable, leading to two equivalent PNE networks.

that if start-up firms have enough resources, they would rather avoid firms with mature technologies and instead dedicate to in-house production or specific collaborations with other start-up's. The second case, CORE-PERIPHERY networks, would mean that if firms with mature technology have high levels of resources, they would be able to attract and maintain relationships with start-ups. Moreover, the start-up firms would put all their efforts in their collaborations with the mature firms. However, given mature firms are better off diversifying, they would also invest in collaborations with other mature firms as well as with other start-ups.

5 Conclusion

We have examined how the problem of establishing collaboration projects in a network is impacted by the interplay between resource heterogeneity and heterogeneity in production functions. Our main findings indicates that different network structures emerge depending on whether mature firms (those with decreasing marginal returns to own effort) have abundant or limited resources. In the latter, they become unattractive partners to firms with the capacity to innovate (those with increasing marginal returns to own effort), which results in DOMINANT GROUP networks where actors are segregated by the type of production functions they have. However, if mature firms have large amounts of resources, they are able to make a high impact on the collaborative projects they establish, and thus, are able to attract different innovative firms. This is portrayed by a CORE-PERIPHERY architecture.

We conclude by pointing out opportunities for further research. Empirical tests of our model constitute an important next step to advance our insights into the impact of heterogeneity in production functions on emergent network structures. Laboratory experiments offer powerful techniques to do so (see Choi et al. 2016; Kosfeld 2014). Particularly, by studying how experimental subjects interact, we can discover in more depth how certain network structures are more likely to emerge than others, while controlling the distribution of players with respect to their production functions and resources.

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