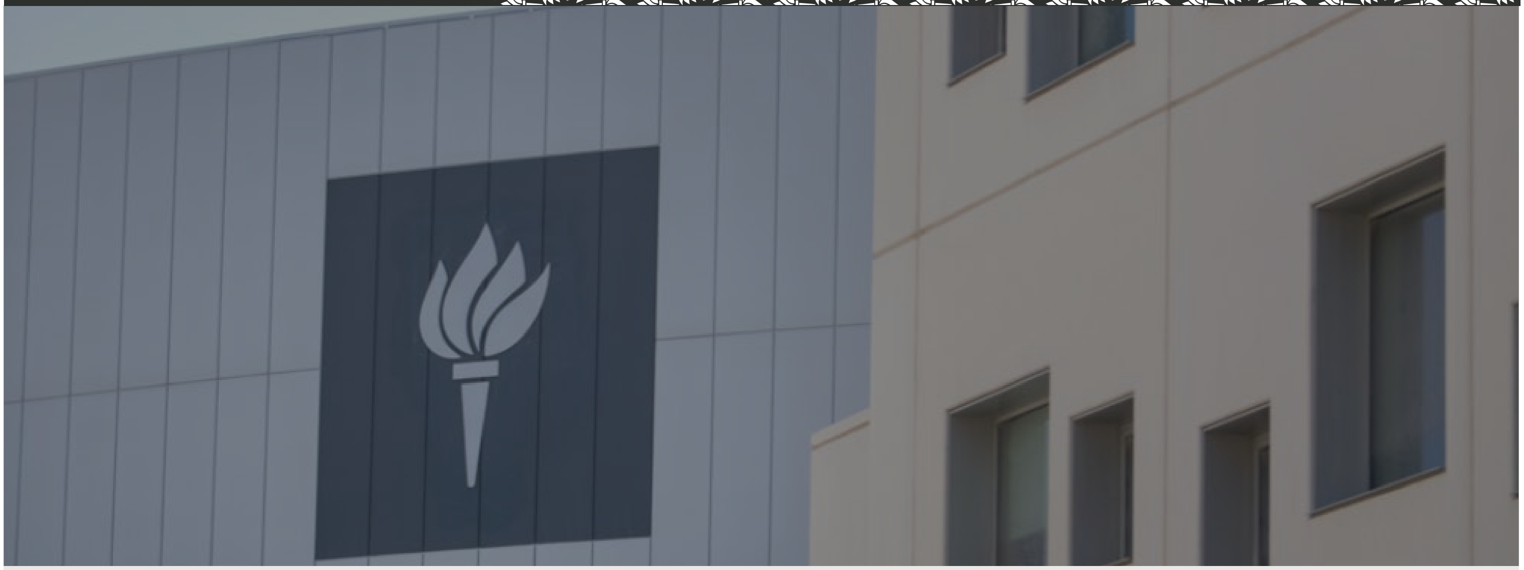

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Collaborative production networks among unequal actors

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Abstract

We develop a model of strategic network formation in productive exchanges to analyze the consequences of an understudied but consequential form of heterogeneity: differences between actors in the form of their production functions. We also address how this interacts with resource heterogeneity. Some actors (e.g. start-up firms) may exhibit accelerating returns to investment in joint projects, while others (e.g. established firms) may face decelerating returns. We show that if there is a direct relation between acceleration and resources, actors form exchange networks segregated by type of production function. On the other hand, if there is an inverse relation between acceleration and resources, networks emerge allowing all types of actors to collaborate, especially high-resource decelerating actors with multiple low-resource accelerating actors.

JEL Codes:

Keywords: Social exchange, Inequality, heterogeneity

1 Introduction

Collaboration is key to realize outcomes that are difficult to achieve alone. Examples of mutually beneficial collaboration range from scientific co-authorships (Jackson and Wolinsky 1996), across R&D joint ventures between firms (Goyal and Moraga-González 2001), mutual help and advice in organizations (Agneessens and Wittek 2012), to mutual support of students in higher education (Tomás-Miquel et al. 2015). A key question therefore is under which conditions engaging in a cooperative exchange with a specific partner becomes mutually beneficial. Two such conditions and their interplay are the focus of the present study: actors' resource endowments and the production functions governing the relationship between their efforts and their outputs.

Resource endowments play a key role in how attractive actors are to potential exchange partners (Blau 1964; Homans 1958; Cook and Emerson 1978; Molm 1994). Wealthier potential partners are more appealing than poorer ones to form alliances with (Cook et al. 1983; Emerson 1962). Yet, screening potential partners only for the size of their resource endowment neglects another key source of productivity: their ability to put their resources to productive use. This ability is captured by an actor's *production function*, i.e. the relation between the output of a production process and the inputs, such as physical resources or time investments (Robinson 1953). We focus on two types of production functions. For actors with an "accelerating" production function, increasing investments yield accelerating rates of return, whereas actors with a "decelerating" production function generate decreasing returns with increasing investments.

We show that differences in production functions can temper or exacerbate the effects that unequal resource endowments have on an actor's attractiveness as an exchange partner. As a consequence, differences in production functions are likely to affect the structures and outcomes of exchange networks. Differences in production functions can arise from differences between actors in terms of skills, talents, or available technology (Collins 1990; Sellinger and Crease 2006). For example, a startup with an innovative technology that is in its early stages of development represents an actor with an accelerating production function, because further investments into it yield increasingly fast progress. An example of an actor with a decelerating production function would be a firm operating with a mature technology, for which investments into new technology do not yield significant productivity gains. For example, in the realm of inter-firm cooperation, consider Campbell Soup Co., which invested $125 million in January 2016 to finance food start-ups, hoping that this would allow them to keep up with small companies increasingly dominating the food trends in the United States.[1] Campbell has ample resources, and yet aimed for alliances with

[1] "Campbell Invests $125 Million in Project to Fund Food Startups". The Wall Street Journal. February 17, 2016.

smaller partners. Thanks to the smaller partners' "start-up" production functions, collaborating with them promised higher returns on investment than collaboration with another equally large firm, or scaling up Campbell's own business. Notably, in a case such as Campbell's having large available resources (i.e. being 'big') compensates for having a decelerating production function (i.e. for being 'slow').

Similarly, in collaboration among scientists, a senior researcher may have broad expertise and experience in a research field (i.e. they are 'big') which can be very important to develop the research idea for a joint publication project with a junior colleague. But as the project progresses, further investments of the senior researcher's time may yield little additional benefit (i.e., he is 'slow'), because carrying out and elaborating the research idea may require the more specialist and up-to-date knowledge of new methods possessed by the more specialized junior colleague. The contributions of the more specialized junior researcher, lacking broad experience (i.e. being 'small'), instead increase in value if he invests more time into this project (i.e. he is 'fast'), because only with sufficient time investment can he demonstrate the added value of the new method in which he specializes.

Given that both available resources and actors' production functions can be key to understand productive exchange relationships, our work contributes to the understanding of how these two central aspects of collaborative exchange combine. We formalize how different combinations of the two factors shape the emergent network of exchange relationships. To give a preview of possible results of such an analysis: for the realm of interfirm collaboration our analysis could for example explain why larger firms with more resources choose investing in collaborations with smaller start-ups with fewer resources but an accelerating production function. Our study can also highlight why an environment in which start-ups have large amounts of resources may entail a network structure in which each start-up individually develops their own project and chooses to not collaborate with other start-ups. Finally, our analysis shows how big decelerators can play a beneficial social role by freeing up the potential of small accelerators.

In the remainder, we first highlight our contribution to the existing literature and then outline the model. Subsequently, we analyze the network structures emerging from the interactions of actors with different resources and production functions in Nash equilibrium. We then move further to analyze 'pairwise stable' Nash equilibria, a class of equilibria describing conditions under which emergent exchange network structures are robust to unilateral and bilateral incentives to change the existing patterns of exchange. We conclude with a discussion of the implications and limitations of the study.

2 Background and Contribution

Our study draws on and contributes to two interrelated fields of research. First, its theoretical point of departure is social exchange theory (Blau 1964; Homans 1958; Cook and Emerson 1978), and in particular the theory on *productive exchange* (Molm 1994, 1997). Productive exchanges are social interactions in which actors join their resources, aiming at outcomes greater than the aggregation of what each could have gotten separately (for a survey see Cook and Cheshire 2013). The productive exchange literature has singled out *resource heterogeneity* as a major antecedent of exchange network structure: the larger an actor's resource endowment, the more attractive this actor becomes as an exchange partner (Cook and Whitmeyer 1992; Galeotti et al. 2006; Goyal and Sumit 2006). Earlier research has modeled the resulting dynamics for example for social support networks (Flache and Hegselmann 1999; Hegselmann 1998). In this model, resource rich actors need little help but can give a lot of help to those in need, while resource poor actors need a lot of help but have little to give. Resource rich actors seeking to optimize their exchange relations prefer to exchange with other resource rich actors, thereby indirectly excluding resource poor actors from their exchanges. For the latter, only other resource poor actors remain as exchange partners, leaving resource poor actors with less favorable exchange (see also Flache 2001).

An implicit assumption behind these resource heterogeneity approaches is that everyone has the same production function. Whereas in such cases resource rich actors may indeed be the most attractive exchange partners, this may change once heterogeneity of exchange partners' *production functions* is taken into account. The effects of production functions have been studied in mathematical sociology before, especially in Marwell and Oliver's work on critical mass in collective action (Marwell et al. 1985; Marwell and Oliver 1993). In their work, however, the shape of a production function is a property of the collective good, rather than a property of (potential) *individual* contributors, as in our study. In our approach, both partners' production functions jointly affect the output of the productive exchange.

Second, we model the exchange network as a dynamic structure resulting from actors strategically optimizing their investments across several competing collaborative exchange relations. By doing so, we build on the literature on *endogenous network formation* (Jackson and Wolinsky 1996; Snijders and Doreian 2010), investigating which structures (i.e., patterns of relations) emerge from rational actors' attempts to optimize their exchange relations (Buskens and van de Rijt 2008; Jackson and Wolinsky 1996; Jackson and Watts 2001; Braun and Gautschi 2006; Dogan and van Assen 2009; Dogan et al. 2011; Doreian 2006; Hummon 2000; Raub et al. 2014). Most of these models treat actors as homogenous and disregard differences in attributes. For example, Buskens and van de Rijt (2008)'s simulations showed that when all actors pursue structural holes (Burt 1992), bi-partite networks emerge. Buechel and Buskens (2013) modeled, via simulations, the emergence of different network structures when actors pursued closeness centrality, betweenness centrality or both. An exception to the homogeneity assumption is a study by Anjos and Reagans (2013), who modeled network emergence when actors pursued different levels of commitment strategies: weak, moderate or strong. They show how actors' differences in how to commit to their relations played a key role in the partner selection and tie formation process.

We build on and extend research on productive exchange and endogenous network formation in four ways. First, we consider effects of emergent (Snijders 2013; Raub et al. 2014) rather than static network structures on inequality of exchange outcomes (Cook and Emerson 1978; Bienenstock and Bonacich 1992; Molm and Cook 1995; Dijkstra and van Assen 2006). Second, we advance strategic network formation models by conceptualizing actors' investment in collaborative exchange relations as a continuous rather than a dichotomous variable. This allows modeling how actors make decisions about the allocation of their resources (e.g. time, effort or money) across a range of potentially competing partners. Third, in contrast to some important work in the sociological exchange literature (cf. Willer 1999), we consider situations in which actors can simultaneously maintain more than one exchange relationship, as is the case e.g. in co-authorship or R&D collaborations. Finally, we assume actor heterogeneity in terms of both resource endowments and production functions (e.g. expertise, skills, creativity, talent, or technology).

3 The Model

The model rests on two general assumptions. First, actors differ in their *resource endowments* and can have an *accelerating* or a *decelerating* production function. Second, actors can form collaborations with others, in pairs, by pooling resources with their partners. They can establish multiple exchanges at a time, distributing their resources across partners. We elaborate on both assumptions below, proceeding to the game theoretic analysis thereafter.

3.1 Heterogeneity in resources and in production functions

Whether or not entering into a productive exchange is mutually attractive for a pair of actors depends on their resource endowments, their production functions, and the production functions of possible alternative exchange partners. Following (Marwell and Oliver 1993, see also Marwell et al. (1985)), we distinguish *decelerating* and *accelerating* production functions, which can be seen as decomposition of a more general S-shaped structure of production processes (see Figure 1.a). In contrast to their approach, our model specifies how the production functions of two collaborating

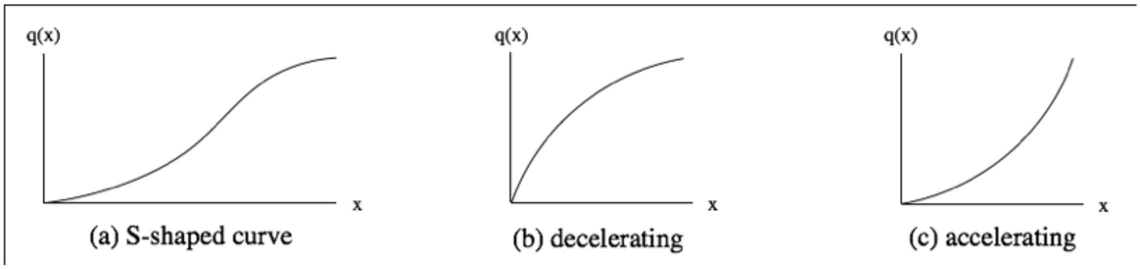

Figure 1. Production functions. The horizontal axis represents units of resources allocated by an actor to a productive exchange and the vertical axis levels of outputs achieved with these resources, given a fixed and strictly positive allocation by an exchange partner.

partners combine into one function for the return on investments of their joint project.

Decreasing marginal returns to own investments (decelerating). The decelerating case captures actors for which each extra unit of resources allocated to a productive exchange relation will be less valuable than the previous unit, keeping fixed the allocation of the exchange partner. That is, the first units of resources invested in a project have the greatest impact and subsequent units invested in the same project are less valuable (see Figure 1.b).

Increasing marginal returns to own investments (accelerating). The accelerating case captures actors for which an extra unit of resources allocated to a productive exchange relation will be *more* valuable than the previous unit, keeping fixed the allocation of the exchange partner. That is, the first units of resources invested in a project have negligible impact, and only after a certain amount of resources have been invested, the additional investments make a big difference (see Figure 1.c).

3.2 Strategic link formation

Our model represents an exchange network as a weighted graph. A link in this graph represents a dyadic productive exchange. The weight of a link represents the output of the productive exchange relation. The size of this output is determined by the partners' allocations to the relation and the combined effect of their production functions. We integrate two choices actors make: *whom* they connect to and *how much of their resources* they allocate to each of their connections. These choices are decided simultaneously by the pair of allocation decisions made by two (potential) exchange partners. If at least one of them allocates no resource to the productive exchange with the other, the exchange does not take place. If both allocate resources to the exchange, the output of these allocations determines the link weights and the outcome of the productive exchange for each partner. The total amount of resources an actor possesses puts a constraint on how much can be invested in a single project.

Decision making about link formation and investments is modeled in terms of a one-shot noncooperative game. The set $N = \{1, \ldots, n\}$, where $|N| \geq 2$, represents the actors, or players, in the productive exchange network game, denoted by Γ. Every player $i \in N$ is ex-ante and exogenously endowed with a fixed individual amount of resources $\Omega_i > 0$, that can vary across players i, and with an individual production function defined by parameter $\delta_i > 0$.[2]

Prior to the start of the game, players are informed about the size of the set of players, which is fixed throughout the analysis, and the endowments and production functions of all players. We represent the network by the set of dyadic links, g, denoting joint projects between connected players. A productive exchange between two players i and j is denoted by $ij \in g$, whereas $ij \notin g$ indicates that there is no exchange. Resources not invested in productive exchange relations are used by players in individual production, denoted by the self-link $ii \in g$. The set of partners a player i has is $N_i(g) = \{j : ij \in g\}$, for all $j \in N$. The cardinality of $N_i(g)$ is n_i (the degree of node i in the network), and is endogenously determined through the simultaneous choices of all players.

Each player can have more than one connection simultaneously and at most n. A player i simultaneously chooses whom to exchange with and the amount of resources to allocate to each of her projects, expressed by the vector of allocations $x_i = \{x_{i1}, \ldots, x_{ii}, \ldots, x_{in}\}$, where Ω_i constrains the size of total investments player i can make. The allocation of resources by i can be made to two types of projects: individual, x_{ii}, and joint with a partner j, x_{ij}. We denote $x_{(N_i(g))}$ as the vector of allocations made to i by i's partners. When a player j does not wish to exchange with i she simply allocates no resources to i.

Payoffs in the game are determined by a Cobb-Douglas production function, $u_i(\Gamma)$, which depends on the allocation choices made by all players and the shapes of their individual production functions, as follows:

$$u_i(\delta_i, \delta_{-i}, x_i, x_{N_i(g)}) = \rho x_{ii}^{\delta_i} + \sum_{j \neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j} \tag{1}$$

where δ_{-i} is the vector of parameters of the production functions of players other than i, and $\rho > 0$ is a premium on individual production, weighting the relation between individual and joint outputs.[3] Note how this production function captures the essential feature of productive exchange,

[2]Following the functions in Figure 1, players with $\delta_i < 1$ are decelerating players, players with $\delta_i = 1$ are linear players, and players with $\delta_i > 1$ are accelerating players. We focus our analysis on accelerating and decelerating players. However, proofs account for linear players as well.

[3]For two players i and j, if $x_{ij} > 0$ and $x_{ji} = 0$, no link is formed between them and the resources invested by i in the failed exchange are lost. However, the resources invested by a player in individual production are multiplied by ρ. Coleman (1990), in his study of social exchange, assumes $\rho = 1$. In our case, by allowing for multiple values of the premium on individual production we cover a wider set of productive scenarios. For details of the analysis see the Appendix.

in which players cannot produce any value unless both partners to an exchange contribute.[4] We assume that players' payoffs are identical to the summed productiveness of their projects, $u_i(\Gamma)$. Note that, in our setup, one and the same player can be part of multiple joint projects without necessarily symmetrically distributing her resources between them (as in Jackson and Wolinsky 1996).

To facilitate the analysis of the relation between resources and production functions we partition the set of players into three mutually exclusive and exhaustive subsets, which will prove useful for illustrating the main findings of the paper. We denote as $G = \{i : \Omega_i^{\delta_i} > \rho\}$ the set of players for whom the maximal impact they can make to a project is greater than the premium on individual production. Similarly, we denote by $E = \{i : \Omega_i^{\delta_i} = \rho\}$ those for whom the maximal impact is equal to the premium, and by $L = \{i : \Omega_i^{\delta_i} < \rho\}$ those for whom the maximal impact is lower than the premium. The sets categorize players by their maximal impact, which not only depends on available resources, Ω_i, but also on the shape of their production function, given by δ_i. This implies that decelerators need more resources than accelerators to end up in the set G, for the latter have higher δ_i.

We call the collection of allocation vectors of all players (one for each player) an *allocation profile*, and denote it by $(x_1, \ldots, x_n)$. When no player has incentives to unilaterally deviate from a given allocation profile $(x_1^*, \ldots, x_n^*)$, this profile is a Nash equilibrium. Formally:

$$u_i(\delta_i, \delta_{-i}, x_1^*, \ldots, x_n^*) \geq u_i(\delta_i, \delta_{-i}, x_1^*, \ldots, x_i', \ldots, x_n^*) \ \forall \ x_i^* \neq x_i', \quad i \in N.$$

The Nash equilibrium requirement can be seen as a minimal condition for an exchange network outcome to be consistent with the rational self-interest of the players involved. If the outcome is not a Nash equilibrium, then at least some players could gain from reallocating their resources and would do so.

4 Equilibrium

In this section, we describe the Nash equilibria for the one-shot network game with complete information, $NE(\Gamma)$. We first define the set of strategies players have and discuss the 2-person game. The 2-person game serves to explain which partners a player would prefer, given their available resources and production functions, and illustrates the best response logic. Then we extend the analysis to networks of size $n \geq 2$, and provide a characterization of the Nash equilibria.

[4] Note that players do not bargain or negotiate the exchange of resources but participate in reciprocal (and contingent) acts of giving resources (Lawler 2001; Molm 1990, 1994).

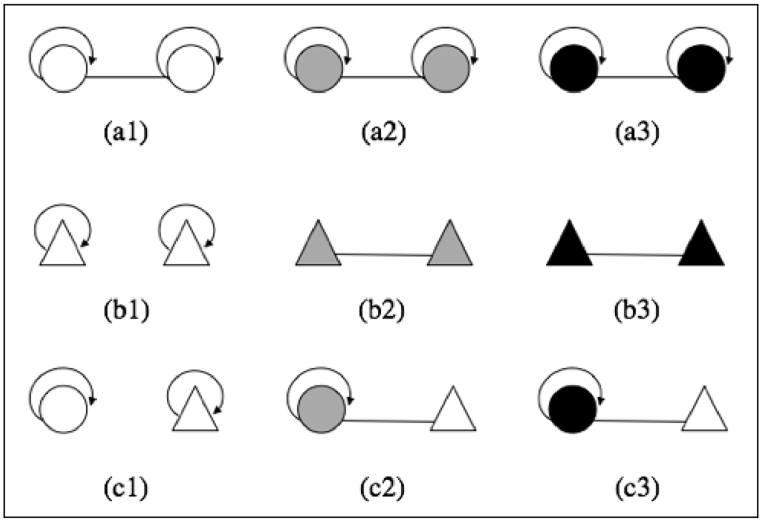

Figure 2. Dyadic interactions. Circles represent decelerators and triangles represent accelerators. The color represents the set they belong to: white for players in L, gray for players in E, and black for players in G. A loop around a node shows that a player invests resources on his individual project, and a link between two nodes shows that both players invest resources on a joint project.

This is the main goal of our study, to model the formation of weighted networks in a context of resource and production heterogeneity.

4.1 Strategies and the 2-person game

A player in the network game Γ chooses an allocation vector x_i. He either invests only in his *individual* project ($x_{ii} = \Omega_i; \sum_{j \neq i}^{n} x_{ij} = 0$), only in *joint* projects with others ($x_{ii} = 0; \sum_{j \neq i}^{n} x_{ij} = \Omega_i$), or in both individual and joint projects ($x_{ii} > 0; \sum_{j \neq i}^{n} x_{ij} > 0$), where always $x_{ii} + \sum_{j \neq i}^{n} x_{ij} = \Omega_i$. To facilitate illustration, we define three types of networks structures based on the players' strategies: Figure 2 visualizes which of these exchange structures in a dyad are compatible with best-response behavior according to Lemma 1 (see below). No Exchange (see Figures 2.b1 and 2.c1), Full Exchange (see Figures 2.b2 and 2.b3), and Hybrid Exchange (see Figures 2.a1, 2.a2, 2.a3, 2.c2, 2.c3). We refer to a *No Exchange* network when all players use their entire endowment in their individual projects. In *Full Exchange* networks, all players use their resources in joint exchanges with others and none invest anything in their individual projects. Finally, in *Hybrid Exchange* networks both individual and joint projects occur.

Lemma 1 shows how a player i's best response to j's level of investment, x_{ij}, depends on i's and j's production functions. Note that the best response is expressed in terms of what player i

invests in his own individual project.[5]

Lemma 1. ***Optimal allocation in a dyad:*** *The optimal allocation in a dyadic interaction for an accelerating player $i(\delta_i > 1)$, is:*

$$x_{ii}^* = \begin{cases} 0, & \text{if } x_{ji}^{*\delta_j} > \rho, \\ \Omega, & \text{otherwise.} \end{cases} \tag{2}$$

with indifference between the two responses when $x_{ji}^{\delta_j} = \rho$. The optimal allocation in a dyadic interaction for a decelerating player $i(\delta_i < 1)$, is:

$$x_{ii}^* = \Omega_i[1 + (\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{-1} > 0 \ \ \forall \ x_{ji}^{\delta_j} \geq 0 \tag{3}$$

Proof. Lemma 1 presents the optimal allocations for the interaction between two players in the productive exchange game Γ. For this proof, we denote the set of resources a player i has as $\hat{\Omega}$, where $\hat{\Omega} \leq \Omega_i$. This means that we can generalize the proof for any proportion of resources considered from the entire endowment Ω_i. This is a useful consideration for the extension of the results to networks of any size $n \geq 2$. Consider the optimization problem below, where a player i decides on the optimal way of allocating her resources between an individual and a joint project:

$$\max_{x_{ii}} \quad u_i = \rho x_{ii}^{\delta_i} + (\hat{\Omega} - x_{ii})^{\delta_i} x_{ji}^{\delta_j}$$

Note that the maximization is phrased in terms of the resources i keeps for herself. The First Order Condition (FOC) implies:

$$\frac{\partial u_i}{\partial x_{ii}} = \rho \delta_i x_{ii}^{(\delta_i - 1)} - \delta_i (\hat{\Omega} - x_{ii})^{(\delta_i - 1)} x_{ji}^{\delta_j} = 0,$$

and the Second Order Condition (SOC) implies:

$$\frac{\partial^2 u_i}{\partial x_{ii}^2} = \rho \delta_i (\delta_i - 1) x_{ii}^{(\delta_i - 2)} \mp \delta_i (\delta_i - 1)(\hat{\Omega} - x_{ii})^{(\delta_i - 2)} x_{ji}^{\delta_j} \gtreqless 0$$

so that:

$$\begin{cases} u_i^{''} > 0 & \text{if } \delta_i > 1 : \nexists \text{ internal maximum} \\ u_i^{''} = 0 & \text{if } \delta_i = 1 : u_i' = \rho - x_{ji}^{\delta_j} \gtreqless 0 \\ u_i^{''} < 0 & \text{if } \delta_i < 1 : \text{internal maximum is feasible} \end{cases}$$

[5] As shown in the proof of Lemma 1, this analysis can be made, with no loss of generality, for $\hat{\Omega}_i \leq \Omega_i$ available resources.

For the case of a player who has an accelerating production function ($\delta_i > 1$) no interior point can be a local maximum, thus neither a global one. Therefore, only the corner solution ($x_{ii} = 0; x_{ii} = \Omega_i$) are candidates for a global solution. The payoff functions for each are $u_i(x_{ii} = \Omega_i) = x_{ii} = \Omega_i^{\delta_i} x_{ji}^{\delta_j}$ and $u_i(x_{ii} = 0) = \rho\Omega_i^{\delta_i}$, respectively. Thus, the best response (RB) is:

$$BR = \begin{cases} x_{ii}^* = 0 & \text{if } x_{ji}^{\delta_j} > \rho \\ x_{ii}^* = \Omega_i & \text{if } x_{ji}^{\delta_j} \leq \rho \end{cases} \tag{4}$$

with indifference between the two possibilities if $x_{ji}^{\delta_j} = \rho$.

If a player has a linear production function ($\delta_i = 1$) it follows immediately from the FOC that:

$$BR = \begin{cases} x_{ii}^* = 0 & \text{iff } x_{ji}^{*\delta_j} > \rho \\ x_{ii}^* \in [0, \Omega_i] & \text{iff } x_{ji}^{*\delta_j} = \rho \\ x_{ii}^* = \Omega_i & \text{iff } x_{ji}^{*\delta_j} < \rho \end{cases} \tag{5}$$

If a player is a decelerating production function ($\delta_i < 1$) from the FOC we know that $\rho\delta_i x_{ii}^{\delta_i - 1} = \delta_i(\Omega_i - x_{ii})^{\delta_i - 1} x_{ji}^{\delta_j}$, where $\rho x_{ii}^{\delta_i - 1} = (\Omega_i - x_{ii})^{\delta_i - 1} x_{ji}^{\delta_j}$, so that $\Omega_i = x_{ii}[1 + (\frac{1}{\rho})^{(\frac{1}{1-\delta_i})} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]$:

$$BR = \Big\{ \; x_{ii}^* = \Omega_i[1 + (\tfrac{1}{\rho})^{(\frac{1}{1-\delta_i})} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{-1} \quad \text{if } x_{ji}^{*\delta_j} \gtreqless \rho \tag{6}$$

To ascertain that Eq. 6 leads to a global BR we compare to the two corner solutions. Substituting Eq. 6, in u_i yields:

$$u_i(x_{ii}^*) = \rho(\Omega_i[1 + (\tfrac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{-1})^{\delta_i} + [\Omega_i - (\Omega_i[1 + (\tfrac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{-1})]^{\delta_i} x_{ji}^{\delta_j}$$

$$u_i(x_{ii}^*) = \frac{\rho\Omega_i^{\delta_i}}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}} + [\Omega_i - \frac{\Omega_i}{1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}}]^{\delta_i} x_{ji}^{\delta_j}$$

$$u_i(x_{ii}^*) = \frac{\rho\Omega_i^{\delta_i} + \Omega_i^{\delta_i}[(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i} x_{ji}^{\delta_j}}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}} = \frac{\Omega_i^{\delta_i}(\rho + \rho^{\frac{\delta_i}{\delta_i - 1}} x_{ji}^{\frac{\delta_j\delta_i}{1-\delta_i} + \delta_j})}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}} = \frac{\Omega_i^{\delta_i}\rho(1 + \rho^{\frac{\delta_i}{\delta_i - 1}} x_{ji}^{\frac{\delta_j}{1-\delta_i}})}{[1+(\frac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{\delta_i}}$$

$$u_i(x_{ii}^*) = \rho\Omega^{\delta_i}[1 + (\tfrac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{1-\delta_i}$$

Now, the question is when is $u_i(x_{ii}^*) \geq u_i(x_{ii} = \Omega)$. We say this condition is satisfied when:

$$\rho\Omega_i^{\delta_i}[1 + (\tfrac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}]^{1-\delta_i} \geq \Omega_i^{\delta_i} x_{ji}^{\delta_j}$$

$$\rho^{\frac{1}{1-\delta_i}}[1 + (\tfrac{1}{\rho})^{\frac{1}{1-\delta_i}} x_{ji}^{\frac{\delta_j}{1-\delta_i}}] \geq x_{ji}^{\frac{\delta_j}{1-\delta_i}}$$

$$\rho^{\frac{1}{1-\delta_i}} \geq 0$$

which is always true. □

Lemma 1 formalizes how accelerating and decelerating players best respond to their partners in a dyadic interaction. Accelerators, on the one hand, have *all-or-nothing* best responses. If their partner is in E or G, accelerators will put their entire endowment in the exchange (see Figures 2.b2, 2.b3, 2.c2, 2.c3). Otherwise, accelerators will rather stay alone (see Figures 2.b1 and 2.c1). That is, only if actors are in E or G they are attractive to an accelerator partner. Notice that accelerators' best responses to their partners' choices are independent of their own resources.

On the other hand, players with a decelerating production function are better off not putting all eggs in one basket. Their best responses are always *fractions* of their total resources (given as the fraction of total resources retained, in Eq. 3). These facts imply that no matter whether they are in G, E or L, a decelerating player can always best respond (see Figures 2.a1, 2.a2, 2.a3, 2.c1, 2.c2, 2.c3). The intersections of the best responses presented in Lemma 1 result in the Nash equilibria of the 2-person game (which are not necessarily unique).

Let us briefly return to our earlier example of an R&D collaboration between a start-up firm with a quickly advancing technology, and a partner with a more mature technology for whom progress slows down with further investments. Lemma 1 shows that the start-up would require from the *mature* partner a certain minimal investment before devoting any level of effort to the joint project. The mature firm would not require such a minimum investment before investing into the collaboration. On top of that, once the start-up has found that the partner guarantees the minimum level of investment, his best choice is to invest all his resources in that project. The mature firm, however is better off diversifying, because there is only so much this firm can contribute to a single project before its marginal productivity falls below that of investing into alternative projects. This example also demonstrates how heterogeneity in resources interacts with heterogeneity in production functions. Only if a mature firm has enough resources to make an investment that passes the minimal investment required by the start-up, exchange between them is at all possible. Technically, exchange is precluded if the mature firm is a member of L (having few resources relative to productivity).

Correspondingly, a senior researcher would not invest all his resources and attention into a single research project, for he can gain more by devoting his generalist expertise to different papers. However, the junior researcher, being a specialist, will be willing to invest all he has into a single project, provided the senior researcher allocates enough resources to the project for them to work together. However, if the senior researcher cannot guarantee a greater impact in resources than what a junior researcher could acquire on his own, then the junior scientist would rather work on a project alone than collaborate with the senior researcher.

4.2 Connectivity in the n-person network

The results of Lemma 1 generalize to n-person networks, because the optimization problem solved in it can be applied to any part $\hat{\Omega} \leq \Omega_i$ of i's resources, i's utility being additive across all projects i is engaged in (see Eq. 1).

Lemma 2 below contains the results this approach yields for the conditions under which different types of equilibria are obtained. First of all, note that the 'No Exchange' configuration is always a Nash equilibrium, regardless of the players' production function and resources (see the first bullet of Lemma 2). Second, consider an accelerating player i ($\delta_i > 1$). By Lemma 1, if there is no other player j such that $x_{ji}^{\delta_j \geq \rho}$, player i will work alone. Note that this is by definition the case if all other actors are members of L. If there is exactly one player j such that $x_{ji}^{\delta_j \geq \rho}$, player i allocates all his resources to j. Note that this is possible because a project is assumed to be always big enough to absorb all of a player's resources. If there are two or more players j in E or G, such that $x_{ji}^{\delta_j \geq \rho}$, player i allocates all his resources to the player j with the highest $x_{ji}^{\delta_j}$, since this yields i the maximum total utility. If the maximum allocation received by i is not unique, i picks one of them. Thus, accelerating players have at most one joint project, and if they have such a project, they allocate all their resources to it. These considerations lead to the second and third bullets of Lemma 2.

Third, consider a decelerating player i ($\delta_i < 1$), and suppose i is involved in k joint projects. If $k = 0$ the fourth bullet of Lemma 2 is trivial. If $k > 0$, since the utility function (equation (1)) of player i is additive in the k projects, we can consider any of the k projects as an independent 2-person game, conditional on the $k - 1$ other projects. By Lemma 1, player i will best respond in any of the k projects according to equation (3). In particular, player i will have a non-zero self-allocation in any of the k projects. Hence, overall player i has $x_{ii}^* > 0$, which leads to the fourth bullet in Lemma 2.

Lemma 2. ***Nash equilibria in n-person game:*** *In the n-person game Nash equilibria are allocation profiles such that there are:*

- *No Exchange configurations, for any composition of players' production functions and resources.*

- *No Exchange configurations if all players are accelerating and are in L.*

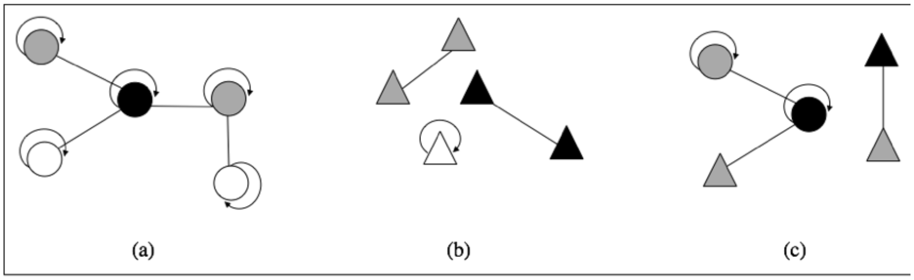

Figure 3. n-person networks. Circles represent decelerators and triangles represent accelerators. The color represents the set they belong to: white for players in L, gray for players in E, and black for players in G. A loop around a node shows that a player invests resources on his individual project, and a link between two nodes shows that both players invest resources on a joint project.

- *Full exchange configurations formed by components of size 2 in which both players allocate all their resources to the exchange, if all players are accelerating and are in E or G.*
- *Hybrid Exchange configurations, in which all players always create an individual project if all players are decelerating, regardless of whether they are in L, E or G.*

Lemma 2 characterizes network configurations in equilibrium, given the players' production functions and the allocations by their neighbors. Note how for expositional convenience the statements in Lemma 2 concern only homogeneous networks in which all players have the same type of production function. However, in the proof above it, we are concerned only with individual best responses depending on the player's production function, not on his partner's. Hence, using Lemmas 1 and 2 we can characterize the Nash equilibria for any network, whether homogeneous or heterogeneous. The bottom line is that decelerating players create both individual and joint projects in equilibrium, resulting in a hybrid exchange network. Accelerating players create but a single (joint or sole) project, possibly resulting in a full exchange network composed of dyads if their partners are in E or G, or of isolated nodes if their partners are in L.

Thus, we can study the way Nash equilibria depend on the distribution of production functions and resources across players in the network. Since a Nash equilibrium is any combination of best responses, it is clear there will be very many different equilibria in any given network. Examples of these equilibria are illustrated in Figure 3.

If all players have a decelerating production function, there is a wide range of equilibrium configurations, from No Exchange to any Hybrid Exchange where all players perform an individual task and they also perform up to $\frac{(n(n-1))}{2}$ joint tasks (see Figure 3.a). On the other hand, accelerating players will either form No Exchange or Full Exchange networks where players only have one joint project and no individual. Thus, the network is composed of dyads (see Figure 3.b).[6] If there is more than one other player who would be both willing to invest into a project with the accelerating player and has enough resources to meet the minimum investment, then the accelerating player would prefer the partner who makes the biggest investment. For accelerating potential partners this implies that the partner with the largest amount of resources is preferred, while for decelerating partners, resources only affect their attractiveness if they have less resources to invest than their optimal investment would require. In that case, they would invest only a fraction of the optimal investment, the size of which depends on the amount of resources at their disposal. More generally, when there is heterogeneity in production functions and resources, the resulting networks are a combination of the homogeneous cases (see Figure 3.c).

In our example of R&D collaboration between firms with mature technology (i.e. decelerating players) and start-up firms with quickly advancing technology (i.e. accelerating players), this would mean that if all firms were mature, it would be optimal for them to diversify into various R&D collaborations. If all firms were start-ups, they would either be working alone if the partners did not have enough resources to exchange with them, or put all their efforts into a single collaboration if they found a partner who can guarantee the necessary contribution.

More commonly, and as our initial example of Campbell Soup Co. and the food start-ups suggests, if there is a single big decelerator and multiple small accelerators, the big decelerator produces a large social surplus that would be forfeited if instead there was a big accelerator, or no big actor at all. For instance, suppose all accelerators are in L. Absent the big decelerator there would be no exchanges (i.e. no joint production and low payoffs), and social efficiency would be negatively affected. However, the single big decelerator could free this potential and make it flourish, by connecting to a number of the start-ups who would all invest all of their resources into their projects with the big decelerator. This is what the empirical examples of mature firms and start-ups suggests.

Similarly, junior scientists may in an early stage of their career not be able to find other junior scientists to collaborate with, because all of them lack the experience and breadth of knowledge to develop fruitful research problems for a joint project. However, as soon as an experienced senior scientist invests some fraction of her time into collaboration with each of the juniors, all joint projects could become sufficiently productive to justify the full efforts of the juniors working on them.

In the following subsection, we provide a characterization of the Nash equilibria in terms of the

[6] If n is odd, there is one player who is excluded from exchanging with others and who will use her resources to produce alone.

shares of their resources players devote to each productive project.

4.3 Characterization of Nash equilibrium

To describe the set of Nash equilibria in terms of the resources players allocate, $NE(\Gamma)$, we consider the general problem of optimizing the payoff function $u_i(\Gamma)$, subject to the constraint $x_{ii} + \sum_{j\neq i}^{n} x_{ij} = \Omega_i$.

Proposition 1. ***Best Responses in*** Γ***:*** *For a productive exchange network game, the proportion of resources a player i allocates to a project is equal to the proportional productivity of the given project compared to her total productive output in equilibrium. Therefore, the best response of player i to the given allocations x_{ji} in terms of his allocation to his individual project, x^*_{ii}, must satisfy the condition:*

$$x^*_{ii} = \frac{\rho x_{ii}^{*\delta_i}}{\rho x_{ii}^{*\delta_i} + \sum_{j\neq i}^{n} x_{ij}^{*\delta_i} x_{ji}^{*\delta_j}} \Omega_i \tag{7}$$

The best response of player i in terms of his allocation to a joint project with j, x^*_{ij}, must satisfy the condition:

$$x^*_{ij} = \frac{x_{ij}^{*\delta_i} x_{ji}^{\delta_j}}{\rho x_{ii}^{*\delta_i} + \sum_{j\neq i}^{n} x_{ij}^{*\delta_i} x_{ji}^{\delta_j}} \Omega_i \tag{8}$$

Proof. Proposition 1 presents the best response functions in the genera n-person productive exchange game. The proof is the solution to the optimization problem of the payoff function in Equation 1:

$$\max_{x_{ii}} \quad u_i(\delta_i, \delta_j, x_i, x_{N_i}(g)) = \rho x_{ii}^{\delta_i} + \sum_{j\neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j} \tag{9}$$

$$\text{s.t.} \;\; x_{ii} + \sum_{j\neq i}^{n} x_{ij} \leq \Omega_i$$

The First Order Conditions (FOCs; Eq. 10 and Eq. 11) and the complementary slackness condition (C.S.C; Eq. 12) imply:

$$\frac{\partial L}{\partial x_{ii}} = \rho \delta_i x_{ii}^{(\delta_i - 1)} - \lambda = 0,$$

$$\rho \delta_i x_{ii}^{\delta_i} = \lambda x_{ii} \tag{10}$$

$$\frac{\partial L}{\partial x_{ij}} = \delta_i x_{ij}^{(\delta_i - 1)} x_{ji}^{\delta_j} - \lambda = 0,$$

$$\delta_i x_{ij}^{\delta_i} x_{ji}^{\delta_j} = \lambda x_{ij} \tag{11}$$

$$\lambda(x_{ii} + \sum_{j \neq i}^{n} x_{ij} - \Omega_i) = 0 \tag{12}$$

where L is the *Lagrange function* and $\lambda \geq 0$ is the *Lagrange multiplier.* From Eq. 10 and Eq. 11 it follows that $\lambda = 0$ implies $x_{ii} = 0$ and $x_{ij} x_{ji} = 0$ for all pairs i and j, yielding a total utility equal to zero. Since any player i can produce a strictly positive utility by working alone, this is never a best reply. So, we must have $\lambda > 0$ and according to Eq. 12 the constraint must be binding: $x_{ii} + \sum_{j=1}^{n_i} x_{ij} = \Omega_i$.

Summing Equation 11 in j:

$$\delta_i \sum_{j=1}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j} = (\Omega - x_{ii})\lambda \tag{13}$$

Adding Equation 10 and 13:

$$\delta_i(\rho x_{ii}^{\delta_i} + \sum_{j=1}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j}) = \lambda \Omega_i \tag{14}$$

Dividing Equation 10 by Equation 14, we obtain the best response of player i to the allocations of the other players, in terms of her allocation to an individual project, x_{ii}^*:

$$x_{ii}^* = \frac{\rho x_{ii}^{\delta_i}}{\rho x_{ii}^{\delta_i} + \sum_{j \neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j}} \Omega_i \tag{15}$$

Dividing Equation 11 by Equation 14, we obtain the best response of player i on her allocation to a combined project with j, x_{ij}^*:

$$x_{ij}^* = \frac{x_{ij}^{\delta_i} x_{ji}^{\delta_j}}{\rho x_{ii}^{\delta_i} + \sum_{j \neq i}^{n} x_{ij}^{\delta_i} x_{ji}^{\delta_j}} \Omega_i \tag{16}$$

□

The best response functions in Proposition 1 show that in the optimum the proportion of resources a player i invests in a productive project equals the proportional productivity of the given project compared to his total productive output. In other words, the greater the output of a productive project, the more resources i allocates to such project.[7]

In the following section, we narrow the set of network configurations that emerge in equilibrium down by imposing a dyadic rationality constraint. This will conclude our analysis.

4.4 Pairwise stable Nash equilibria

In the previous sections, we have used Nash equilibrium as the solution concept. However, in social and economic settings such as the productive exchanges studied here, *relations require mutual agreement to be created* and we want to take this dyadic nature of interactions into account. Because players are assumed to behave rationally (i.e. they are utility maximizers), they can be expected to bilaterally form relationships that are mutually beneficial and to unilaterally sever relationships that are not. For instance, one would expect players to talk to each other and form a productive exchange if it is in their mutual interest. Nash equilibrium, however, does not properly account for this. Nash equilibrium only accounts for unilateral deviations, which allows equilibria that are 'unreasonable', such as the *No Exchange* network in our model. Whenever two players could complete a productive exchange to the benefit of both, we would expect them to talk to each other and create a joint project.

To realign models of strategic network formation with this 'coalition behavior' (e.g. Emerson 1972), Jackson and Wolinsky (1996) proposed pairwise stability as an alternative to the Nash concept that captures mutual consent (see also Jackson and Watts 2001, 2002). A network is pairwise stable if it meets the following two requirements: (i) no player can strictly improve his utility by severing a relationship he has and (ii) for any link that does not exist, whenever one player would strictly improve his utility by forming the link, the potential partner would *not* experience a strict increase in utility. This concept leads to the notion of pairwise stable Nash equilibrium (PNE), so that a network is PNE if it is Nash and pairwise stable.

PNE is related to a game theoretic solution concept widely used for social exchange: the core (see Bienenstock and Bonacich 1992, 1993; Bonacich and Bienenstock 1995; Dijkstra 2009). Like the core, PNE is based on two conceptions of rationality: individual and dyadic rationality (Rapoport 1970). Individual rationality is needed to ensure Nash equilibrium outcomes where no player will choose an allocation profile that gives him a lower outcome than what he could maximally achieve, given the allocation of the other players. Dyadic rationality is the same assumption with respect

[7] A unilateral deviation by a neighbor j, increasing her investment to a common project with i, gives incentives to i to make an increasing unilateral deviation as well. Given $x_{ij} > 0$ and $x_{ik} > 0$, we get $\frac{x_{ii}}{x_{ij}} = \frac{x_{ii}^{\delta_i}}{x_{ij}^{\delta_i} x_{ji}^{\delta_j}}$ and $\frac{x_{ij}}{x_{ik}} = \frac{\rho x_{ij}^{\delta_i} x_{ji}^{\delta_j}}{x_{ik}^{\delta_i} x_{ki}^{\delta_k}}$. Then, $\frac{\partial x_{ii}}{\partial x_{ji}} \leq 0,\ \frac{\partial x_{ik}}{\partial x_{ji}} \leq 0$, and $\frac{\partial x_{ij}}{\partial x_{ji}} \geq 0 \ \ \forall\, i \in N$ and $j, k \in N_i(g) : j \neq k, j \neq i,$ and $i \neq k$.

to pairs of players.

Note that PNE has been widely used in strategic network formation models, where links are either present or not. However, in our productive exchanges, players decide how much of their resources to devote to various collaborations, so that it is not only a matter of *whether* a connection exists, but also what its *weight* is. Particularly, in our setting of weighted networks, we need to adapt the notion of pairwise stability from a binary choice set to a continuous choice set, by considering that changes are not restricted only to formation or deletion of links, but include also variations in allocations even for links that have already been formed (see Definition 1 below).

Definition 1. ***PNE in productive exchange:*** *A network is PNE if no player i would strictly benefit by any reallocation of her resources in vector x_i, and no pair of players i and j would both strictly benefit by a reallocation in x_i and x_j.*

Proposition 2 characterizes the PNE configurations in our model.

Proposition 2. ***Pairwise stable Nash equilibria:*** *The set of $PNE(\Gamma)$ is a subset of $NE(\Gamma)$.*

Under a homogeneous distribution of production functions ($\delta_i = \delta_j, \forall\ i \in N$). If all players have accelerating production functions:

1. *For players in L the unique PNE is No Exchange.*
2. *For players in E or G the unique PNE is Full exchange, for which pairs are formed by matching adjacent players, where players are ranked by their maximal impact, $\Omega_i^{\delta_i}$, from highest to lowest.*

   If all players have decelerating production functions:
3. *there is no PNE, regardless of whether players are in L, E or G*

   And under a heterogeneous distribution of production functions. If there is heterogeneity in production functions and resources are homogeneous
4. *In any PNE network there is at most one exchange between an accelerating and a decelerating player.*

   If there is heterogeneity in production functions and resources are such that accelerating players have strictly larger endowments than decelerating players
5. *PNE configurations are as in (iv)*

   If there is heterogeneity in production functions and resources are such that accelerating players belong to L and decelerating players belong to E or G

6. *Any PNE configuration is a core-periphery structure in which decelerating players are linked between them, as the core, and also act as hubs to which accelerating players link*

Proof. We present the proof for each item in Proposition 2.

1. Given that $\Omega_i^{\delta_i} \leq \rho, \ \forall \, i \in \{E \cup L\}$, players respond by working alone, as shown in Lemma 2. This is pairwise stable because no player strictly prefers creating a non-existing link to not creating it, since $\rho\Omega^{\delta_i} \geq \Omega^{\delta_i}\Omega^{\delta_j}$.

2. Consider that $\Omega_i^{\delta_i} > \rho, \ \forall \, i \in G$, let the cardinality of G be k. Rank and label all members of G from 1 to k, such that $\Omega_1^{\delta_1} \geq \Omega_2^{\delta_2} \geq \Omega_3^{\delta_3} \geq \ldots \geq \Omega_{k-1}^{\delta_{k-1}} \geq \Omega_k^{\delta_k}$. Let pairs of players $\{1, 2\}, \{3, 4\}, \{5, 6\}$, etc., engage in *full exchange* in network g. If k is uneven, player k is left without a partner. All players outside K work alone. By Lemma 2 this is a Nash configuration. To see that it is pairwise stable first observe that Nash equilibrium guarantees that no player will individually want to reallocate resources. Second, consider non-existing links between layers in K. Consider players i, j, l, m such that $\Omega_i^{\delta_i} \geq \Omega_j^{\delta_j} \geq \Omega_l^{\delta_l} \geq \Omega_m^{\delta_m}$, and $i, j \in g$ and $l, m \in g$. Suppose i proposes a ink to player l, by allocating $x_{il}^{\delta_i} > \Omega_m^{\delta_m}$, than player l is better off reciprocating i and allocating $x_{li}^{\delta_l} = \Omega_l^{\delta_l}$. However, following the construction of network g, $\Omega_j^{\delta_j} \geq \Omega_l^{\delta_l}$, which does not make i better off allocating any resources to player l. Notice this is also true if player l is the k^{th} player and is working alone, because $\Omega_j^{\delta_j} > \rho$. Moreover, it is also true when considering a player n such that $\Omega_n^{\delta_n} \leq \rho$. Thus, network g is PNE.[8]

3. Consider $\delta_i < 1, \ \forall \, i \in N$, and $\Omega_j^{\delta_j} \lesseqqgtr \rho$. Let g be any network. Suppose that under g some joint projects occur, and suppose g is a Nash equilibrium. Then by Lemma 2 we know that all players i involved in a joint project have $x_{ii}^* > 0$. By the proof of Proposition 1 we know that the marginal utilities of all the projects any such i is involved in equal $\lambda > 0$. Now consider $ij \in g$. Taking the cross partial derivate we get $\frac{\partial^2 u_i}{\partial x_{ij}^2} = \delta_j \delta_i x_{ij}^{\delta_i - 1} x_{ji}^{\delta_j - 1} > 0$. Hence, a margina increase in x_{ji} gives i an incentive to strictly increase x_{ji}. Since the same is true mutatis mutandis for player j, i and j can both strictly improve their utilities through a marginal increase in x_{ij} and x_{ji}, respectively. Hence, g is not PNE.

   Now suppose there are no joint projects in g. Then $x_{ii} = \Omega_i$ for all i, and this is a Nash equilibrium. Now, propose a link between i and j. The dyadic best response behavior for players with decelerating production functions analyzed in Lemma 1 then shows that both i

[8]Note that since some players in K might have identical levels of production functions, g is not a unique *network*, but a unique *configuration*. In other words, if two players have identical production functions they are interchangeable, leading to two equivalent PNE networks.

and j strictly improve their utilities by both allocating strictly positive amounts of resources to their exchange. Hence, g is not PNE.

4. Suppose in the network g there are two joint projects between an accelerating player and a decelerating player. By Lemma 2 the accelerating players allocate all their resources in their single exchange. In particular, accelerating player i earns $u_i(g) \leq \Omega_i^{\delta_i}\Omega_j^{\delta_j} < \Omega_i^{\delta_i}\Omega_k^{\delta_k}$, where j is the decelerating player exchanging with i, and k is the other accelerating player. The same is true for the other accelerating player k. hence, i and k can both strictly improve their utilities by engaging in full exchange with each other and g is not PNE.

5. The same arguments give in the proof for item 4 hold in item 5.

6. Consider a heterogenous distribution of production functions and heterogeneity in resources, such that accelerating players belong to L and decelerating players belong to E or G. Then, from the arguments in item 1 there are no exchanges between players with an accelerating production function. Regarding players with decelerating production functions, suppose that under g some joint projects occur, and suppose g is a Nash equilibrium. Then, by Lemma 2 we know that all players j involved in a joint project have $x_{jj}^* > 0$. Following the arguments in item 3 we know that a player j can form a link with a player $i : \delta_i \geq 1$. The matching is such as in item 2 but instead of ranking $\Omega_j^{\delta_j}$ (given that players were fully exchanging all their endowments), we rank $x_{ji}^{\delta_j}$ for all $j : \delta_j < 1$, such that $x_{ji}^{\delta_j *}$ is best response to $\Omega_i^{\delta_i}$ for the highest $\Omega_i^{\delta_i}$. the structure is one in which $\delta_j < 1$ players are in the core and $\delta_i \geq 1$ players are linked, as in a star, to a decelerating player. Thus, resulting in a core-periphery structure in which decelerating payers are linked between them, as in item 3, and each is linked to a set of accelerating players, following the arguments in items 2 and 3.

□

If a network is PNE it is also a Nash equilibrium. Thus, it is straightforward that the set of $PNE(\Gamma)$ is a subset of $NE(\Gamma)$. Now we discuss the specific pattern of productive relations and resource allocations that emerges when assuming that players will pursue, bilaterally, relationships that if formed will benefit both parts (PNE).

Although there are various combinations of conditions for which different social outcomes may emerge, we focus on two relevant cases that arise as PNE configurations: (i) segregation between players by their production function, and (ii) core-periphery networks with decelerators as hubs. A third outcome results from a combination of the two above-mentioned.

Case 1. *Segregation between players by their production function: Consider a setting in which accelerators have more resources than decelerators, say accelerators are in E or G while decelerators are in L.*[9] *In such a case of heterogeneity in resources and production functions, segregation by the degree to which players' production functions accelerate generally arises. Accelerators pair up with other accelerators if they are in E or G, or work alone if all of them are in L. In fact, PNE configurations are characterized by perfect assortativity in terms of the shape of the production function: provided that not everyone is in L, the two players with the most accelerating production functions fully exchange with each other, the next two players fully exchange with each other, etc. The decelerators cannot compete against 'richer' accelerators and, thus, only collaborate between them (see Figure 4.a).*

Decelerating players have the intention and availability to collaborate with other decelerating players as well as with accelerating players, but the accelerating players would only be interested in joining efforts with other accelerating players, excluding decelerating players from their collaborations. According to this logic, start-up firms with quickly advancing technologies would prefer to have R&D collaborations with each other if they can choose between mature partners and start-ups, when the accelerators are in G or E and the decelerators are in L. Generally, as long as decelerators do not have more resources than accelerators (i.e. decelerators in G or E and accelerators in L), in our model dyadic rationality considerations (such as embodied in PNE) lead to segregation between players with different types of production functions.

Case 2. *Core-periphery networks with decelerators as hubs: Consider a setting in which accelerators have less resources than decelerators; they are in L or E, and decelerators are in G. In such a case, decelerators with large amounts of resources, although with decelerating production functions, will become more attractive partners for accelerators than other accelerators. Therefore, big decelerators will link to various accelerators, and each accelerator will use all their resources to invest in their productive exchange with the decelerator. Moreover, decelerators would also collaborate between them, forming a core group of decelerators, each linked to one or more accelerators depending on their available resources (see Figure 4.b). Note that absent the big decelerator much of the innovative potential of the start-ups would lie dormant. It takes a big decelerator's involvement to activate the productivity of the decelerators, resulting in larger overall payoffs.*

Going back to our example of how firms can decide on the R&D collaborations, this would mean that if firms with mature technology have high levels of resources, they would be able to attract

[9] This case holds even when players are in the same set (L, E or G) as long as the maximal impact of the accelerators is strictly greater than that of the decelerators.

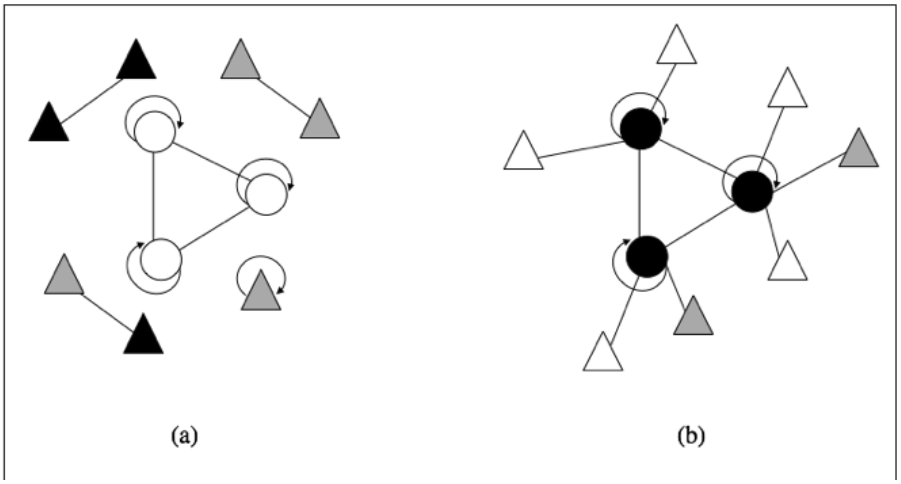

Figure 4. PNE networks. Circles represent decelerators and triangles represent accelerators. The color represents the set they belong to: white for players in L, gray for players in E, and black for players in G. A loop around a node shows that a player invests resources on his individual project, and a link between two nodes shows that both players invest resources on joint project.

and maintain relationships with start-ups. Moreover, the start-up firms would put all their efforts in their exchange with the mature firm. However, given mature firms are better off diversifying, they would invest in R&D collaborations with other mature firms as well as with other start-ups. For instance, a mature firm such as Campbell Soup Co., with a large amounted wealth but little capacity to create growth on its own, would seek to invest from its resources in various startups that can guarantee greater returns to its investment.

5 Discussion and Conclusion

Our study enriches the insights into the structure of emergent exchange networks by pointing to the importance of the interplay between resource heterogeneity and heterogeneity in production functions. Four key findings stand out.

First, actors with accelerating production functions (e.g. start-ups) are better off following an *all-or-nothing* strategy, whereas actors with decelerating production functions are better off diversifying and *not putting all eggs in one basket*. Accelerators require a minimum level of investment from their partner to establish an exchange, otherwise they will prefer to work alone. Therefore, the combination of partners' production functions and available resources must make enough impact to motivate an accelerator to establish an exchange. *Decelerators exchanging with accelerators need large resource endowments to compensate for their slow production function.*

Second, the composition of the population affects the network structure that emerges. *Segregated networks* (i.e. accelerators do not exchange with decelerators) emerge in populations where decelerators are poor (sets L or E). Decelerators will establish collaborations with different partners regardless of their own impact, but will only establish projects with accelerators if their maximal impact is large enough to compensate with resources what they lack in acceleration (i.e., decelerators are in G). *Core-periphery networks* (i.e. decelerators are hubs tied to several accelerators) emerge in settings where decelerators are rich and accelerators have too small resources of their own to impact a relationship with another accelerator (i.e. accelerators are in L or E). Such structures reflect situations in which big firms, like Campbell Soup Co., benefit from the growth capacity of small new firms, because the latter are willing to devote all their capacity into making the best out of the joint project. Similarly, this exemplifies a key structure in scientific collaborations, where experienced researchers collaborate with each other, but also benefit from collaboration with junior scholars because the latter are prepared to fully engage in the joint research project. Such exchanges between senior researches with large 'resource' endowments (i.e. skill, experience) and junior scientists with accelerating production functions but less experience also benefit scientific production as a whole.

Third, our study delineates the conditions under which resource rich actors do *not* acquire a *central position* in the emergent exchange network structure. Consider a well-endowed actor with a decelerating production function. This actor could in principle serve as a hub in the network. However, if there is a less well-endowed actor with an accelerating production function, whose *total impact* is nonetheless larger, the resource-rich decelerator may end up exchanging only with other decelerators (if any) and not with any accelerator. The result that resources can be inversely related to exchange outcomes is a new possibility that previous work on heterogeneity in exchange networks has overlooked (Friedkin 1992; Markovsky et al. 1988, 1993).

Finally, our study contributes to the literature on strategic link formation by extending the analysis to weighted networks, which allows incorporating variations in the intensity of the relationships (e.g. involvement, use of resources, strength of ties).

We conclude with pointing out two opportunities for further research. First, whereas previous work on productive exchange models group interactions with more than three actors (see Lawler et al. 2000), we use game theory to model collaborations as dyadic interactions. Though dyadic collaborations are common, future research may benefit from extending the game theoretical analyses to larger groups.

Second, empirical tests of our model constitute an important next step to advance our insights into the impact of heterogeneity in production functions on emergent network structures. Laboratory experiments offer powerful techniques to do so (for a survey of early works see Kosfeld 2014, see also Falk and Kosfeld (2012); van Dolder D. and Buskens (2014)). Particularly, by studying how experimental subjects interact, we can discover in more depth how certain network structures are more likely to emerge than others (Corten and Buskens 2010), while controlling how the type of production functions and resources are distributed.

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