Ultra Unification and Categorical Symmetry of the Standard Model from Gravitational Anomaly

Juven Wang

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Previous work: 1910.14668 [JHEP], 2012.15860 [PRD], 2106.16248 [PRD], 2112.14765 [PRD], 2204.08393 [PRD],

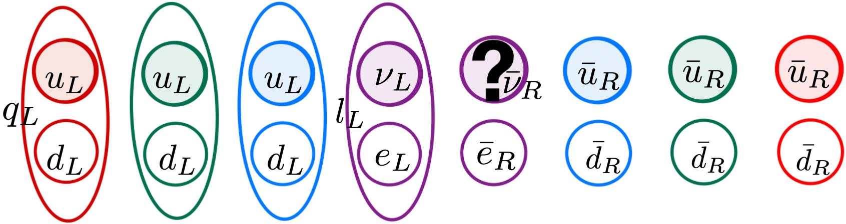
w/ **Pavel Putrov** (ICTP),

Zheyan Wan (YMSC), **Yi-Zhuang You** (UCSD).

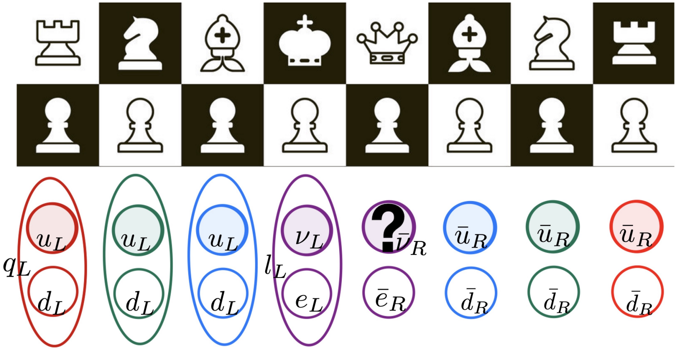
NYU Abu Dhabi, QI and QMatter, Fri, May 26, 2023

What is the missing piece on the Chessboard?

“left-handed” King $\Leftrightarrow$ “right-handed” King?



Where is the missing
“sterile right-handed neutrino $\bar{\nu}_R$”?



Queen replaces the "sterile right-handed" King.

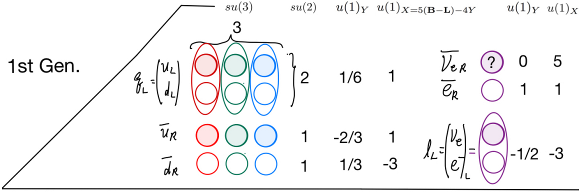
What can replace the missing "sterile right-handed" (SU(2) singlet) neutrino $\bar{\nu}_R$?

Ultra Unification (2020): Nonperturbative discrete global anomaly cancellation permits to replace some of 0d particle $\bar{\nu}_R$ in 4d QFT to

a fermionic
4d or 5d Topological Quantum Field Theory (TQFT)
or 4d Conformal Field Theory (CFT) sector.

Introduction

4d Standard Model (SM) with $(15{+}1)N_f$ Weyl fermions coupled to Yang-Mills gauge $su(3)_c \times su(2)_{\rm L} \times u(1)_{\tilde Y}$ in representation (rep):



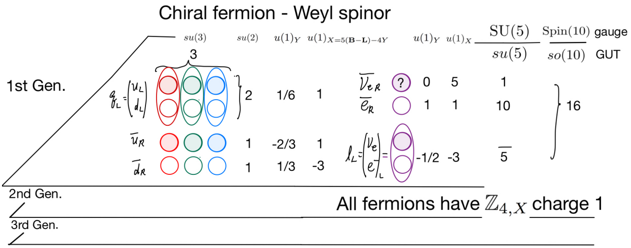
$\times(N_f = 3)$ families.

Each **spin-1/2** Weyl fermion $\mathbf{2}_L$ of Spin(3,1)

$\bar d_R \oplus l_L \oplus q_L \oplus \bar u_R \oplus \bar e_R \oplus ?\bar\nu_R$
$= (\overline{\mathbf{3}}, \mathbf{1})_{2,L} \oplus (\mathbf{1}, \mathbf{2})_{-3,L} \oplus (\mathbf{3}, \mathbf{2})_{1,L} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-4,L} \oplus (\mathbf{1}, \mathbf{1})_{6,L} \oplus ?(\mathbf{1}, \mathbf{1})_{0,L}.$

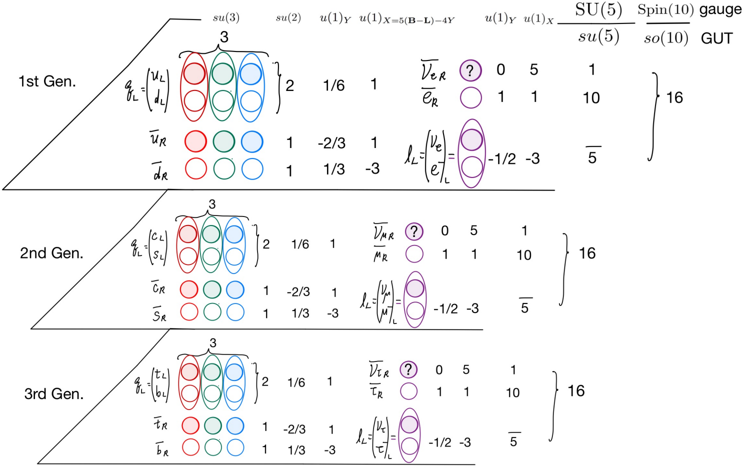
- There are also 8 of $su(3)_c$, 3 of $su(2)_{\rm L}$, and 1 of $u(1)$ **spin-1** gauge bosons, and a **spin-0** Higgs boson ϕ_H.
- Top t quark found in 1995. Higgs found in 2012.
- What's next? Where is "sterile right-handed" neutrino $\bar\nu_R$?

Standard Model and GUT anomaly cancellation



SM fermion spinor	SU(3)	SU(2)	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{\tilde{Y}}$	$\mathrm{U}(1)_{\mathrm{EM}}$	$\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$	$\mathrm{U}(1)_{\mathbf{Q}-N_c\mathbf{L}}$	$\mathbb{Z}_{2N_cN_f,\mathbf{Q}+N_c\mathbf{L}}$ as $\mathrm{U}(1)_{\mathbf{Q}+N_c\mathbf{L}}$ mod $2N_cN_f$	$\mathrm{U}(1)_X$	$\mathbb{Z}_{5,X}$	$\mathbb{Z}_{4,X}$ or $\mathbb{Z}_2^F$
$\bar{d}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	2	1/3	−1/3	−1	−1	−3	−3	1
l_L	$\mathbf{1}$	$\mathbf{2}$	−1/2	−3	0 or −1	−1	−3	+3	−3	−3	1
q_L	$\mathbf{3}$	$\mathbf{2}$	1/6	1	2/3 or −1/3	1/3	1	1	1	1	1
$\bar{u}_R$	$\bar{\mathbf{3}}$	$\mathbf{1}$	−2/3	−4	−2/3	−1/3	−1	−1	1	1	1
$\bar{e}_R = e_L^+$	$\mathbf{1}$	$\mathbf{1}$	1	6	1	1	3	−3	1	1	1
$\bar{\nu}_R = \nu_L$	$\mathbf{1}$	$\mathbf{1}$	0	0	0	1	3	−3	5	0	1

Check: discrete **Baryon** minus **Lepton** ($\mathbf{B}-\mathbf{L}$, precisely $\mathbf{Q}-N_c\mathbf{L}$) and $X \equiv 5(\mathbf{B}-\mathbf{L}) - 4Y \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}\tilde{Y}$. symmetries, local vs global anomalies, index theorem and cobordism.

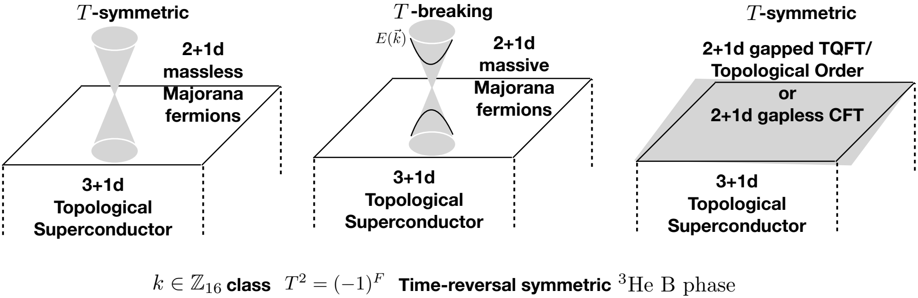


An important **Anomaly Index**: The family/generation number N_f and the total "sterile right-handed" neutrino number n_{ν_R} difference:

$$-N_f + n_{\nu_R} \equiv -N_f + \sum_j n_{\nu_{j,R}} = -3 + n_{\nu_{e,R}} + n_{\nu_{\mu,R}} + n_{\nu_{\tau,R}} + \dots$$

Anomaly Cancellation

2+1d boundaries of 3+1d topological superconductor (TSC) protected by an anti-unitary time-reversal symmetry T. (Bulk-Boundary correspondence.)



Fermion dispersion $E(\vec{k})$ in momentum space $\vec{k}$.

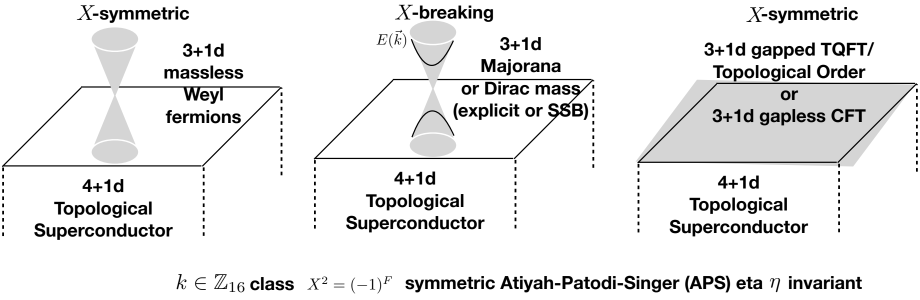
$\mathbb{Z}_{16}$ class TSC: 4d Atiyah-Patodi-Singer eta invariant $\eta_{4\mathrm{d}}$.

Anomaly Cancellation via Topological Green-Schwartz mechanism.

Condensed matter review: Senthil 1405.4015.

Anomaly Cancellation

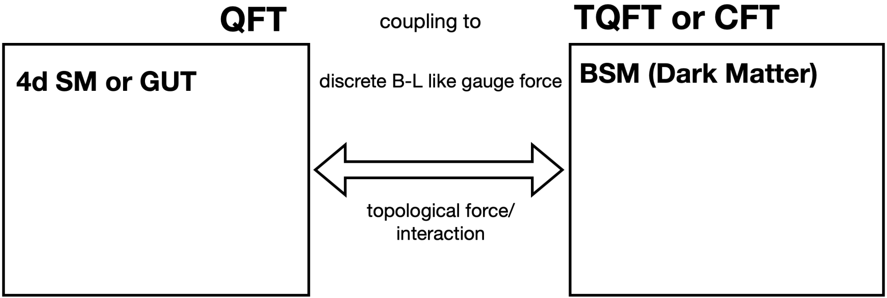
3+1d boundaries of 4+1d "topological superconductor (TSC)" protected by a unitary discrete $\mathbb{Z}_{4,X}$ symmetry: **Baryon** minus **Lepton** and electroweak $\mathrm{U}(1)_{\tilde{Y}}$-hypercharge $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}\tilde{Y}$.



$\mathbb{Z}_{16}$ class 5d eta invariant $\eta_{4\mathrm{d}}(\mathrm{PD}(A_{\frac{\mathbb{Z}_{4,X}}{\mathbb{Z}_2^F}}))$. Decorate 3+1d TSC on X-symmetry breaking domain wall in 4+1d, and condense the domain wall configuration to restore X symmetry.

Wilczek-Zee '79, Garcia-Etxebarria-Montero, Hsieh '18, Wan-JW '19, JW '20.

Ultra Unification (2020)



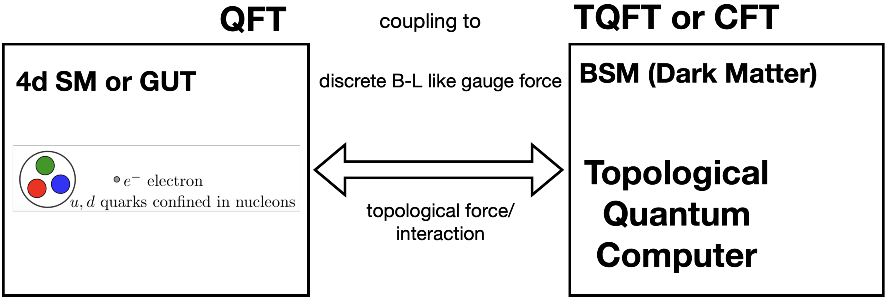
$\mathbb{Z}_{16}$ nonperturbative global gauge-gravitational anomaly cancellation

A unitary discrete $\mathbb{Z}_{4,X}$ gauge force: **Baryon** minus **Lepton** and electroweak $\mathrm{U}(1)_{\tilde{Y}}$-hypercharge $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}\tilde{Y}$.

QFT+TQFT/CFT on a curved spacetime under gravity.

(Lieb-Schultz-Mattis thm analogy: topological order in cond matt pseudogap)

Ultra Unification (2020)



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QFT+TQFT/CFT on a curved spacetime under gravity.

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Ultra Unification (2020)

QFT — coupling to — **TQFT or CFT**

4d SM or GUT

$15 \times 3 = -3 \mod 16$

Yang-Mills Lie group gauge theory,
Anderson-Higgs **symmetry breaking**,
Weyl fermions with Dirac or Majorana masses.
Characteristic class and fiber bundle.

- **4d sterile fermion (0d $\bar{\nu}_R$)**

  0d ψ

discrete B-L like gauge force

$\mathcal{A}_{\frac{\mathbb{Z}_{4,X}}{\mathbb{Z}_2^F}}$

topological force/ interaction

fractional / category statistical interaction

BSM (Dark Matter)

- **4d noninvertible TQFT**

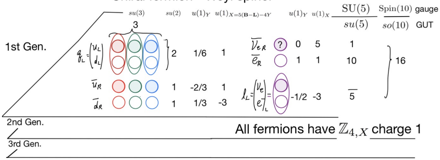
  5d invertible TFT

Cohomology, Cobordism, Category,
symmetry extension,
symmetric/topological mass generation.
(SMG/TMG)

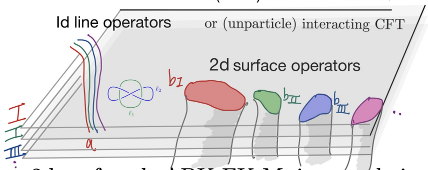
- **4d interacting CFT**

  1d line a, 2d surface b,
  ABK FK Majorana chain

$\mathbb{Z}_{16}$ nonperturbative global gauge-gravitational anomaly cancellation



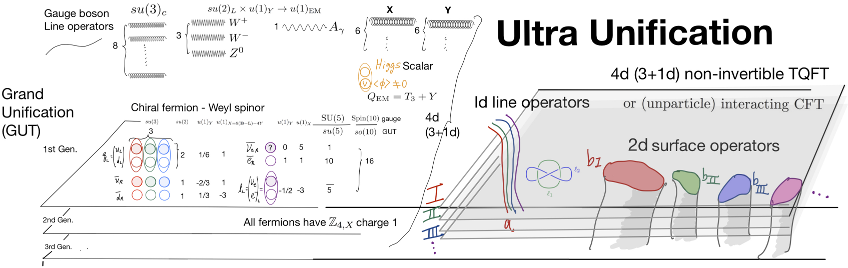
4d (3+1d) non-invertible TQFT



1d line a, 2d surface b, ABK FK Majorana chain

Ultra Unification (2020) 4d and 5d coupled quantum system

A sterile neutrino (massless/massive) carries a $\mathbb{Z}_{16}$ class mixed gauge-gravitational global anomaly index, which could be replaced by interacting 4d or 5d gapped topological quantum field theory, or 4d gapless conformal field theory.



Chiral fermion and chiral gauge sector $(-(N_{\rm gen} = 3) + n_{\nu_{e,R}} + n_{\nu_{\mu,R}} + n_{\nu_{\tau,R}} + \nu_{\rm 4d} - \nu_{\rm 5d}) = 0 \mod 16$

5d (4+1d) invertible TQFT (Cobordism invariant, SPT phase, topological superconductor.)

$\exp(\frac{2\pi \mathrm{i}}{16} \nu_{\rm 5d} \int_{M^5} \eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2})))$ with APS eta invariant

$\mathcal{A}_{\mathbb{Z}_4} \equiv \mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathbb{Z}_{4,X}$-background gauge field
$\mathcal{A}_{\mathbb{Z}_2} \equiv$ background gauge field in $\mathrm{H}^1(M, \frac{\mathbb{Z}_{4,X}}{\mathbb{Z}_2^F})$

$\nu_{\rm 5d} \in \mathbb{Z}_{16}$ from $\Omega_5^{\mathrm{Spin} \times_{\mathbb{Z}_2^F} \mathbb{Z}_{4,X}}$

Mirror fermion and chiral gauge sector, or TQFT

Outline

1. 4d Standard Model (SM) and 5d invertible topological field theory (iTFT):

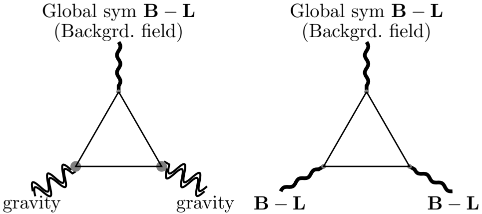
- $\mathbb{Z}^2$-class perturb local and $\mathbb{Z}_{16}$-class nonperturb global anomalies.
- Index theorem (Characteristic class) vs Cobordism.
- **Ultra Unification**: Anomaly cancellation/Cobordism constraint.
- Beyond SM and GUT + 4d TQFT/CFT + 5d iTFT sector.

2. Noninvertible Categorical Symmetry of the SM

- 4d QED example: $\mathrm{U}(1)_A$-$\mathrm{U}(1)_V^2$ anomaly, of $\mathbb{Z}$ class.
- 4d SM: $\mathrm{U}(1)$-gravity^2 and $\mathrm{U}(1)^3$ anomaly, of $\mathbb{Z}^2$ classes.
- 4d SM: $\mathbb{Z}_4$-gravity anomaly, of $\mathbb{Z}_{16}$ class.

$(\mathbf{B}-\mathbf{L})$-(gravity)^2 and $(\mathbf{B}-\mathbf{L})^3$ as $\mathbb{Z}$-class ABJ anomaly or 't Hooft anomaly in SM

$\bar{d}_R \oplus l_L \oplus q_L \oplus \bar{u}_R \oplus \bar{e}_R \oplus n_{\nu_R} \bar{\nu}_R = (\overline{\mathbf{3}}, \mathbf{1})_{2,L} \oplus (\mathbf{1}, \mathbf{2})_{-3,L} \oplus (\mathbf{3}, \mathbf{2})_{1,L} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-4,L} \oplus (\mathbf{1}, \mathbf{1})_{6,L} \oplus n_{\nu_R} (\mathbf{1}, \mathbf{1})_{0,L}.$



$$
\begin{aligned}
(\mathbf{B}-\mathbf{L})\text{-(gravity)}^2 \Rightarrow j_{\mathbf{B}} &: \quad 0. \\
j_{\mathbf{L}} &: \quad -N_f + n_{\nu_R}. \\
(\mathbf{B}-\mathbf{L})^3 \Rightarrow j_{\mathbf{B}} &: \quad 0. \\
j_{\mathbf{L}} &: \quad -N_f + n_{\nu_R}.
\end{aligned}
$$

The $\mathrm{d} \star j_{\mathbf{B}} = 0$ but $\mathrm{d} \star j_{\mathbf{L}} \neq 0$ $\mathrm{d} \star (j_{\mathbf{B}} - j_{\mathbf{L}}) \neq 0$ only when $N_f \neq n_{\nu_R}$.

- Leptogenesis: Gravitation instanton generates unbalanced leptons.

Just 't Hooft anomaly? Require the 16th Weyl fermion ν_R? or break $(\mathbf{B}-\mathbf{L})$, or?

- We will propose new scenarios: Ultra Unification.

Anomaly polynomial of Weyl fermions: Atiyah-Singer index theorem. Anomaly of a single Weyl fermion in 4d is the degree 6 part of $\hat{\mathrm{A}}\,\mathrm{ch}(\mathcal{E})$:

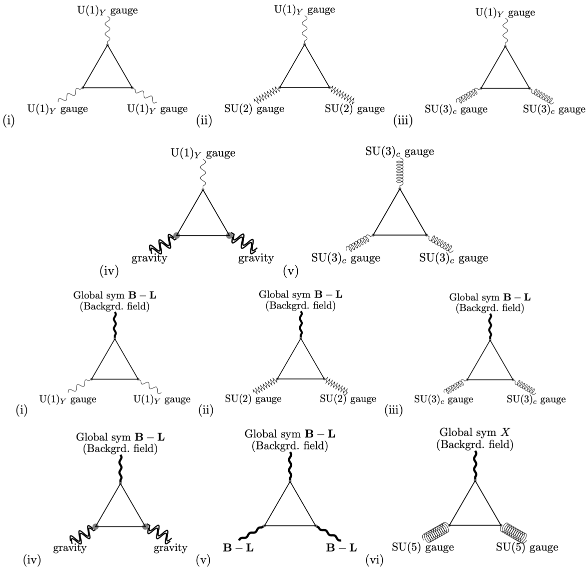
$$\begin{aligned} \hat{\mathrm{A}} &= 1 - \frac{p_1}{24} + \frac{7p_1^2 - 4p_2}{5760} + \dots, \\ \mathrm{ch}(\mathcal{E}) &= \mathrm{rk}\,\mathcal{E} + c_1(\mathcal{E}) + \frac{\left(c_1^2(\mathcal{E}) - 2c_2(\mathcal{E})\right)}{2} + \frac{\left((c_1^3(\mathcal{E}) - 3c_1(\mathcal{E})c_2(\mathcal{E}) + 3c_3(\mathcal{E})\right)}{6} + \end{aligned}$$

- $\hat{\mathrm{A}}$: A-roof genus of spacetime tangent bundle TM. p_j: jth Pontryagin class.
- ch: the total Chern character. c_j: jth Chern class.
- $\mathcal{E}$ is the complex vector bundle associated w/ fermion rep.
- $p_1 = -\frac{1}{8\pi^2}\mathrm{Tr}[R \wedge R]$. $c_1 = \frac{\mathrm{Tr}F}{2\pi}$. $c_2 = \frac{1}{8\pi^2}(-\mathrm{Tr}(F \wedge F) + (\mathrm{Tr}F) \wedge (\mathrm{Tr}F))$.

For the 4d SM, the explicit $\exp(\mathrm{i}\theta \int_{M^6} I_6)$ in terms of Pontryagin p_j and Chern c_j characteristic classes can be obtained using the expansions of $\hat{\mathrm{A}}$ and $\mathrm{ch}(E)$.

$$\begin{aligned} I_6 \equiv (N_c c_1(\mathrm{U}(1)_{\mathbf{Q}}) + c_1(\mathrm{U}(1)_{\mathbf{L}}))\, N_f \left(18\, \frac{c_1(\mathrm{U}(1)_{\tilde{Y}})^2}{2} + c_2(\mathrm{SU}(2))\right) \\ + (-N_f + n_{\nu_R}) \left(\frac{c_1(\mathrm{U}(1)_{\mathbf{L}})^3}{6} - \frac{c_1(\mathrm{U}(1)_{\mathbf{L}}) p_1(TM)}{24}\right) \end{aligned}$$

Feynman diagram interpretations of anomaly polynomial



$\mathrm{U}(1)_{\mathbf{Q}\pm N_c\mathbf{L}}$-symmetry violation

$\mathbf{B}\pm\mathbf{L}$ (precisely $\mathbf{Q}\pm N_c\mathbf{L}$) current j nonconservation:

$$
\begin{aligned}
\mathrm{d}\star j_{\mathbf{Q}} &= -N_c N_f\big(18\tfrac{c_1(\mathrm{U}(1)_{\tilde{Y}})^2}{2} + c_2(\mathrm{SU}(2))\big).\\
\mathrm{d}\star j_{\mathbf{L}} &= -N_f\big(18\tfrac{c_1(\mathrm{U}(1)_{\tilde{Y}})^2}{2} + c_2(\mathrm{SU}(2))\big) - (-N_f + n_{\nu_R})\big(\tfrac{c_1(\mathrm{U}(1)_{\mathbf{L}})^2}{6} - \tfrac{p_1(TM)}{24}\big).\\
\mathrm{d}\star j_{\mathbf{Q}+N_c\mathbf{L}} &= -2N_c N_f\big(18\tfrac{c_1(\mathrm{U}(1)_{\tilde{Y}})^2}{2} + c_2(\mathrm{SU}(2))\big) - (-N_f + n_{\nu_R})\big(N_c^3\tfrac{c_1(\mathrm{U}(1)_{\mathbf{L}})^2}{6} - N_c\tfrac{p_1(TM)}{24}\big)
\end{aligned}
$$

$$
\boxed{\mathrm{d}\star j_{\mathbf{Q}-N_c\mathbf{L}} = (-N_f + n_{\nu_R})\big(N_c^3\frac{c_1(\mathrm{U}(1)_{\mathbf{Q}-N_c\mathbf{L}})^2}{6} - N_c\frac{p_1(TM)}{24}\big).}
$$

4d anomaly written as 5d invertible TQFT (iTFT) partition function $\mathbf{Z}_5^{\mathrm{U}(1)} = \exp(\mathrm{i}\, S_5)$:

$$
S_5 \equiv \int_{M^5} (N_c A_{\mathbf{Q}} + A_{\mathbf{L}}) N_f \left(18\,\frac{c_1(\mathrm{U}(1)_{\tilde{Y}})^2}{2} + c_2(\mathrm{SU}(2))\right) + (-N_f + n_{\nu_R})\, A_{\mathbf{L}} \left(\frac{c_1(\mathrm{U}(1)_{\mathbf{L}})^2}{6} - \frac{p_1(TM)}{24}\right).
$$

$$
\boxed{S_5 \equiv (-N_f + n_{\nu_R}) \int_{M^5} A_{\mathbf{Q}-N_c\mathbf{L}} \left(N_c^3\frac{c_1(\mathrm{U}(1)_{\mathbf{Q}-N_c\mathbf{L}})^2}{6} - N_c\frac{p_1(TM)}{24}\right).}
$$

4d Anomaly (5d iTQFT) of 15,16N_f-fermion $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y}{\mathbb{Z}_q}$

$-N_f + n_{\nu_R} \equiv -N_f + \sum_j n_{\nu_{j,R}} = -3 + n_{\nu_{e,R}} + n_{\nu_{\mu,R}} + n_{\nu_{\tau,R}} + \dots$.

- $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B}-\mathbf{L} \text{ or } X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}$-class perturbative local anomaly $\mathbf{B}-\mathbf{L}$ or $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}Y$: $\mathrm{U}(1)^3$ and $\mathrm{U}(1)$-grav^2:

$$\mathbf{Z}_5^{\mathrm{U}(1)} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} A_{\mathbf{Q}-N_c\mathbf{L}} \left(N_c^3 \frac{c_1(\mathrm{U}(1)_{\mathbf{Q}-N_c\mathbf{L}})^2}{6} - N_c \frac{p_1(TM)}{24} \right)).$$

Index $(-N_f + n_{\nu_R}) \in \mathbb{Z}$.

- $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}$-symmetry. With $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}\tilde{Y}$.

$\mathbb{Z}_{16}$-class nonperturbative global anomaly of $\mathbb{Z}_{4,X}$-gravity:

$$\mathbf{Z}_5^{\mathbb{Z}_{4,X}} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} \frac{2\pi}{16} \eta_{4\mathrm{d}}(\mathrm{PD}(A_{\frac{\mathbb{Z}_{4,X}}{\mathbb{Z}_2^F}}))).$$

Index $(-N_f + n_{\nu_R}) \mod 16 \in \mathbb{Z}_{16}$ and $\eta_{4\mathrm{d}} \in \mathbb{Z}_{16}$.
4d bdry of 5d X-symmetric topological superconductor

Spacetime-internal symmetry $(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B-L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B+L}} \times G_{\mathrm{SM}_q})$.

Spacetime-internal symmetry $(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_f,\mathbf{B+L}} \times G_{\mathrm{SM}_q})$.

4d Anomaly (5d iTQFT) contained in the cobordism group ($N_f = 3$):

$$\begin{aligned}
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B-L}} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{\mathrm{SM}_q}) &= (\mathbb{Z}^{11}) \times (\mathbb{Z}_9 \times \mathbb{Z}_3^7).\\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{\mathrm{SM}_q}) &= \begin{cases} (\mathbb{Z}^5 \times \mathbb{Z}_2 \times \mathbb{Z}_4^2 \times \mathbb{Z}_{16}) \times (\mathbb{Z}_9 \times \mathbb{Z}_3^4), & q = 1,3. \\ (\mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}) \times (\mathbb{Z}_9 \times \mathbb{Z}_3^4), & q = 2,6. \end{cases}\\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B-L}} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{[1]}) &= (\mathbb{Z}^2 \times \mathbb{Z}_{6/q}) \times (\mathbb{Z}_9 \times (\mathbb{Z}_3)^2 \times (\mathbb{Z}_3)^{3n_3}).\\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathbb{Z}_{3,\mathbf{B+L}} \times G_{[1]}) &= (\mathbb{Z}_{16} \times (\mathbb{Z}_4)^{n_2} \times \mathbb{Z}_{6/q}) \times (\mathbb{Z}_9 \times (\mathbb{Z}_3)^{2n_3}).\\
\mathrm{TP}_5(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{Spin}(10)) &= \mathbb{Z}_2.
\end{aligned}$$

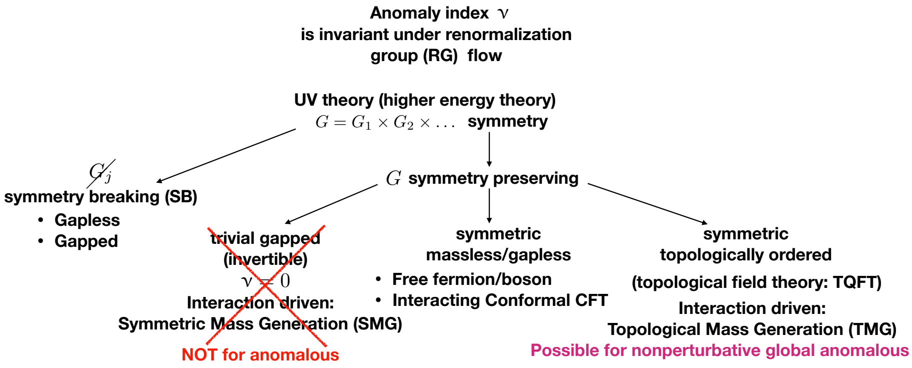
Freed-Hopkins cobordism TP_D contains bordism groups $\Omega_D^{\mathrm{torsion}}$ and $\Omega_{D+1}^{\mathrm{free}}$.

Wan-JW-You, arXiv:1910.14668, 2112.14765, 2204.08393

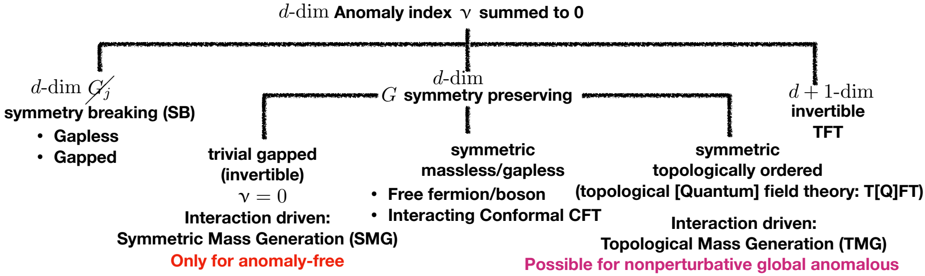
See also Garcia-Etxebarria-Montero 1808.00009, Davighi-Gripaios-Lohitsiri1910.11277

Anomaly Matching

('tHooft Anomaly Matching: Global G symmetry)



Anomaly Cancellation



Anomaly Cancellation

d-dim **Anomaly index** ν **summed to 0**

- d-dim $\not{G_j}$ **symmetry breaking (SB)**
  - **Gapless**
  - **Gapped**

  Dirac or Majorana mass

- d-dim G **symmetry preserving**
  - **trivial gapped (invertible)** $\nu = 0$

    Interaction driven: Symmetric Mass Generation (SMG)

    Only for anomaly-free
  - **symmetric massless/gapless**
    - **Free fermion/boson**

      Weyl fermions (15Nf +...)
    - **Interacting Conformal CFT**

      New 4d unparticle CFT physics
  - **symmetric topologically ordered (TO) (topological [Quantum] field theory: T[Q]FT)**

    Interaction driven: Topological Mass Generation (TMG)

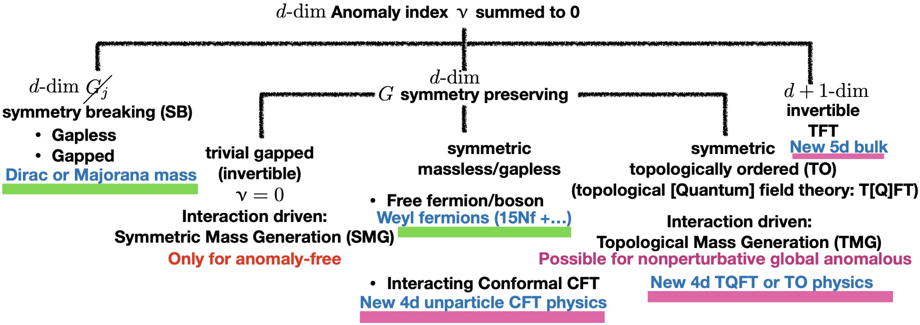
    Possible for nonperturbative global anomalous

    New 4d TQFT or TO physics

- $d+1$-dim **invertible TFT**

  New 5d bulk

Anomaly Cancellation



Logic to Ultra Unification

- $\mathbb{Z}_{16}$ global anomaly cancellation application.

Assumptions:

1. Standard Model (SM) G_{internal}: Lie algebra $su(3) \times su(2) \times u(1)$.
   $G_{\text{SM}_q} \equiv \frac{\text{SU}(3)\times\text{SU}(2)\times\text{U}(1)}{\mathbb{Z}_q}, \quad q = 1, 2, 3, 6.$
2. $15 \times (N_f = 3)$ Weyl fermions (spacetime Weyl spinors) observed, applicable to both SM and SU(5) GUT.
3. Discrete Baryon−Lepton number preserved (or not) at high energy: $\mathbb{Z}_{4,X\equiv 5(\mathbf{B}-\mathbf{L})-4Y} \supset \mathbb{Z}_2^{\text{F}}$, so $X^2 = (-1)^F$, also dynamically gauged at higher energy due to no global symmetry in quantum gravity (if we embed the theory into quantum gravity).

Check: Perturbative local & nonperturbative global anomalies via cobordism.

Logic to Ultra Unification

Consequences: $\mathbb{Z}_{16}$ anomaly index as total $(N_f = 3)\cdot(15 = -1 \mod 16)$.

$$(-(N_f = 3) + \sum_{j=e,\mu,\tau,\ldots} n_{\nu_{j,R}} + \nu_{\text{new hidden sectors}}) = 0 \mod 16.$$

Anomaly-cancellation?

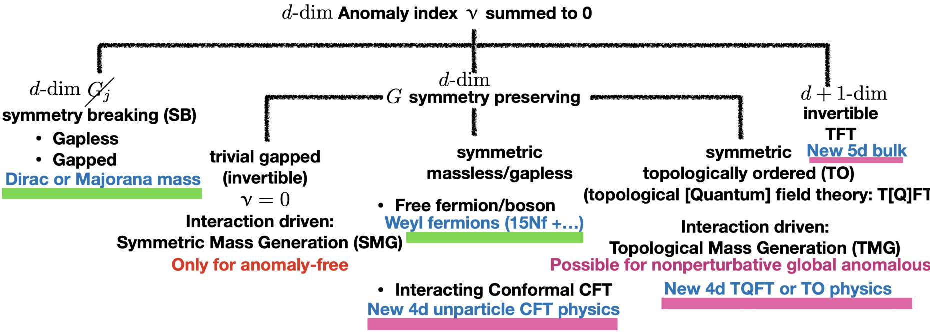
(1) **Standard Lore:** R-handed sterile neutrino (16th Weyl) $n_{\nu_{j,R}} = 1$.
$\mathbb{Z}_{4,X}$ preserved (gapless fermion) vs broken (gap) by **Dirac** or **Majorana** mass.

(2) **My proposal:** New hidden sectors beyond SM (∼ Lieb-Schultz-Mattis thm) :

1. $\mathbb{Z}_{4,X}$-symmetry-preserving anomalous gapped 4d TQFT (**Topological** Mass). (Topo.Green-Schwarz mechanism. Boundary topological order [2+1d Vishwanath-Senthil'12].)
2. $\mathbb{Z}_{4,X}$-5d invertible TQFT (SPTs) by cobordism invariant $\eta(\text{PD}(\mathcal{A}_{\frac{\mathbb{Z}_{4,X}}{\mathbb{Z}_2^F}}))$.
3. $\mathbb{Z}_{4,X}$-gauged-5d-noninvertible TQFT (SETs) + gravity.
4. $\mathbb{Z}_{4,X}$-symmetry-breaking gapped phase (e.g. Landau phase or 4d TQFT).
5. $\mathbb{Z}_{4,X}$-symmetry-preserving gapless or breaking gapless (e.g., extra CFT).

HEP-PH **Gapped Extended 1d/2d Objects beyond 0d Particle Physics**.
HEP-PH **Gapless Unparticle CFT Physics**.

Anomaly Cancellation



Math Physics Equations: **Ultra Unification** Path (Functional) Integral example

$$\mathbf{Z}_{\mathrm{UU}}[\mathcal{A}_{\mathbb{Z}_4}] \equiv \mathbf{Z}_{\substack{\text{5d-iTQFT}/\\ \text{4d-SM+TQFT}}}[\mathcal{A}_{\mathbb{Z}_4}] \equiv \mathbf{Z}^{(-\nu_{5\mathrm{d}})}_{\text{5d-iTQFT}}[\mathcal{A}_{\mathbb{Z}_4}] \cdot \mathbf{Z}^{(\nu_{4\mathrm{d}})}_{\text{4d-TQFT}}[\mathcal{A}_{\mathbb{Z}_4}] \cdot \mathbf{Z}^{(n_{\nu_{j,R}})}_{\mathrm{SM}}[\mathcal{A}_{\mathbb{Z}_4}].$$

$$\mathbf{Z}_{\mathrm{SM}}[\mathcal{A}_{\mathbb{Z}_4}] \equiv \int [\mathcal{D}\psi][\mathcal{D}\bar{\psi}][\mathcal{D}A][\mathcal{D}\phi]\ldots \exp(\mathrm{i}\ S_{\mathrm{SM}}[\psi,\bar{\psi},A,\phi,\ldots,\mathcal{A}_{\mathbb{Z}_4}]\big|_{M^4})$$

$$S_{\mathrm{SM}} = \int_{M^4}\Big(\mathrm{Tr}(F_I\wedge\star F_I) - \frac{\theta_I}{8\pi^2}g_I^2\mathrm{Tr}(F_I\wedge F_I)\Big) + \int_{M^4}\Big(\bar{\psi}(\mathrm{i}\,\not{D}_{A,\mathcal{A}_{\mathbb{Z}_4}})\psi$$

(Gauge) *symmetry breaking* $\quad +|D_{\mu,A,\mathcal{A}_{\mathbb{Z}_4}}\phi|^2 - \mathrm{U}(\phi) - (\psi_L^\dagger\phi(\mathrm{i}\sigma^2\psi_L'^*) + \mathrm{h.c.})\Big)\,\mathrm{d}^4x.$

$$(-(N_f=3) + (\sum_{j=e,\mu,\tau,\ldots} n_{\nu_{j,R}}) + \nu_{4\mathrm{d}} - \nu_{5\mathrm{d}}) = 0 \mod 16.$$

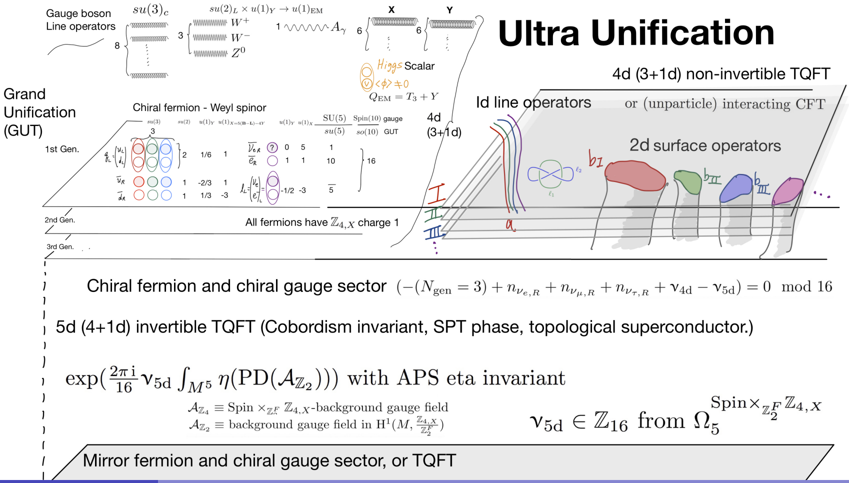
$$\mathbf{Z}^{(-\nu_{5\mathrm{d}}=-2)}_{\text{5d-iTQFT}}[\mathcal{A}_{\mathbb{Z}_4}] \cdot \mathbf{Z}^{(\nu_{4\mathrm{d}}=2)}_{\text{4d-TQFT}}[\mathcal{A}_{\mathbb{Z}_4}] = \sum_{c\in\partial'^{-1}(\partial[\mathrm{PD}(\mathcal{A}^3)])} \mathrm{e}^{\frac{2\pi\,\mathrm{i}}{8}\mathrm{ABK}(c\cup\mathrm{PD}(\mathcal{A}^3))}$$

$$\cdot \frac{1}{2^{|\pi_0(M^4)|}} \sum_{\substack{a\in C^1(M^4,\mathbb{Z}_2),\\ b\in C^2(M^4,\mathbb{Z}_2)}} (-1)^{\int_{M^4} a(\delta b+\mathcal{A}^3)} \cdot \mathrm{e}^{\frac{2\pi\,\mathrm{i}}{8}\mathrm{ABK}(c\cup\mathrm{PD}'(b))}.$$

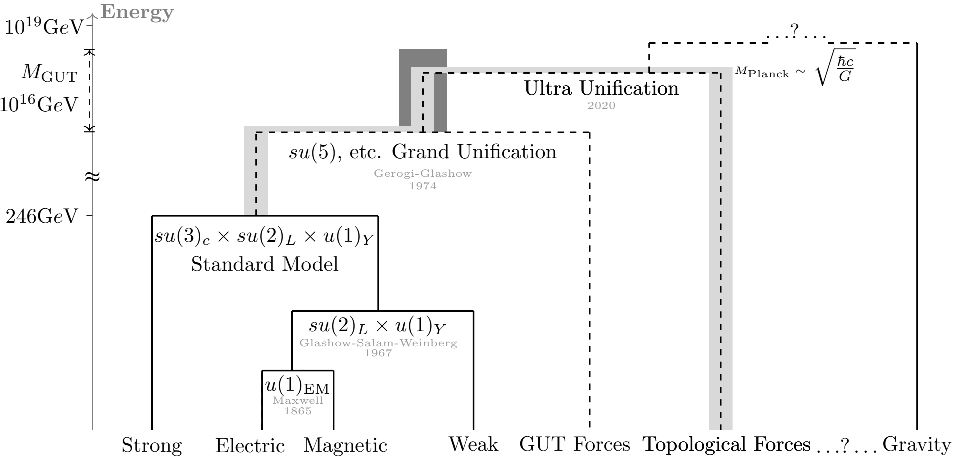
Symmetry extension trivialize anomaly (JW-Wen-Witten'17 1705.06728). Fermionic *non-abelian* TQFT.

Ultra Unification (2020) 4d and 5d coupled quantum system

A sterile neutrino (massless/massive) carries a $\mathbb{Z}_{16}$ class mixed gauge-gravitational global anomaly index, which could be replaced by interacting 4d or 5d gapped topological quantum field theory, or 4d gapless conformal field theory.



Fundamental Physics embodies Ultra Quantum Matter



HEP-phenomenology: beyond 0d particle physics (to gapped extended TQFT objects (topological order) or gapless unparticle CFT).
Quantum Matter in Math/Physics.

Outline

1. 4d Standard Model (SM) and 5d invertible topological field theory (iTFT):
- $\mathbb{Z}^2$-class perturb local and $\mathbb{Z}_{16}$-class nonperturb global anomalies.
- Index theorem (Characteristic class) vs Cobordism.
- **Ultra Unification**: Anomaly cancellation/Cobordism constraint.
- Beyond SM and GUT + 4d TQFT/CFT + 5d iTFT sector.

2. Noninvertible Categorical Symmetry of the SM
- 4d QED example: $\mathrm{U}(1)_A$-$\mathrm{U}(1)_V^2$ anomaly, of $\mathbb{Z}$ class.
- 4d SM: $\mathrm{U}(1)$-gravity^2 and $\mathrm{U}(1)^3$ anomaly, of $\mathbb{Z}^2$ classes.
- 4d SM: $\mathbb{Z}_4$-gravity anomaly, of $\mathbb{Z}_{16}$ class.

Modern view on "Symmetry": G-K-Seiberg-W 1412.5148

Symmetry generator = charge operator = topological defect U

Invertible symmetry (group): Symmetry group G implies that the fusion rules of the charge operators U (a.k.a. topological defects) is described by the corresponding group law.
$\alpha_1, \alpha_2 \in G$ and $\alpha_1 + \alpha_2 \in G$. (e.g., $G = \mathrm{U}(1)$)

$$U_{\alpha_1} \times U_{\alpha_2} = U_{\alpha_1+\alpha_2}, \qquad U_{\alpha_1} \times U_{-\alpha_1} = U_0 = 1$$

Categorical or **noninvertible symmetry** (fusion category):
Charge operators obey fusion rules described by a fusion category.
For the full (i.e. closed under fusion) noninvertible symmetry, however, there is no longer a one-to-one correspondence —

$$\alpha = 2\pi p/N \in 2\pi \cdot \mathbb{Q}/\mathbb{Z} \subset 2\pi \cdot \mathbb{R}/\mathbb{Z} \cong \mathrm{U}(1)$$

between $\mathbb{Q}/\mathbb{Z}$ elements and the charge operators. Operators labelled by elements of a certain commutative monoid $\mathbf{M}$, such that the noninvertible fusion rules

$$U_{\alpha_1} \times U_{\alpha_2} = \sum_j U_{\alpha_j}$$

correspond to the monoid's binary operation and there is surjective homomorphism of monoids $\mathbf{M} \to \mathbb{Q}/\mathbb{Z}$.

Snowmass White Paper review: 2205.09545 e.g., Putrov 2208.12071

4d QED example: $\mathrm{U}(1)_{\mathrm{A}}$-$\mathrm{U}(1)_{\mathrm{V}}^2$ anomaly, of $\mathbb{Z}$ class

- **Adler-Bell-Jackiw anomaly (ABJ)** (1969)

axial A-vector V mixed anomaly

$$\mathrm{d} \star j_{\mathrm{A}} = \mathrm{d} \star (\bar{\Psi}\gamma_5\gamma_\mu\Psi \,\mathrm{d} x^\mu) = \frac{1}{4\pi^2} F_{\mathrm{V}} \wedge F_{\mathrm{V}}, \qquad F_{\mathrm{V}} = F = \mathrm{d} A$$

Axial current j_{A} non-conserved under dynamical F_{V}.

- **Choi-Lam-Shao** (2205.05086), **Cordova-Ohmori** (2205.06243)

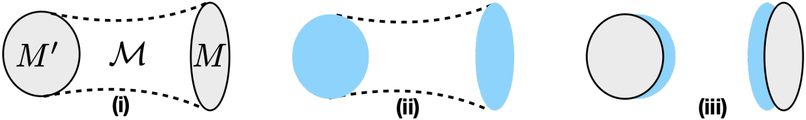
$$\text{try to make sense } \mathrm{d}(\star j_{\mathrm{A}} - \frac{1}{4\pi^2} A \wedge \mathrm{d} A) = 0.$$

Although the original $\mathrm{U}(1)_{\mathrm{A}} \cong \mathbb{R}/\mathbb{Z}$ **invertible symmetry** is broken by ABJ anomaly, there is a $\mathbb{Q}/\mathbb{Z}$ subgroup that can be revived as **noninvertible symmetry** by decorating the 3d charge operator with 3d abelian Chern-Simons TQFT (fractional quantum Hall states).

Stoke's theorem vs Anomaly on Noether theorem

• $U_\alpha(M) = \exp\left(\frac{i\alpha}{2}\oint_M \star j_{\rm A}\right)$ at $\alpha \in [0, 2\pi) \cong \mathrm{U}(1)_{\rm A}$.

$$U_\alpha(M')\, U_\alpha(M)^{-1} = \mathrm{e}^{\frac{\mathrm{i}\,\alpha}{2}\left(\int_{M'} \star j_{\rm A} - \int_M \star j_{\rm A}\right)} = \mathrm{e}^{\frac{\mathrm{i}\,\alpha}{2}\int_{\mathcal{M}} \mathrm{d}\star j_{\rm A}}$$
$$= \mathrm{e}^{\frac{\mathrm{i}\,\alpha}{8\pi^2}\int_{\mathcal{M}} F\wedge F} = \mathrm{e}^{\frac{\mathrm{i}\,\alpha}{2}\int_{\mathcal{M}} c_1^2}.$$



• $\hat{U}_\alpha(M) = \exp\left[\frac{\mathrm{i}\,\alpha}{2}\oint_M \left(\star j_{\rm A} - \frac{1}{4\pi^2} A \wedge \mathrm{d}A\right)\right]$ at $\alpha \in \mathrm{U}(1)_{\rm A}$.

$\hat{U}_\alpha(M)$ topological, but not invariant under large gauge transf.

• $\mathcal{D}_{\frac{1}{N}}(M) = \int [\mathcal{D}a] \exp\left[\mathrm{i}\oint_M \left(\frac{2\pi}{2N}\star j_{\rm A} + \frac{N}{4\pi} a\wedge \mathrm{d}a + \frac{1}{2\pi} a \wedge \mathrm{d}A\right)\right]$

Define 3d noninv TQFT on the bdry of 4d **inv FT**.

$\mathcal{D}_{\frac{1}{N}}(M) \times \mathcal{D}^\dagger_{\frac{1}{N}}(M) = \exp\left[i\oint_M \left(\frac{N}{4\pi} a \wedge da - \frac{N}{4\pi}\bar{a}\wedge d\bar{a} + \frac{1}{2\pi}(a-\bar{a})\wedge dA\right)\right] \neq 1$

rational angle $\alpha = 2\pi/N \subset \mathrm{U}(1)_{\rm A}$ Noninvertible and nonunitary.

4d QED: Anomalous invertible $\mathrm{U}(1)_{\mathrm{A}}$ broken,

but noninvertible symmetry is revived Choi-Lam-Shao (2205.05086)

	$U_\alpha(M)$	$\hat{U}_\alpha(M)$	$\mathcal{D}_{\frac{p}{N}}(M)$
Conserved (Topological)	✗	✓	✓
Gauge-invariant	✓	✗	✓
Invertible	N/A	✓	✗

- $U_\alpha(M) = \exp\left(\frac{i\alpha}{2}\oint_M \star j_{\mathrm{A}}\right)$ at $\alpha \in [0, 2\pi) \cong \mathrm{U}(1)_{\mathrm{A}}$
- $\hat{U}_\alpha(M) = \exp\left[\frac{\mathrm{i}\alpha}{2}\oint_M \left(\star j_{\mathrm{A}} - \frac{1}{4\pi^2} A \wedge \mathrm{d}A\right)\right]$ at $\alpha \in \mathrm{U}(1)_{\mathrm{A}}$
- $\mathcal{D}_{\frac{1}{N}}(M) = \int [\mathcal{D}a] \exp\left[\mathrm{i}\oint_M \left(\frac{2\pi}{2N} \star j_{\mathrm{A}} + \frac{N}{4\pi} a \wedge \mathrm{d}a + \frac{1}{2\pi} a \wedge \mathrm{d}A\right)\right]$

Anomalous invertible $\mathrm{e}^{\mathrm{i}\alpha} \in \mathrm{U}(1)_{\mathrm{A}}$ symmetry broken by ABJ anomaly, but the noninvertible counterpart survives at the

rational angle $\alpha = 2\pi p/N \in 2\pi \cdot \mathbb{Q}/\mathbb{Z} \subset 2\pi \cdot \mathbb{R}/\mathbb{Z} \cong \mathrm{U}(1)_{\mathrm{A}}$

4d SM suffers $\mathrm{U}(1)^3_{\mathbf{B-L}}$ and $\mathrm{U}(1)_{\mathbf{B-L}}$-gravity^2 anomaly

- Chern (1946), Pontryagin (1947), Eguchi-Freund (1976)

$$\boxed{\mathrm{d} \star j_{\mathbf{Q}-N_c\mathbf{L}} = (-N_f + n_{\nu_R})\,(N_c^3 \frac{c_1(\mathrm{U}(1)_{\mathbf{Q}-N_c\mathbf{L}})^2}{6} - N_c \frac{p_1(TM)}{24}).}$$

$$\text{Pontryagin class}: \; p_1(TM) = -\frac{1}{8\pi^2}\mathrm{Tr}[R \wedge R].$$

$R = \mathrm{d}\omega + \omega \wedge \omega$ is the 2-form curvature of the Levi-Cevita connection 1-form ω.

Local current density:

$$j_{\mathbf{Q}} = j_{\mathbf{Q}\mu}\mathrm{d}x^\mu = q_{\mathbf{Q}}(\psi^\dagger_{L\mathbf{Q}}\bar{\sigma}_\mu\psi_{L\mathbf{Q}})\mathrm{d}x^\mu, \quad j_{\mathbf{L}} = j_{\mathbf{L}\mu}\mathrm{d}x^\mu = q_{\mathbf{L}}(\psi^\dagger_{L\mathbf{L}}\bar{\sigma}_\mu\psi_{L\mathbf{L}})\mathrm{d}x^\mu,$$

$$\boxed{\mathrm{d} \star j = k_1 \frac{c_1^2}{3!} + k_2\, p_1}$$

Putrov-JW 2302.14862

4d SM's **B-L** suffers Mixed Gravity (Spin^c $\mathrm{U}(1)^3_{\mathbf{B-L}}$ and $\mathrm{U}(1)_{\mathbf{B-L}}$-gravity^2) anomaly, but noninvertible symmetry is revived

	U(1) invertible sym	$2\pi \cdot (\mathbb{Q}/\mathbb{Z})$ noninvertible sym
Background Grav	Ambiguous	Preserved
Semi Dynamical Grav	Broken (by Grav Anom)	Preserved
Full Quantum Grav	Broken (by Grav Anom)	Broken, or (e.g. by wormhole) Dynamically Gauged

Background vs Semiclassical Dynamical vs UV-Complete Full Quantum Gravity.

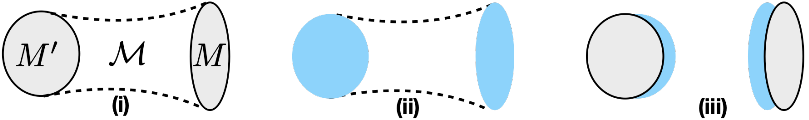
To be consistent with "No global symmetry in Full Quantum Gravity."

Stoke's theorem vs Framing Anomaly on Noether theorem

- $U_\alpha(M')\, U_\alpha(M)^{-1} = e^{i\alpha\left(\int_{M'} \star j - \int_M \star j\right)} = e^{i\alpha \int_{\mathcal{M}} d\star j}$

$$= e^{\frac{-ik\alpha}{24}\int_{\mathcal{M}} p_1} = e^{\frac{ik\alpha}{24}\int_{\mathcal{M}} \frac{1}{8\pi^2}\mathrm{Tr}[R\wedge R]} = e^{\frac{ik\alpha}{24\cdot 2\pi}\int_{\mathcal{M}} d\,\mathrm{GCS}}.$$

$$\mathrm{GCS} := \frac{1}{4\pi}\mathrm{Tr}[\omega\wedge d\omega + \frac{2}{3}\omega\wedge\omega\wedge\omega]$$



- $\tilde{U}_\alpha(M) = e^{i\alpha\int_M\left(\star j - \frac{k\,\mathrm{GCS}}{24\cdot 2\pi}\right)}$ same form as $\tilde{U}_\alpha(M')$
- $D_{(c,\mathbf{T})}(M) := e^{i\,c\int_M\left(\frac{2\pi}{k}\star j - \frac{1}{24}\mathrm{GCS}\right)} \cdot \mathbf{Z_T}[M].$

Fix the Framing Anomaly: $\boxed{f \in \pi_3(\mathrm{SO}(3)) \cong \mathbb{Z}}$.

Witten-Reshetikhin-Turaev-type 3d TQFT **T** with

$\boxed{c/k = \alpha/(2\pi) \mod 1}$. $\boxed{k\in\mathbb{Z}.\ c \text{ and } \alpha/(2\pi)\in\mathbb{Q}/\mathbb{Z}}$:

$$\tilde{U}_\alpha(M)\mathbf{Z_T}[M] \to \tilde{U}_\alpha(M)\mathbf{Z_T}[M]\, e^{-\frac{i\alpha k f}{24}} e^{\frac{2\pi i f c}{24}}.$$

Fix the Framing Anomaly

$\tilde{U}(\mathcal{Y})$ on 3-manifold $\mathcal{Y}$ above requires spin-connection ω and GCS defined by some vierbein ${e_\mu}^a$ (a choice of basis of orthonormal tangent spacetime, trivialization of tangent bundle $T\mathcal{Y}$).

A change of the vierbein: The homotopy classes of trivializations of $T\mathcal{Y}$, i.e. framings of the tangent bundle, form an integer class (Witten '89)

$$f \in \pi_3(\mathrm{SO}(3)) \cong \mathbb{Z}$$

(or a torsor over $\mathrm{H}^3(\mathcal{Y}, \mathbb{Z}) \cong \mathbb{Z}$, Atiyah '90). Large gauge transformation of tangent bundle (whose structure group $\mathrm{SO}(3)$) changes the framing by $f \in \mathbb{Z}$ units, it shifts GCS by $2\pi f$, then the charge operator changes as follows:

$$\tilde{U}_\alpha(\mathcal{Y}) \;\to\; \tilde{U}_\alpha(\mathcal{Y})\, \mathrm{e}^{-\frac{\mathrm{i}\,\alpha\, k\, f}{24}}.$$

When $\alpha/(2\pi)$ is rational, we compensate it by 3d WRT TQFT with a 2d rational CFT of chiral central charge $c_- \equiv c_L - c_R \in \mathbb{Q}$. Under the change of framing of $\mathcal{Y}$:

$$\mathbf{Z_T}[\mathcal{Y}] \;\to\; \mathbf{Z_T}[\mathcal{Y}]\, \mathrm{e}^{\frac{2\pi\,\mathrm{i}\;f\,c_-}{24}}.$$

4d SM's **B-L** suffers Mixed Gravity (Spin^c structure $\mathrm{U}(1)^3_{\mathbf{B-L}}$ and $\mathrm{U}(1)_{\mathbf{B-L}}$-gravity^2) anomaly, but noninvertible symmetry is revived

	$U_\alpha(M)$	$\tilde{U}_\alpha(M)$	$D_{(c_-,\mathbf{T})}(M)$	$D_{(c_-,\mathbf{T},\Lambda,n)}(M)$
Topological (w/ Grav)	✗	✓	✓	✓
Topological (w/ Grav + U(1))	✗	✗	✗	✓
Grav general-covariant	✓	✗	✓	✓
U(1) gauge-invariant	✓	✗	✗	✓
Unitary	N/A	✓	✗	✗
Invertible	N/A	✓	✗	✗

"w/ Grav" means under (semi-classical dynamical or background) gravity.

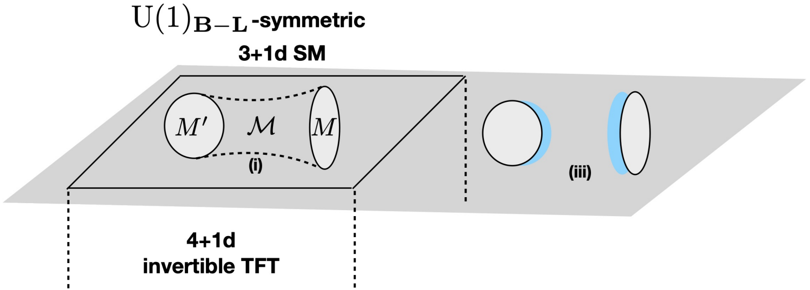
• $U_\alpha(M) = \mathrm{e}^{\mathrm{i}\,\alpha\left(\int_M \star j\right)}$.

• $\tilde{U}_\alpha(M) = \mathrm{e}^{\mathrm{i}\,\alpha \int_M(\star j - \frac{k\,\mathrm{GCS}}{24\cdot 2\pi})}$.

• $D_{(c,\mathbf{T})}(M) := \mathrm{e}^{\mathrm{i}\ c \int_M \left(\frac{2\pi}{k}\star j - \frac{1}{24}\,\mathrm{GCS}\right)} \cdot \mathbf{Z_T}[M]$.

• $D_{(c,\mathbf{T},\Lambda,n)}(M) := \mathrm{e}^{\mathrm{i}\ c \int_M \left(\frac{2\pi}{k}\star j - \frac{1}{24}\,\mathrm{GCS}\right)} \cdot \mathbf{Z_T}[M] \cdot \mathbf{Z}^{\mathrm{abCS}}_{(\Lambda,n)}[M; A]$.

p.s. abCS is some abelian CS theory. $\because$ the anomaly is in Spin^c, so the Table is only loosely speaking.

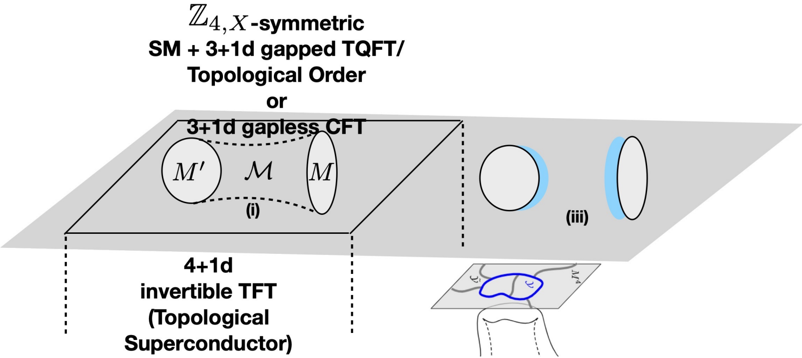


- Spin $\times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B}-\mathbf{L} \text{ or } X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}$-class perturbative local anomaly $\mathbf{B}-\mathbf{L}$ or $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}Y$: $\mathrm{U}(1)^3$ and $\mathrm{U}(1)$-grav^2:

$$\mathbf{Z}_5^{\mathrm{U}(1)} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} A_{\mathbf{Q}-N_c\mathbf{L}} \left(N_c^3 \frac{c_1(\mathrm{U}(1)_{\mathbf{Q}-N_c\mathbf{L}})^2}{6} - N_c \frac{p_1(TM)}{24} \right)).$$

Index $(-N_f + n_{\nu_R}) \in \mathbb{Z}$.



- Spin $\times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}$-symmetry. With $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}\tilde{Y}$.

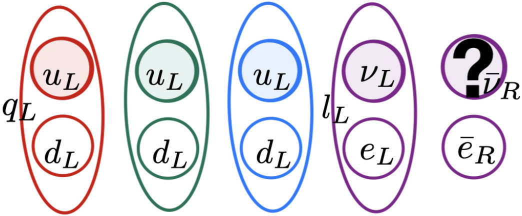
$\mathbb{Z}_{16}$-class nonperturbative global anomaly of $\mathbb{Z}_{4,X}$-gravity:

$$\mathbf{Z}_5^{\mathbb{Z}_{4,X}} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} \frac{2\pi}{16} \eta_{4\mathrm{d}}(\mathrm{PD}(A_{\mathbb{Z}_{2,X}}))).$$

Index $(-N_f + n_{\nu_R}) \mod 16 \in \mathbb{Z}_{16}$.
4d bdry of 5d X-symmetric topological superconductor (TSC).
3d bdry topological order of 4d $\mathbb{Z}_4^{TF}$-TSC (Pin$^+$), $T^2 = (-1)^F$.
See Putrov's talk at UK Symmetry Seminar.

Ultra Unification

www.youtube.com/@juvenwang1



Ultra Unification (2020-) arXiv: 2012.15860, 1809.11171, 1910.14668, 2006.16996, 2008.06499, 2112.14765, 2204.08393
Gauge-Enhanced Quantum Criticality beyond the Standard Model (2021-): 2106.16248, 2111.10369
Strong CP problem and Symmetric Mass Generation (2022-): 2204.14271, 2207.14813, 2212.14036
Categorical Symmetry of the Standard Model from Gravitational Anomaly (2022-23): 2302.14862, 2111.10369

Conclusion

1. An invertible $\mathrm{U}(1)$ symmetry can suffer from mixed grav anomalies under gravitational backgrounds (such as gravitational instantons), still a certain noninvertible counterpart of discrete subgroup of $\mathrm{U}(1)$ can be revived as a noninvertible categorical symmetry. SM's $\mathbf{B} - \mathbf{L}$.

2. No global sym in quantum gravity: Fate of Topological Defects? Categorical sym is broken or dynamically gauged.

3. Leptogenesis and Baryogenesis, Dirac vs Majorana masses vs exotic-BSM TQFT/CFT sectors — **Ultra Unification**. **THANK YOU - NYU Abu Dhabi**

جامعة نيويورك ابوظبي
NYU ABU DHABI

Back Up Slides:

Design Topological Operator: Compare w/ 2205.05086, 2205.06243

Let us compare two cases.
To make the defect topological, add by hand

- an improperly quantized CS term $\frac{1}{4\pi N} A \mathrm{d} A$
- an improperly quantized GCS term $\frac{c_-}{24}\mathrm{GCS}$

but then it will be non-invariant under large gauge-diffeomorphism transformations (which is the framing dependence of GCS in our story).

- For the former CS $\frac{1}{4\pi N} A \mathrm{d} A$, it can be fixed by considering a 3d TQFT **T** with anomalous discrete magnetic 1-form $\mathbb{Z}_N$ symmetry. Combine to get $\frac{N}{4\pi} a \mathrm{d} a + \frac{1}{2\pi} a \mathrm{d} A$ similar to our **T**' (fractional quantum Hall state).

- For the latter $\frac{c_-}{24}\mathrm{GCS}$ (that depends on metric and framing), it can be fixed by a 3d WRT TQFT **T** that depends on framing with an opposite framing anomaly. The combined theory is QFT **T**' (that depends on metric). But the outcome is Topological Operator that is independent of metric and framing:

$$\mathbf{Z}_{\mathbf{T}'}[\mathcal{Y}] := \mathrm{e}^{-\mathrm{i}\,\frac{c_-}{24}\int_{\mathcal{Y}}\mathrm{GCS}} \cdot \mathbf{Z}_{\mathbf{T}}[\mathcal{Y}].$$

$$D_{(c_-,\mathbf{T}',\Lambda,n)}(\mathcal{Y}) := \mathrm{e}^{\mathrm{i}\,c_-\int_{\mathcal{Y}}\left(\frac{2\pi}{k}\star j\right)} \cdot \mathbf{Z}_{\mathbf{T}'}[\mathcal{Y}] \cdot \mathbf{Z}^{\mathrm{abCS}}_{(\Lambda,n)}[\mathcal{Y};A]$$

where $D_{(c_-,\mathbf{T},\Lambda,n)}(M) := \mathrm{e}^{\mathrm{i}\,c_-\int_M\left(\frac{2\pi}{k}\star j-\frac{1}{24}\mathrm{GCS}\right)} \cdot \mathbf{Z}_{\mathbf{T}}[M] \cdot \mathbf{Z}^{\mathrm{abCS}}_{(\Lambda,n)}[M;A]$.

Modern view on "Symmetry"

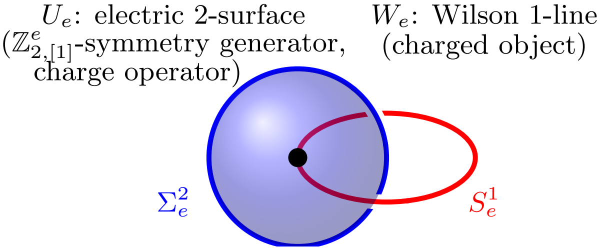
Gaiotto-Kapustin-Seiberg-Willett (1412.5148) and many others.

For the measurement of global symmetry:
charge operator U that measures,
charged object $\mathcal{O}$ that is being measured.

Symmetry generator = charge operator = topological defect U

Measurement: charge operator linked with charged object in Dd.

p-symmetry (e.g., p-form):
pd **charged object**. Codim-$p+1$ thus $(D-p-1)$d **charge operator**.



Modern view on “Symmetry”

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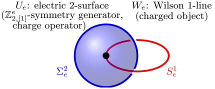
Noether theorem: Continuous global symmetry labeled by α has a conservation $\mathrm{d} \star j = 0$. Charge conservation $Q = \int \mathrm{d}x^{D-1} j_0$ integrated over the spatial slice is conserved over time evolution. Define charge operator $U_\alpha \equiv \exp(\mathrm{i}\alpha Q) = \exp(\mathrm{i}\alpha \int \mathrm{d}x^{D-1} j_0)$, then $[\hat{H}, \hat{U}] = 0$.
Consider more general operator on a closed codim-$p+1$ manifold,

$$U_\alpha \equiv \exp(\mathrm{i}\alpha \int \star j)$$

such that $\mathrm{d} \star j = 0$ makes U_α topological operator in path integral.



If $\mathrm{d} \star j \neq 0$ depends on external background $\Rightarrow$ anomaly

Modern view on "Symmetry"

Measurement: charge operator linked with charged object in Dd.

p-symmetry (e.g., p-form):
pd **charged object**. Codim-p thus $(D-p-1)$d **charge operator**.

4d U(1) gauge theory, $\int \mathrm{d}A \wedge \star \mathrm{d}A$ with $\mathrm{d}A = \star \mathrm{d}A_m$.

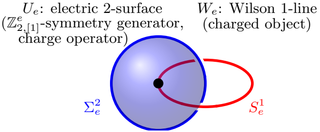
	charged object	charge operator
$\mathrm{U}(1)^e_{[1]}$	$\mathrm{e}^{\mathrm{i}\, q_e \oint A}$	$\mathrm{e}^{\mathrm{i}\, \frac{\theta_e}{2\pi} \oint (\star \mathrm{d}A)\rvert_{= \mathrm{d}A_m}}$
$\mathrm{U}(1)^m_{[1]}$	$\mathrm{e}^{\mathrm{i}\, q_m \oint A_m}$	$\mathrm{e}^{\mathrm{i}\, \frac{\theta_m}{2\pi} \oint (\star \mathrm{d}A_m)\rvert_{= \mathrm{d}A_e}}$

$\left\{ \begin{array}{l} q_e, q_m \in \mathbb{Z} \\ \theta_e, \theta_m \in \mathrm{U}(1) \end{array} \right. .$

4d $\mathbb{Z}_N$ gauge theory $\int \frac{N}{2\pi} b \,\mathrm{d}a$.

	charged object	charge operator
$\mathbb{Z}^e_{N[1]}$	$\mathrm{e}^{\mathrm{i}\, q_e \oint a}$	$\mathrm{e}^{\mathrm{i}\, \frac{N\theta_e}{2\pi} \oint b}$
$\mathbb{Z}^m_{N[2]}$	$\mathrm{e}^{\mathrm{i}\, q_m \oint b}$	$\mathrm{e}^{\mathrm{i}\, \frac{N\theta_m}{2\pi} \oint a}$

$\left\{ \begin{array}{l} q_e, q_m \in \mathbb{Z}_N. \\ \theta_e, \theta_m = \frac{2\pi}{N} k,\ k \in \mathbb{Z}_N. \end{array} \right. .$



Spacetime and internal symmetry of the Standard Model (SM)?

0-symmetry

• Spacetime symmetry: Spin group. Diffeomorphism/grav. background. $\frac{\mathrm{Spin}}{\mathbb{Z}_2^{\mathrm{F}}} = \mathrm{SO}$.

$G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_{\tilde{Y}}}{\mathbb{Z}_q}$.

• Internal symmetry: $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_cN_f,\mathbf{Q}+N_c\mathbf{L}} \times G_{\mathrm{SM}_q}$.

• Internal symmetry after gauging G_{SM_q}: $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{2N_cN_f,\mathbf{Q}+N_c\mathbf{L}} \times \mathbb{Z}^e_{6/q,[1]} \times \mathrm{U}(1)^m_{[1]}$.

1-symmetry

Denote 1-form symmetry $G_{[1]} = G^e_{[1]} \times G^m_{[1]} = \mathbb{Z}^e_{6/q,[1]} \times \mathrm{U}(1)^m_{[1]}$.

	$Z(G_{\mathrm{SM}_q})$	$\pi_1(G_{\mathrm{SM}_q})^\vee$	1-form e sym $G^e_{[1]}$	1-form m sym $G^m_{[1]}$
G_{SM_q}	$\mathbb{Z}_{6/q} \times \mathrm{U}(1)$	$\mathrm{U}(1)$	$\mathbb{Z}^e_{6/q,[1]}$	$\mathrm{U}(1)^m_{[1]}$

• No C, P, T discrete symmetry within SM.

• We can replace the $\mathrm{U}(1)_{\mathbf{B}-\mathbf{L}}$ to a discrete $\mathbb{Z}_{4,X}$ (Wilczek-Zee '79) that is more robust and preserves the 4n fermion interactions (quarks and leptons with $\mathbb{Z}_{4,X}$ charges 1):

$$X \equiv 5(\mathbf{B}-\mathbf{L}) - 4Y \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}\tilde{Y}.$$

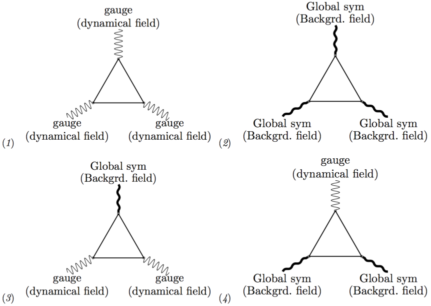
Tong 1705.01853, Anber-Poppitz 2110.02981. our 1912.13504, 2111.10369, 2112.14765.

Interpretation of Adler-Bell-Jackiw (axial or chiral) anomaly

$$\mathbf{Z}_{\text{Dirac}\Psi}[A] \text{ or } \mathbf{Z}_{\psi_L,\psi_R}[A] \xrightarrow{\psi_{L/R}\to \mathrm{e}^{\pm\,\mathrm{i}\,\alpha}\psi_{L/R}}$$

$$\int[D\bar{\Psi}][D\Psi]\exp\Big(\mathrm{i}\int_{M^d}\mathrm{d}^d x(\bar{\Psi}(\mathrm{i}\,\not{D}_A)\Psi+\alpha\Big(\partial_\mu J^{\mu,\text{Chiral}}+2N_f\,\frac{(q\;g)^{d/2}\epsilon^{\mu_1\dots\mu_d}}{(d/2)!(4\pi)^{d/2}}F_{\mu_1\mu_2}\dots F_{\dots\mu_d}\Big)\Big)\Big),$$

- 't Hooft anomaly of background (Backgrd.) fields. Mixed anomaly between $\mathrm{U}(1)_V$, $\mathrm{U}(1)_A$.
- Continuous $\mathrm{U}(1)_A$ anomalous, but its discrete $\mathbb{Z}_{2N_f,A}$ can be anomaly-free with $\mathrm{U}(1)_V^2$
- ABJ: 4d anomaly $\mathrm{U}(1)_A$-$\mathrm{U}(1)_V^2$. Dynamical gauging $\mathrm{U}(1)_V$, the $\mathrm{U}(1)_A$ broken to $\mathbb{Z}_{2N_f,A}$.



(*1*) Dynamical gauge anomaly. (*2*) 't Hooft anomaly of background fields. (*3*) Adler-Bell-Jackiw (ABJ) type of anomalies. (*4*) Anomaly that involves two background fields of global symmetries and one dynamical gauge field. The charge $q\in\mathbb{Z}$ is quantized, thus $\mathbb{Z}$ class **perturbative local anomaly**.

Classify all invertible anomalies of QFT in $d$dim via cobordism $\mathrm{TP}_{d+1}(G)$:

(Quantum) Anomaly in Physics: **Boundary** Phenomenon vs **Bulk**.

adjective for anomalies:

- (1) $\mathbb{Z}$ vs $\mathbb{Z}_n$-class: **perturbative local** vs **nonperturbative global** anomaly.
- (2) probes: *gauge* anomaly vs mixed *gauge-grav.* vs *gravitational* anomaly.

- chiral internal symmetry: chiral or axial anomaly.

- (3) **bosonic** (SO/O/E) vs **fermionic** (Spin/Pin$^{\pm}$/DPin/EPin-structure).
- (4) **background** fields ('t Hooft anomalies) or **dynamical** fields

R.B.Laughlin '81, Witten '83-85, Callan-Harvey '84-'85, Dai-Freed '94, etc.

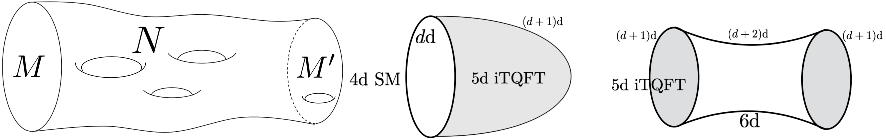
Cobordism: Kapustin'14, Kapustin-Thorngren-Turzillo-Wang'14 (proposed), Freed-Hopkins'16 (systematic), Wan-JW'18 arXiv:1812.11967: Encode higher-sym/classifying space. Wan-JW-Zheng'19 arXiv:1912.13504

Application to SM: Garcia-Etxebarria-Montero'18, JW-Wen'18, Davighi-Gripaios-Lohitsiri'19, Wan-JW'19 arXiv:1910.14668

Classify dd Anomalies and $(d+1)$d iTQFT/SPTs via Cobordism

iTQFT: invertible topological field theory. Invertible path integral $\mathbf{Z}(M)$.
SPTs: Symmetry-protected topological state.
Bordism group (discrete and abelian group $\mathbb{Z}$ or $\mathbb{Z}_n$ class): Ω^G_{d+1}

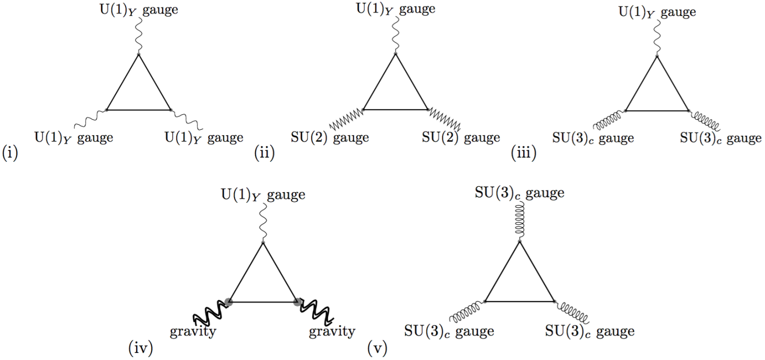
- $+$: the disjoint union.
- Closure: Disjoint union of manifolds is a manifold.
- Identity: 0 is the empty manifold.
- Inverse: $[M] + [\bar{M}] = 0$ since $\partial(M \times [0,1]) = M \sqcup \bar{M}$.
- Associativity and commutativity: true for disjoint union.



Spin cobordism:Kapustin-Thorngren-Turzillo-Wang'14 (proposed), Freed-Hopkins'16 (systematic), Wan-JW'18 arXiv:1812.11967: Encode higher-symmetry/classifying space.

Here we only concern a cobordism group $\mathrm{TP}_{d+1}(G)$,
$(\mathrm{TP}_{d+1}(G))_{\text{free}} = (\Omega^G_{d+2})_{\text{free}}$: local anomaly.
$(\mathrm{TP}_{d+1}(G))_{\text{tors}} = (\Omega^G_{d+1})_{\text{tors}}$: global anomaly.

I. (Local) Anomalies of $\mathrm{Spin}(d) \times G_{\mathrm{SM}_q}\big|_{(\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1))/\mathbb{Z}_q}$



1. $\mathrm{U}(1)_Y^3$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_1(\mathrm{U}(1))^2$ and 6d $c_1(\mathrm{U}(1))^3$
2. $\mathrm{U}(1)_Y$-$\mathrm{SU}(2)^2$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_2(\mathrm{SU}(2))$, 6d $c_1(\mathrm{U}(1)) c_2(\mathrm{SU}(2))$
3. $\mathrm{U}(1)_Y$-$\mathrm{SU}(3)_c^2$: 4d anomaly from 5d $\mathrm{CS}_1^{\mathrm{U}(1)} c_2(\mathrm{SU}(3))$, 6d $c_1(\mathrm{U}(1)) c_2(\mathrm{SU}(3))$
4. $\mathrm{U}(1)_Y$-(gravity)^2: 4d anomaly from 5d $\mu(\mathrm{PD}(c_1(\mathrm{U}(1))))$, 6d $\frac{c_1(\mathrm{U}(1))(\sigma - \mathrm{F}\cdot\mathrm{F})}{8}$
5. $\mathrm{SU}(3)_c^3$: 4d anomaly from 5d $\frac{1}{2}\mathrm{CS}_5^{\mathrm{SU}(3)}$, 6d $\frac{1}{2} c_3(\mathrm{SU}(3))$
6. 4d global Witten SU(2) anomaly from 5d $c_2(\mathrm{SU}(2))\tilde{\eta}$, 6d $c_2(\mathrm{SU}(2))\mathrm{Arf}$.
   It becomes part of local anomaly in $\mathbb{Z}$ when $q = 2, 6$.

Davighi-Gripaios-Lohitsiri1910.11277, Wan-JW 1910.14668, Davighi-Lohitsiri 2001.07731 JW 2006.16996.

II. (Local+Global) Anomalies: $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\text{internal/gauge}}$

Focus on $\mathbb{Z}_{4,X} = Z(\mathrm{Spin}(10)) \subset \mathrm{U}(1)_X$ where $X = 5(\mathbf{B}-\mathbf{L}) - 4Y$.

$G = \mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}$ and $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathrm{SU}(5)$:

$$\mathrm{TP}_{d=5}(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}) = \begin{cases} \mathbb{Z}^5 \times \mathbb{Z}_2 \times \mathbb{Z}_4^2 \times \mathbb{Z}_{16}, & q = 1,3. \\ \mathbb{Z}^5 \times \mathbb{Z}_2^2 \times \mathbb{Z}_4 \times \mathbb{Z}_{16}, & q = 2,6. \end{cases}$$

$$\mathrm{TP}_{d=5}(\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times \mathrm{SU}(5)) = \mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_{16}.$$

$\mathcal{A}_{\mathbb{Z}_2} \in \mathrm{H}^1(M, \mathbb{Z}_{4,X}/\mathbb{Z}_2^{\mathrm{F}})$ is a $\mathbb{Z}_2$-gauge field of $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X}$-manifold.

1. Mutated Witten SU(2) anomaly $c_2(\mathrm{SU}(2))\tilde{\eta}$:
   4d $\mathbb{Z}_2$ to $\mathbb{Z}_4$ global anomaly free ($q = 1,3$): $c_2(\mathrm{SU}(2))\eta'$.
   4d $\mathbb{Z}_2$ to $\mathbb{Z}$ local anomaly free ($q = 2,6$): $\frac{1}{2}\mathrm{CS}_1^{\mathrm{U}(2)} c_2(\mathrm{U}(2)) \sim \frac{1}{2} c_1(\mathrm{U}(2))\mathrm{CS}_3^{\mathrm{U}(2)}$.
2. $(\mathcal{A}_{\mathbb{Z}_2})c_2(\mathrm{SU}(2))$: 4d $\mathbb{Z}_2$ global anomaly free ($q = 2,6$)
3. $(\mathcal{A}_{\mathbb{Z}_2})c_2(\mathrm{SU}(3))$: 4d $\mathbb{Z}_2$ global anomaly free
4. $c_1(\mathrm{U}(1))^2\eta'$: 4d $\mathbb{Z}_4$ global anomaly free
5. $(\mathcal{A}_{\mathbb{Z}_2})c_2(\mathrm{SU}(5))$: 4d $\mathbb{Z}_2$ global anomaly free
6. $\eta(\mathrm{PD}(\mathcal{A}_{\mathbb{Z}_2}))$: $\Omega_5^{\mathrm{Spin} \times_{\mathbb{Z}_2} \mathbb{Z}_4} \cong \Omega_4^{\mathrm{Pin}^+} = \mathbb{Z}_{16}$.
   4d $\mathbb{Z}_{16}$ global anomaly not canceled for $15 N_f$ Weyl fermions. Alternative stories?

Wan-JW 1910.14668

Logic to Ultra Unification

$$\boxed{\Big(-(N_f = 3) + \sum_{j=e,\mu,\tau,\dots} n_{\nu_{j,R}} + \nu_{\text{new hidden sectors}}\Big) = 0 \mod 16}$$

$$\Big(-(N_f = 3) + \sum_{j=e,\mu,\tau,\dots} n_{\nu_{j,R}} + \nu_{4\mathrm{d,TQFT}} + \nu_{4\mathrm{d,others}} - \nu_{5\mathrm{d}}\Big) = 0 \mod 16.$$

• $\nu_{4\mathrm{d,odd}} = 1, 3, 5, 7, \dots \in \mathbb{Z}_{16} \Rightarrow$ Obstruction to symmetry-preserving gapped phase. No 4d TQFTs constructible.

Cordova-Ohmori'19 1912.13069.

• $\nu_{4\mathrm{d,even}} = 2, 4, 6, 8, \dots \in \mathbb{Z}_{16} \Rightarrow$ Symmetry-preserving gapped phase. 4d TQFTs constructible. Based on a symmetry-extension method.

JW-Wen-Witten'17 1705.06728.
Hsieh'18 1808.02881, Wan-JW-Zheng 1912.13504, JW 2006.16996, 2012.15860.

Math Physics Equations: **Ultra Unification** Path (Functional) Integral example

$$\mathbf{Z}_{\rm UU}[\mathcal{A}_{\mathbb{Z}_4}] \equiv \mathbf{Z}_{\substack{\text{5d-iTQFT}/\\ \text{4d-SM+TQFT}}}[\mathcal{A}_{\mathbb{Z}_4}] \equiv \mathbf{Z}^{(-\nu_{5\rm d})}_{\text{5d-iTQFT}}[\mathcal{A}_{\mathbb{Z}_4}] \cdot \mathbf{Z}^{(\nu_{4\rm d})}_{\text{4d-TQFT}}[\mathcal{A}_{\mathbb{Z}_4}] \cdot \mathbf{Z}^{(n_{\nu_{j,R}})}_{\rm SM}[\mathcal{A}_{\mathbb{Z}_4}].$$

$$\mathbf{Z}_{\rm SM}[\mathcal{A}_{\mathbb{Z}_4}] \equiv \int [\mathcal{D}\psi][\mathcal{D}\bar{\psi}][\mathcal{D}A][\mathcal{D}\phi]\dots \exp(\mathrm{i}\ S_{\rm SM}[\psi, \bar{\psi}, A, \phi, \dots, \mathcal{A}_{\mathbb{Z}_4}]\big|_{M^4})$$

$$S_{\rm SM} = \int_{M^4} \Big(\mathrm{Tr}(F_I \wedge \star F_I) - \frac{\theta_I}{8\pi^2} g_I^2 \mathrm{Tr}(F_I \wedge F_I)\Big) + \int_{M^4} \Big(\bar{\psi}(\mathrm{i}\,\not{D}_{A,\mathcal{A}_{\mathbb{Z}_4}})\psi$$

(Gauge) *symmetry breaking* $\quad + |D_{\mu,A,\mathcal{A}_{\mathbb{Z}_4}}\phi|^2 - \mathrm{U}(\phi) - (\psi_L^{\dagger}\phi(\mathrm{i}\sigma^2\psi_L'^{*}) + \mathrm{h.c.})\Big)\, \mathrm{d}^4x.$

$$(-(N_f = 3) + (\sum_{j=e,\mu,\tau,\dots} n_{\nu_{j,R}}) + \nu_{4\rm d} - \nu_{5\rm d}) = 0 \mod 16.$$

$$\mathbf{Z}^{(-\nu_{5\rm d}=-2)}_{\text{5d-iTQFT}}[\mathcal{A}_{\mathbb{Z}_4}] \cdot \mathbf{Z}^{(\nu_{4\rm d}=2)}_{\text{4d-TQFT}}[\mathcal{A}_{\mathbb{Z}_4}] = \sum_{c\in\partial'^{-1}(\partial[\mathrm{PD}(\mathcal{A}^3)])} \mathrm{e}^{\frac{2\pi\,\mathrm{i}}{8}\mathrm{ABK}(c\cup \mathrm{PD}(\mathcal{A}^3))}$$
$$\cdot \frac{1}{2^{|\pi_0(M^4)|}} \sum_{\substack{a\in C^1(M^4,\mathbb{Z}_2),\\ b\in C^2(M^4,\mathbb{Z}_2)}} (-1)^{\int_{M^4} a(\delta b + \mathcal{A}^3)} \cdot \mathrm{e}^{\frac{2\pi\,\mathrm{i}}{8}\mathrm{ABK}(c\cup \mathrm{PD}'(b))}.$$

Symmetry extension trivialize anomaly (JW-Wen-Witten'17 1705.06728). Fermionic *non-abelian* TQFT.

Anderson-Higgs symmetry breaking:

PHYSICAL REVIEW VOLUME 130, NUMBER 1 1 APRIL 1963

Plasmons, Gauge Invariance, and Mass

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 8 November 1962)

Schwinger has pointed out that the Yang-Mills vector boson implied by associating a generalized gauge transformation with a conservation law (of baryonic charge, for instance) does not necessarily have zero mass, if a certain criterion on the vacuum fluctuations of the generalized current is satisfied. We show that the theory of plasma oscillations is a simple nonrelativistic example exhibiting all of the features of Schwinger's idea. It is also shown that Schwinger's criterion that the vector field $m \neq 0$ implies that the matter spectrum before including the Yang-Mills interaction contains $m=0$, but that the example of superconductivity illustrates that the physical spectrum need not. Some comments on the relationship between these ideas and the zero-mass difficulty in theories with broken symmetries are given.

VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one

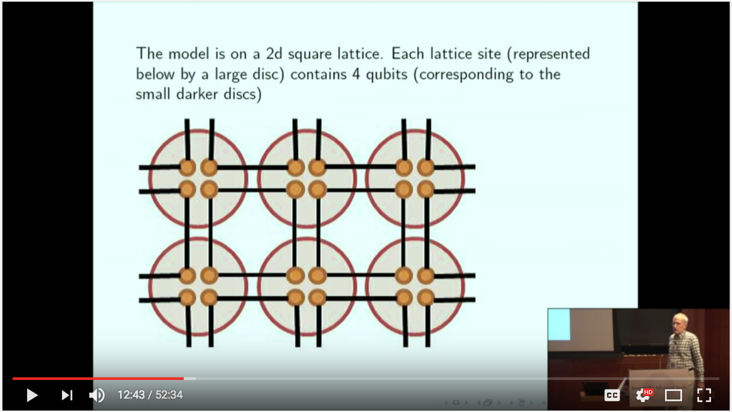
about the "vacuum" solution $\varphi_1(x) = 0$, $\varphi_2(x) = \varphi_0$:

$$\partial^{\mu}\{\partial_{\mu}(\Delta\varphi_1) - e\varphi_0 A_{\mu}\} = 0, \tag{2a}$$

$$\{\partial^2 - 4\varphi_0^{\,2} V''(\varphi_0^{\,2})\}(\Delta\varphi_2) = 0, \tag{2b}$$

$$\partial_{\nu} F^{\mu\nu} = e\varphi_0\{\partial^{\mu}(\Delta\varphi_1) - e\varphi_0 A_{\mu}\}. \tag{2c}$$

Symmetry-breaking (Anderson-Higgs):
G broken down to G_{sub}.
Symmetry-extension:
$1 \to K \to \hat{G} \xrightarrow{r} G \to 1.$



Some Boundary States for Bosons - Edward Witten

Neutrino ν_L vs TQFT sector

(Conventional) Quadratic neutrino mass term:

Dirac mass with some Higgs: $(\bar{\nu}_R \phi_H^\dagger \nu_L + \bar{\nu}_L \phi_H \nu_R)$.

Majorana mass: $\frac{\mathrm{i}\, m_{\text{Maj}}}{2}(\chi^{\mathrm{T}} \sigma^2 \chi + \chi^\dagger \sigma^2 \chi^*)$.

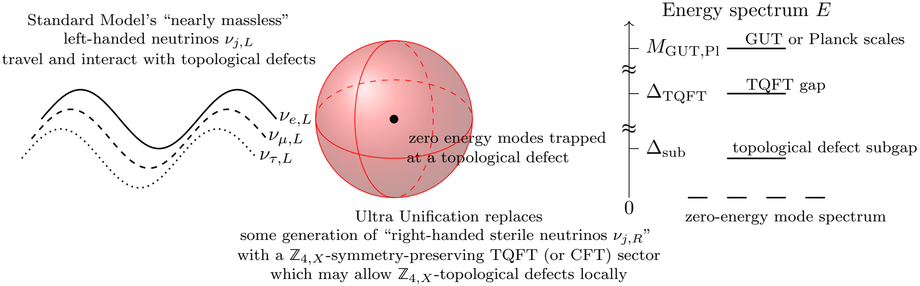
Both Dirac and Majorana masses:

$$\frac{1}{2}\Bigg(\Big(\Big(l_{L\nu_e}, l_{L\nu_\mu}, l_{L\nu_\tau} \Big) \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|}, \chi_{\nu_e}^\dagger, \chi_{\nu_\mu}^\dagger, \chi_{\nu_\tau}^\dagger \Big) \begin{array}{c} \\ 3 \\ 3 \end{array} \!\!\begin{array}{c} \begin{array}{cc} 3 & 3 \end{array} \\ \left(\begin{array}{c|c} 0 & M_{\text{Dirac}} \\ \hline M_{\text{Dirac}} & M_{\text{S}} \end{array} \right) \end{array} \begin{pmatrix} l_{L\nu_e} \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|} \\ l_{L\nu_\mu} \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|} \\ l_{L\nu_\tau} \frac{\langle \phi_H \rangle}{|\langle \phi_H \rangle|} \\ \chi_{\nu_e}^\dagger \\ \chi_{\nu_\mu}^\dagger \\ \chi_{\nu_\tau}^\dagger \end{pmatrix} + h.c. \Bigg).$$

Seesaw mechanism:
3 mass eigenstates have small mass $\simeq \frac{|M_{\text{Dirac}}|^2}{|M_{\text{S}}|} \ll |M_{\text{Dirac}}|$ for ν_L-like.
3 mass eigenstates have large mass M_{S} for ν_R-like.

New Proposal on Neutrino Mass

In contrast to the traditional seesaw mechanism $\frac{M_{\text{Dirac}}^2}{M_{\text{S}}} \ll M_{\text{Dirac}}$.



The energy spectrum near the defect has energy subgap

$$\Delta_{\text{sub}} \lesssim \frac{\Delta_{\text{TQFT}}^2}{M_{\text{GUT,Pl}}}. \qquad \Delta_{\text{small}} \lesssim \frac{\Delta_{\text{sub}}^2}{\Delta_{\text{TQFT}}} \ll M_{\text{Dirac}}.$$

(In analogy with the vortex subgap $\Delta_{\text{sub}} \simeq \frac{\Delta_{\text{SC}}^2}{E_F}$ of superconductor gap Δ_{SC} and Fermi energy E_F.) Both can give the left-handed neutrinos small masses.

(JW, Harvard HEP-String lunch Dec 1, 2020 and arXiv v3: 2012.15860)

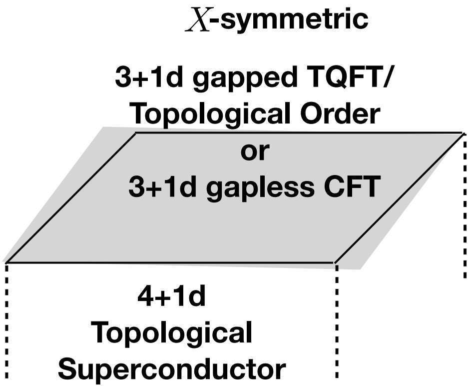
Neutrino ν_L vs TQFT sector

Convention: Neutrino pairs up to get Dirac (left-handed ν_L with right-handed ν_R) or Majorana (right-handed ν_R with itself) mass.

My talk:
Some of right-handed neutrinos ν_R may be replaced by 4d TQFT/5d iTQFT. Left-handed neutrinos ν_L travel and interfere with the zero modes of topological defects of TQFT.

- Δ_{TQFT} gap replaces the right-handed ν_R mass M_{S}.
- Subgap Δ_{sub} or Δ_{small} gives the left-handed ν_L mass.
- Mixed scenarios.

Neutrinos: a (SU(2) singlet) right-handed neutrino (massless/massive) carries a $\mathbb{Z}_{16}$ class mixed gauge-gravitational global anomaly index, which could be replaced by interacting 4d or 5d gapped topological quantum field theory, or 4d gapless conformal field theory. These theories provide new neutrino mass mechanisms [arXiv:2012.15860].



4d Anomaly (5d iTQFT) of 15,16N_f-fermion $G_{\mathrm{SM}_q} \equiv \frac{\mathrm{SU}(3)_c \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y}{\mathbb{Z}_q}$

Part 1:

- $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{U}(1)_{\mathbf{B}-\mathbf{L} \text{ or } X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}$-class local anomaly $\mathbf{B}-\mathbf{L}$ or $X \equiv 5(\mathbf{B}-\mathbf{L}) - \frac{2}{3}Y$: $\mathrm{U}(1)^3$ and $\mathrm{U}(1)$-grav^2:

$$\mathbf{Z}_5^{\mathrm{U}(1)} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R})((\int_{M^5} A c_1^2) + \frac{1}{48}\mathrm{CS}_3^{T(\mathrm{PD}(c_1))})).$$

$-N_f + n_{\nu_R} \equiv -N_f + \sum_j n_{\nu_{j,R}} = -3 + n_{\nu_{e,R}} + n_{\nu_{\mu,R}} + n_{\nu_{\tau,R}} + \dots$.

- $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathbb{Z}_{4,X} \times G_{\mathrm{SM}_q}$-symmetry.

$\mathbb{Z}_{16}$-class global anomaly of $\mathbb{Z}_{4,X}$-grav^2:

$$\mathbf{Z}_5^{\mathbb{Z}_{4,X}} \equiv \exp(\mathrm{i}(-N_f + n_{\nu_R}) \int_{M^5} \frac{2\pi}{16}\eta_{4\,\mathrm{d}}(\mathrm{PD}(A_{\mathbb{Z}_{2,X}}))).$$

Part 2:

- $\mathrm{Spin} \times_{\mathbb{Z}_2^{\mathrm{F}}} \mathrm{Spin}(10)$-symmetry.

$\mathbb{Z}_2$-class global anomaly. $p \in \mathbb{Z}_2$:

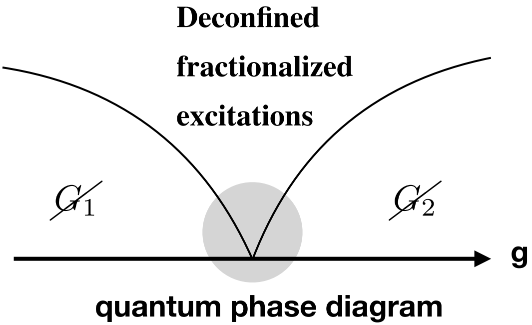
$$\mathbf{Z}_5 \equiv \exp(\mathrm{i}\pi\, \mathrm{p} \int_{M^5} w_2 w_3)|_{w_2 w_3(TM) = w_2 w_3(V_{\mathrm{SO}(n)})}$$

arXiv:2112.14765

- We can also include $\mathbb{Z}_{2N_f, \mathbf{B}+\mathbf{L}}$ background field.

arXiv:2204.08393

Deconfined Quantum Criticalities between Landau-Ginzburg phases



II. Gauge-Enhanced Deconfined Quantum Criticality BSM (2021)